

Cambridge University Press

978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations

Edward F. McClennen

Frontmatter

[More information](#)

---

RATIONALITY AND DYNAMIC CHOICE

Cambridge University Press

978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations

Edward F. McCledden

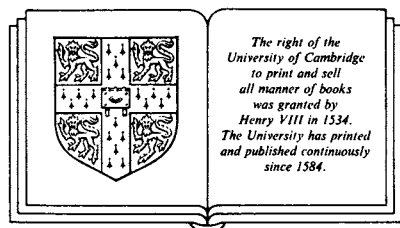
Frontmatter

[More information](#)

# RATIONALITY AND DYNAMIC CHOICE

Foundational explorations

EDWARD F. McCLEDDEN



CAMBRIDGE UNIVERSITY PRESS

*Cambridge*

*New York Port Chester Melbourne Sydney*

Cambridge University Press  
978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations  
Edward F. McClennen  
Frontmatter  
[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521360470](http://www.cambridge.org/9780521360470)

© Cambridge University Press 1990

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 1990  
This digitally printed version 2008

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

McClennen, Edward F. (Edward Francis), 1936–  
Rationality and dynamic choice : foundational explorations / by  
Edward F. McClennen.

p. cm.

ISBN 0-521-36047-1

1. Reasoning. 2. Decision-making. I. Title.

BC177.M224 1990

89-17392

128'.3 – dc20

CIP

ISBN 978-0-521-36047-0 hardback  
ISBN 978-0-521-06391-3 paperback

Cambridge University Press

978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations

Edward F. McClennen

Frontmatter

[More information](#)

---

... 'tis only in two senses, that any affection can be call'd unreasonable. First, When a passion, such as hope or fear, grief or joy, despair or security, is founded on the supposition of the existence of objects, which really do not exist. Secondly, When in exerting any passion in action, we chuse means insufficient for the design'd end, and deceive ourselves in our judgment of causes and effects. Where a passion is neither founded on false supposition, nor chuses means insufficient for the end, the understanding can neither justify it nor condemn it.

David Hume, *Treatise*,  
Bk. II, Pt. III, Sec. III

## Contents

<i>Conditions on orderings and acceptable-set functions</i>	<i>page</i> xi
<i>Acknowledgments</i>	xv
<b>1 Introduction and sketch of the main argument</b>	<b>1</b>
1.1 Two principles of rationality	1
1.2 The focus of the book	2
1.3 The problem of justification	3
1.4 An alternative approach to justification	4
1.5 Hammond's consequentialist argument	5
1.6 Pragmatism and dynamic choice	6
1.7 Do the pragmatic arguments succeed?	11
1.8 Resolute choice	12
1.9 The issue of feasibility	14
1.10 Other implications of resolute choice	15
1.11 The organization of the book	17
<b>2 The ordering principle</b>	<b>20</b>
2.1 Weak preference orderings	20
2.2 Preference and choice	21
2.3 Choice functions and coherence	22
2.4 Weak orderings and choice functions	24
2.5 The minimax risk rule	25
2.6 Reinterpreting the standard results	28
2.7 General preference-based choice functions	33
2.8 Necessary conditions on choice functions	34
2.9 Another generalization	35
2.10 A maximally permissive framework	36
2.11 A more structured framework	40
2.12 Summary	41
<b>3 The independence principle</b>	<b>44</b>
3.1 Alternative formulations of independence	44
3.2 Reduction principles	46
3.3 Independence and noncomplementarity	47
3.4 Independence and sure-thing reasoning	48
3.5 Independence and dominance	50

3.6	Stochastic dominance	53
3.7	A choice-set version of independence	57
3.8	The expected utility result	58
<b>4</b>	<b>The problem of justification</b>	<b>60</b>
4.1	Introduction	60
4.2	The reduction assumption	62
4.3	The weak ordering condition	64
4.4	The transitivity condition	65
4.5	Disjunctive noncomplementarity	67
4.6	The independence principle	73
4.7	Sure-thing reasoning and independence	77
4.8	Summary	80
<b>5</b>	<b>Pragmatic arguments</b>	<b>82</b>
5.1	Introduction	82
5.2	A pragmatic/consequentialist perspective	84
5.3	Refining the pragmatic perspective	86
5.4	Money-pump arguments	89
5.5	Violations of IND and DSO	91
5.6	Dutch books	95
5.7	Summary	96
<b>6</b>	<b>Dynamic choice problems</b>	<b>99</b>
6.1	Dynamic choice and decision trees	99
6.2	Trees and terminal outcomes	100
6.3	Plans	100
6.4	Truncated trees and plans	101
6.5	The evaluation of plans	101
6.6	Evaluation at subsequent nodes	104
6.7	Prospects	107
6.8	An important qualification	111
<b>7</b>	<b>Rationality conditions on dynamic choice</b>	<b>112</b>
7.1	Introduction	112
7.2	Simple reduction	113
7.3	Plan reduction conditions	113
7.4	Dynamic consistency	116
7.5	Separability	120
7.6	Summary of dynamic choice conditions	122
7.7	The relation between SR, NEC, DC, and SEP	123
<b>8</b>	<b>Consequentialist constructions</b>	<b>127</b>
8.1	Introduction	127
8.2	Derivation of the context-free principle	129

Cambridge University Press

978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations

Edward F. McClennen

Frontmatter

[More information](#)

## CONTENTS

ix

8.3	Derivation of the independence principle	131
8.4	Plausibility of the construction	132
8.5	Formalizing restricted feasibility	134
8.6	Theorems for separable feasibility	137
8.7	Results for very separable feasibility	139
8.8	Modifying the separability assumption	141
8.9	Implications for Hammond's construction	142
8.10	An additional problem	143
8.11	A return to a more pragmatic perspective	146
<b>9</b>	<b>Reinterpreting dynamic consistency</b>	<b>148</b>
9.1	Introduction	148
9.2	Strotz on dynamic inconsistency	148
9.3	An interpretive problem	152
9.4	Sophisticated choice	153
9.5	Sophisticated choice and plan reduction	154
9.6	Resolute choice	156
9.7	Resolute choice and separability	158
9.8	Resolute choice and feasibility	159
9.9	Summary and anticipations	160
<b>10</b>	<b>A critique of the pragmatic arguments</b>	<b>162</b>
10.1	Introduction	162
10.2	Context-free conditions and money pumps	163
10.3	Independence violations	167
10.4	Raiffa's argument	168
10.5	Seidenfeld's argument	173
10.6	Some tentative conclusions	182
<b>11</b>	<b>Formalizing a pragmatic perspective</b>	<b>183</b>
11.1	Introduction	183
11.2	The problem with myopic choice	184
11.3	The problem with sophisticated choice	190
11.4	Significance of the pragmatic arguments	195
11.5	Pragmatic impeccability of resolute choice	198
11.6	Conclusions and anticipations	198
<b>12</b>	<b>The feasibility of resolute choice</b>	<b>200</b>
12.1	An argument against resolute choice	200
12.2	Different senses of feasibility	200
12.3	Dynamic choice and R-feasibility	203
12.4	Resolute choice and separability	204
12.5	A closer look at separability	206
12.6	Consequentialism and resolute choice	209

Cambridge University Press

978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations

Edward F. McClennen

Frontmatter

[More information](#)

x

## CONTENTS

12.7	Endogenous preference changes	213
12.8	Another condition of R-feasibility?	215
12.9	Presuppositions concerning the self	216
<b>13</b>	<b>Connections</b>	219
13.1	Other models of dynamic choice	219
13.2	Strotz's article and related literature	219
13.3	Johnsen and Donaldson	223
13.4	Schelling	225
13.5	Thaler and Shefrin	225
13.6	Bratman	226
13.7	Elster	231
<b>14</b>	<b>Conclusions</b>	239
14.1	The standard theory of rationality	239
14.2	A possible fallback position	241
14.3	Consequentialist choice	242
14.4	Symmetry of the arguments presented	243
14.5	The issue of choice versus preference	244
14.6	Final remarks about dominance arguments	246
14.7	The demand for a determinate theory	251
<b>15</b>	<b>Postscript: projections</b>	256
15.1	Resolute choice and game theory	256
15.2	Resolute choice and morality	261
	<i>Notes</i>	265
	<i>Bibliography</i>	300
	<i>Author index</i>	307
	<i>Subject index</i>	309



## Conditions on orderings and acceptable-set functions

**Alpha** An acceptable-set function  $D(\cdot) [= C(\cdot)]$  defined on  $X$  satisfies Alpha just in case for all  $x$  in  $S$ , and all  $S^*$  such that  $S^*$  is a superset of  $S$ , if  $x$  is not in  $D(S)$ , then  $x$  is not in  $D(S^*)$  (p. 23).

**Beta** An acceptable-set function  $D(\cdot) [= C(\cdot)]$  defined on  $X$  satisfies Beta just in case for all  $x$  and  $y$  in  $S$ , and all  $S^*$  such that  $S^*$  is a superset of  $S$ , if both  $x$  and  $y$  are in  $D(S)$ , then either  $x$  and  $y$  are both in  $D(S^*)$  or neither is in  $D(S^*)$  (p. 23).

**CF (context-free choice)** An acceptable-set function  $D(\cdot) [= C(\cdot)]$  satisfies CF just in case  $D(\cdot)$  satisfies both Alpha and Beta (p. 23).

**CFO (context-free ordering)** The ordering  $R$  defined over any set of alternatives  $X$  is not changed by adding new alternatives, that is, by expanding  $X$  to some superset  $Y$  (p. 29).

**CIND (independence for choice)** Let  $g_1, g_2$ , and  $g_3$  be any three gambles, let  $g_{13} = [g_1, p; g_3, 1 - p]$  be a gamble over  $g_1$  and  $g_3$  such that one stands to confront the gamble  $g_1$  with probability  $p$  and the gamble  $g_3$  with probability  $1 - p$ , and let  $g_{23} = [g_2, p; g_3, 1 - p]$  be similarly defined. Then  $g_1$  is in  $D(\{g_1, g_2\})$  iff for  $0 < p \leq 1$ ,  $g_{13}$  is in  $D(\{g_{13}, g_{23}\})$  (pp. 57–8).

**CIND-E (equivalent choice independence)** For any  $g_1, g_2$ , and  $g_3$ , and  $0 < p \leq 1$ , if both  $g_1$  and  $g_2$  are in  $D(\{g_1, g_2\})$  then both  $g_{13}$  and  $g_{23}$  are in  $D(\{g_{13}, g_{23}\})$ , where  $g_{13} = [g_1, p; g_3, 1 - p]$  and  $g_{23} = [g_2, p; g_3, 1 - p]$  (p. 137).

**CIND-S (strict choice independence)** For any  $g_1, g_2$ , and  $g_3$ , and  $0 < p \leq 1$ , if  $g_2$  is not in  $D(\{g_1, g_2\})$  then  $g_{23}$  is not in  $D(\{g_{13}, g_{23}\})$ , where  $g_{23} = [g_2, p; g_3, 1 - p]$  and  $g_{13} = [g_1, p; g_3, 1 - p]$  (p. 139).

**DC (dynamic consistency)** For any choice point  $n_i$  in a decision tree  $T$ , if  $D(S)(n_i)$  is not empty and  $s(n_i)$  is in  $D(S)(n_i)$ , then  $s(n_i)$  is in  $D(S)(n_i)$ ; and if  $s(n_i)$  is in  $D(S)(n_i)$ , then  $s(n_i)$  is in  $D(S)(n_i)$  (p. 120).

**DC-EXC (exclusion)** For any choice point  $n_i$  in a decision tree  $T$ , if  $s(n_i)$  is defined and  $s(n_i)$  is not in  $D(S)(n_i)$ , then  $s$  is not in  $D(S)$  (p. 119).

**DC-INC (inclusion)** For any choice point  $n_i$  in a decision tree  $T$ , if  $D(S)(n_i)$  is nonempty and  $s(n_i)$  is in  $D(S)(n_i)$ , then there is some plan  $s^*$  in  $D(S)$  such that  $s(n_i) = s^*(n_i)$  is the plan continuation of  $s^*$  at  $n_i$ , and hence such that  $s(n_i) = s^*(n_i)$  is in  $D(S)(n_i)$  (pp. 118–19).

**DF (dynamic feasibility)** To assess plan  $p$  at a choice node  $n_i$ , anticipate how you will choose at its (potential) “future” choice nodes  $n_j$  and declare infeasible all future alternatives under  $p$  that are inadmissible at  $n_j$  (p. 174).

**DSO (dominance in terms of sure outcomes)** For  $g = [o_1, E_1; \dots; o_n, E_n]$  and  $g^* = [o_1^*, E_1; \dots; o_n^*, E_n]$ , if  $o_i R o_i^*$  for all  $i$ , then  $g R g^*$ ; and if, in addition,  $o_j P o_j^*$  for some  $j$ , then  $g P g^*$  (p. 50).

**D-SUB (dynamic substitution)** If plans  $s$  and  $r$  differ solely by a substitution of indifferents at some choice point, then  $s$  and  $r$  are indifferent (p. 176).

**FSD (principle of first-order stochastic dominance)** For any two gambles  $g_1$  and  $g_2$  defined over the same set of sure outcomes, if  $g_1$  first-order stochastically dominates  $g_2$ , then  $g_1 P g_2$  (p. 54).

**GDE (general dominance for a fixed partition of events)** For  $g = [g_1, E_1; \dots; g_n, E_n]$  and  $g^* = [g_1^*, E_1; \dots; g_n^*, E_n]$  if  $g_i R g_i^*$  for all  $i$ , then  $g R g^*$ ; and if, in addition,  $g_j P g_j^*$  for some  $j$ , then  $g P g^*$  (p. 49).

**GDP (general dominance for fixed probabilities)** For  $g = [g_1, p_1; \dots; g_n, p_n]$  and  $g^* = [g_1^*, p_1; \dots; g_n^*, p_n]$ , if  $g_i R g_i^*$  for all  $i$ , then  $g R g^*$ ; and if, in addition,  $g_j P g_j^*$  for some  $j$ , then  $g P g^*$  (p. 49).

**ICO (independence for constant outcomes)** For any  $0 < p \leq 1$  and any four gambles  $g_1, g_2, g_3$ , and  $g_4$ ,  $g_{13} = [g_1, p; g_3, 1 - p] R g_{23} = [g_2, p; g_3, 1 - p]$  iff  $g_{14} = [g_1, p; g_4, 1 - p] R g_{24} = [g_2, p; g_4, 1 - p]$  (p. 45).

**IND (independence)** Let  $g_1, g_2$ , and  $g_3$  be any three alternative gambles. Then  $g_1 R g_2$  iff  $g_{13} = [g_1, p; g_3, 1 - p] R g_{23} = [g_2, p; g_3, 1 - p]$ , where  $g_{ij} = [g_i, p; g_j, 1 - p]$  is a complex gamble in which there is  $p$  probability of being exposed to the gamble  $g_i$  and  $1 - p$  probability of being exposed to  $g_j$  and  $0 < p \leq 1$  (p. 44).

**ISO (independence for sure outcomes)** Let  $o_1, o_2$ , and  $o_3$  be any three sure outcomes (monetary prizes, etc.). Then  $o_1 R o_2$  iff, for  $0 < p \leq 1$ ,  $[o_1, p; o_3, 1 - p] R [o_2, p; o_3, 1 - p]$  (p. 44).

**MIC (minimal intelligible choice)** The evaluative method must be such that it generates a nonempty acceptable set for each subset of  $X$ ,

that is, such that there exists an acceptable-set function  $D(\cdot)$  defined over  $X$  (pp. 39–40).

**MD (mixture dominance)** If each of two lotteries  $g_1$  and  $g_2$  is preferred (or dispreferred) to a third gamble  $g_3$ , then so too is any convex combination of  $g_1$  and  $g_2$  (p. 173).

**MO (monotonicity)** If  $o_1$  and  $o_2$  are two sure (nonrisky) prizes such that  $o_1 P o_2$ , then for any two gambles of the form  $g_p = [o_1, p; o_2, 1 - p]$  and  $g_q = [o_1, q; o_2, 1 - q]$ ,  $g_p P g_q$  iff  $p > q$  (p. 53).

**NEC (normal-form/extensive-form coincidence)** Let  $T$  be any decision tree with associated set of plans  $S$ , and let  $T^n$  be the decision problem that results by converting each  $s$  in  $S$  into its normal form, so that each  $s$  in  $S$  is mapped into  $s^n$  in  $S^n$ . Then for any plan  $s$  in  $S$ ,  $s$  is in  $D(S)$  iff  $s^n$  is in  $D(S^n)$  (p. 115).

**PR (plan reduction)** Let  $T$  be any decision tree with associated set of plans  $S$ , and let  $G_S$  be the set of prospects associated with such plans. Then for any plan  $s$  in  $S$  and associated prospect  $g_s$  in  $G_S$ ,  $s$  is in  $D(S)$  iff  $g_s$  is in  $D(G_S)$  (p. 114).

**RD (reduction)** Any compound gamble is indifferent to a simple gamble with  $o_1, \dots, o_r$  as outcomes, their probabilities being computed according to the ordinary probability calculus. In particular, if  $g^{(i)} = [o_1, p_1^{(i)}; o_2, p_2^{(i)}; \dots; o_r, p_r^{(i)}]$  for  $i = 1, \dots, s$ , then  $[g^{(1)}, q_1; g^{(2)}, q_2; \dots; g^{(s)}, q_s] I [o_1, p_1; o_2, p_2; \dots, o_r, p_r]$ , where  $p_i = q_1 p_i^{(1)} + \dots + q_s p_i^{(s)}$  (p. 47).

**RF (restricted feasibility)** A plan  $s$  is feasible iff  $s(n_i)$  is in  $D(S(n_i))$  for every choice point  $n_i$ ,  $i \neq 0$ , for which  $s$  is defined (p. 134).

**RPR (restricted plan reduction)** For any plan  $s$ , such that  $s$  satisfies SF,  $s$  is in  $D(S)$  iff  $g_s$  is in  $D(G_S)$  (p. 135).

**SEP (separability)** For any tree  $T$  and any node  $n_i$  within  $T$ , let  $T(n_i)^d$  be a separate tree that begins at a node that corresponds to  $n_i$  but otherwise coincides with  $T(n_i)$ , and let  $S(n_i)^d$  be the set of plans available in  $T(n_i)^d$  that correspond one to one with the set of truncated plans  $S(n_i)$  available in  $T(n_i)$ . Then  $s(n_i)$  is in  $D(S(n_i))$  iff  $s(n_i)^d$  is in  $D(S(n_i)^d)$  (p. 122).

**SF (separable feasibility)** A plan  $s$  is feasible iff  $s(n_i)^d$  is in  $D(S(n_i)^d)$  for every choice point  $n_i$ ,  $i \neq 0$ , for which  $s$  is defined (p. 134).

**SI (Savage independence)** Let  $E$  and  $-E$  be mutually exclusive and exhaustive events conditioning the various components of four gambles  $g_{13}$ ,  $g_{23}$ ,  $g_{14}$ ,  $g_{24}$ , and let the schedule of consequences be as follows:

	$E$	$-E$
$g_{13}$	$g_1$	$g_3$
$g_{23}$	$g_2$	$g_3$
$g_{14}$	$g_1$	$g_4$
$g_{24}$	$g_2$	$g_4$

Then  $g_{13} R g_{23}$  iff  $g_{14} R g_{24}$  (p. 45).

**SR (simple reduction)** Let  $T$  be any decision tree with associated set of plans  $S$  such that each plan  $s$  in  $S$  requires for its implementation a single choice “up front” by the agent, and let  $G_S$  be the set of prospects associated with such plans. Then for any plan  $s$  in  $S$  and associated prospect  $g_s$  in  $G_S$ ,  $s$  is in  $D(S)$  iff  $g_s$  is in  $D(G_S)$  (p. 113).

**SUB (substitution)** Let  $g_{1x} = [\dots g_1 \dots]$  and  $g_{2x} = [\dots g_2 \dots]$  be two complex gambles that are alike in every respect except that in one or more places where  $g_{1x}$  has  $g_1$  as a component outcome,  $g_{2x}$  substitutes  $g_2$ . Then  $g_1 I g_2$  iff  $g_{1x} I g_{2x}$  (p. 45).

**TR (truncated plan reduction)** Let  $n_i$  be any node in a decision tree  $T$ , and let  $S(n_i)$  be the set of truncated plans that can be associated with  $T(n_i)$ . Then  $s(n_i)$  is in  $D(S(n_i))$  iff  $g_{s(n_i)}$  is in  $D(G_{S(n_i)})$  (p. 121).

**VRPR (very restricted plan reduction)** For any plan  $s$  in any tree  $T$ , such that  $s$  satisfies VSF,  $s$  is in  $D(S)$  iff  $g_s$  is in  $D(G_S)$  (p. 136).

**VSF (very separable feasibility)** If  $s$  and  $r$  are such that both satisfy the “only if” part of SF but there exists some  $n_i, i \neq 0$ , such that both  $s$  and  $r$  are defined at  $n_i$ , then neither  $s$  nor  $r$  itself is a feasible plan; what is feasible are (1) a modified version of  $s$  that is just like  $s$  except that at  $n_i$  it calls for choosing either  $s(n_i)$  or  $r(n_i)$  and (2) a modified version of  $r$  that is just like  $r$  except that at  $n_i$  it calls for choosing either  $s(n_i)$  or  $r(n_i)$  (p. 136).

**WO (weak ordering)** An agent’s preference ordering  $R$  of  $X$  constitutes a weak ordering  $R$  of  $X$  just in case  $R$  is connected, is fully transitive, and satisfies CFO (p. 30).

Cambridge University Press

978-0-521-06391-3 - Rationality and Dynamic Choice: Foundational Explorations

Edward F. McClennen

Frontmatter

[More information](#)

---

## Acknowledgments

I am deeply indebted to the following people for provocative thoughts, useful suggestions, and hardheaded criticisms – all of which have helped, over a period of many years, to shape both the form and the substance of this book: Maurice Allais, Kenneth Arrow, John Bennett, Daniel Ellsberg, Ed Freeman, David Gauthier, Peter Hammond, Bengt Hansson, John Harsanyi, Mark Kaplan, Henry Kyburg, Isaac Levi, R. Duncan Luce, Paul Lyon, Hanan Polansky, Mark Machina, Frederic Schick, and last of all (but only alphabetically) Teddy Seidenfeld. I am also indebted to Mary Nevader for her editing.