RATIONALITY AND DYNAMIC CHOICE

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Foundational explorations

EDWARD F. McCLENNEN

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. . . 'tis only in two senses, that any affection can be call'd unreasonable. First, When a passion, such as hope or fear, grief or joy, despair or security, is founded on the supposition of the existence of objects, which really do not exist. Secondly, When in exerting any passion in action, we chuse means insufficient for the design'd end, and deceive ourselves in our judgment of causes and effects. Where a passion is neither founded on false supposition, nor chuses means insufficient for the end, the understanding can neither justify it nor condemn it.

David Hume, *Treatise,*
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Conditions on orderings and acceptable-set functions

Alpha An acceptable-set function $D(\cdot)$ [$= C(\cdot)$] defined on X satisfies Alpha just in case for all x in S, and all S^* such that S^* is a superset of S, if x is not in $D(S)$, then x is not in $D(S^*)$ (p. 23).

Beta An acceptable-set function $D(\cdot)$ [$=C(\cdot)$] defined on X satisfies Beta just in case for all x and y in S, and all S^* such that S^* is a superset of S, if both x and y are in $D(S)$, then either x and y are both in $D(S^*)$ or neither is in $D(S^*)$ (p. 23).

CF (context-free choice) An acceptable-set function $D(\cdot)$ [$= C(\cdot)$] satisfies CF just in case $D(\cdot)$ satisfies both Alpha and Beta (p. 23).

CFO (context-free ordering) The ordering R defined over any set of alternatives X is not changed by adding new alternatives, that is, by expanding X to some superset Y (p. 29).

CIND (independence for choice) Let g_1, g_2, and g_3 be any three gambles, let $g_{13} = [g_1, p; g_3, 1 - p]$ be a gamble over g_1 and g_3 such that one stands to confront the gamble g_1 with probability p and the gamble g_3 with probability $1 - p$, and let $g_{23} = [g_2, p; g_3, 1 - p]$ be similarly defined. Then g_1 is in $D(\{g_1, g_2\})$ iff for $0 < p \leq 1$, g_{13} is in $D(\{g_{13}, g_{23}\})$ (pp. 57–8).

CIND-E (equivalent choice independence) For any g_1, g_2, and g_3, and $0 < p \leq 1$, if both g_1 and g_2 are in $D(\{g_1, g_2\})$ then both g_{13} and g_{23} are in $D(\{g_{13}, g_{23}\})$, where $g_{13} = [g_1, p; g_3, 1 - p]$ and $g_{23} = [g_2, p; g_3, 1 - p]$ (p. 137).

CIND-S (strict choice independence) For any g_1, g_2, and g_3, and $0 < p \leq 1$, if g_2 is not in $D(\{g_1, g_2\})$ then g_{23} is not in $D(\{g_{13}, g_{23}\})$, where $g_{23} = [g_2, p; g_3, 1 - p]$ and $g_{13} = [g_1, p; g_3, 1 - p]$ (p. 139).

DC (dynamic consistency) For any choice point n_i in a decision tree T, if $D(S)(n_i)$ is not empty and $s(n_i)$ is in $D(S(n_i))$, then $s(n_i)$ is in $D(S)(n_i)$; and if $s(n_i)$ is in $D(S)(n_i)$, then $s(n_i)$ is in $D(S(n_i))$ (p. 120).

DC-EXC (exclusion) For any choice point n_i in a decision tree T, if $s(n_i)$ is defined and $s(n_i)$ is not in $D(S(n_i))$, then s is not in $D(S)$ (p. 119).

DC-INC (inclusion) For any choice point n_i in a decision tree T, if $D(S)(n_i)$ is nonempty and $s(n_i)$ is in $D(S(n_i))$, then there is some plan s^* in $D(S)$ such that $s(n_i) = s^*(n_i)$ is the plan continuation of s^* at n_i, and hence such that $s(n_i) = s^*(n_i)$ is in $D(S)(n_i)$ (pp. 118–19).

DF (dynamic feasibility) To assess plan p at a choice node n_i, anticipate how you will choose at its (potential) "future" choice nodes n_j and declare infeasible all future alternatives under p that are inadmissible at n_j (p. 174).

DSO (dominance in terms of sure outcomes) For $g = [o_1, E_1; \ldots; o_n, E_n]$ and $g^* = [o_1^*, E_1; \ldots; o_n^*, E_n]$, if $o_i \, R \, o_i^*$ for all i, then $g \, R \, g^*$; and if, in addition, $o_j \, P \, o_j^*$ for some j, then $g \, P \, g^*$ (p. 50).

D-SUB (dynamic substitution) If plans s and r differ solely by a substitution of indifferents at some choice point, then s and r are indifferent (p. 176).

FSD (principle of first-order stochastic dominance) For any two gambles g_1 and g_2 defined over the same set of sure outcomes, if g_1 first-order stochastically dominates g_2, then $g_1 \, P \, g_2$ (p. 54).

GDE (general dominance for a fixed partition of events) For $g = [g_1, E_1; \ldots; g_n, E_n]$ and $g^* = [g_1^*, E_1; \ldots; g_n^*, E_n]$ if $g_i \, R \, g_i^*$ for all i, then $g \, R \, g^*$; and if, in addition, $g_j \, P \, g_j^*$ for some j, then $g \, P \, g^*$ (p. 49).

GDP (general dominance for fixed probabilities) For $g = [g_1, p_1; \ldots; g_n, p_n]$ and $g^* = [g_1^*, p_1; \ldots; g_n^*, p_n]$, if $g_i \, R \, g_i^*$ for all i, then $g \, R \, g^*$; and if, in addition, $g_j \, P \, g_j^*$ for some j, then $g \, P \, g^*$ (p. 49).

ICO (independence for constant outcomes) For any $0 < p \leq 1$ and any four gambles g_1, g_2, g_3, and g_4, $g_{13} = [g_1, p; g_3, 1 - p] \, R \, g_{23} = [g_2, p; g_3, 1 - p]$ iff $g_{14} = [g_1, p; g_4, 1 - p] \, R \, g_{24} = [g_2, p; g_4, 1 - p]$ (p. 45).

IND (independence) Let g_1, g_2, and g_3 be any three alternative gambles. Then $g_1 \, R \, g_2$ iff $g_{13} = [g_1, p; g_3, 1 - p] \, R \, g_{23} = [g_2, p; g_3, 1 - p]$, where $g_{ij} = [g_i, p; g_j, 1 - p]$ is a complex gamble in which there is p probability of being exposed to the gamble g_i and $1 - p$ probability of being exposed to g_j and $0 < p \leq 1$ (p. 44).

ISO (independence for sure outcomes) Let o_1, o_2, and o_3 be any three sure outcomes (monetary prizes, etc.). Then $o_1 \, R \, o_2$ iff, for $0 < p \leq 1$, $[o_1, p; o_3, 1 - p] \, R \, [o_2, p; o_3, 1 - p]$ (p. 44).

MIC (minimal intelligible choice) The evaluative method must be such that it generates a nonempty acceptable set for each subset of X, that is, such that there exists an acceptable-set function $D(\cdot)$ defined over X (pp. 39–40).

MD (mixture dominance) If each of two lotteries g_1 and g_2 is preferred (or dispreferred) to a third gamble g_3, then so too is any convex combination of g_1 and g_2 (p. 173).

MO (monotonicity) If o_1 and o_2 are two sure (nonrisky) prizes such that $o_1 P o_2$, then for any two gambles of the form $g_p = [o_1, p; o_2, 1 - p]$ and $g_q = [o_1, q; o_2, 1 - q]$, $g_p P g_q$ iff $p > q$ (p. 53).

NEC (normal-form/extensive-form coincidence) Let T be any decision tree with associated set of plans S, and let T^n be the decision problem that results by converting each s in S into its normal form, so that each s in S is mapped into s^n in S^n. Then for any plan s in S, s is in $D(S)$ iff s^n is in $D(S^n)$ (p. 115).

PR (plan reduction) Let T be any decision tree with associated set of plans S, and let G_S be the set of prospects associated with such plans. Then for any plan s in S and associated prospect g_s in G_S, s is in $D(S)$ iff g_s is in $D(G_S)$ (p. 114).

RD (reduction) Any compound gamble is indifferent to a simple gamble with $o_1, \ldots, o_r$ as outcomes, their probabilities being computed according to the ordinary probability calculus. In particular, if $g^{(i)} = [o_1, p_1^{(i)}; o_2, p_2^{(i)}; \ldots; o_r, p_r^{(i)}]$ for $i = 1, \ldots, s$, then $[g^{(1)}, q_1; g^{(2)}, q_2; \ldots; g^{(s)}, q_s] \, I \, [o_1, p_1; o_2, p_2; \ldots, o_r, p_r]$, where $p_i = q_1 p_i^{(1)} + \ldots + q_s p_i^{(s)}$ (p. 47).

RF (restricted feasibility) A plan s is feasible iff $s(n_i)$ is in $D(S(n_i))$ for every choice point n_i, $i \neq 0$, for which s is defined (p. 134).

RPR (restricted plan reduction) For any plan s, such that s satisfies SF, s is in $D(S)$ iff g_s is in $D(G_S)$ (p. 135).

SEP (separability) For any tree T and any node n_i within T, let $T(n_i)^d$ be a separate tree that begins at a node that corresponds to n_i but otherwise coincides with $T(n_i)$, and let $S(n_i)^d$ be the set of plans available in $T(n_i)^d$ that correspond one to one with the set of truncated plans $S(n_i)$ available in $T(n_i)$. Then $s(n_i)$ is in $D(S(n_i))$ iff $s(n_i)^d$ is in $D(S(n_i)^d)$ (p. 122).

SF (separable feasibility) A plan s is feasible iff $s(n_i)^d$ is in $D(S(n_i)^d)$ for every *choice* point n_i, $i \neq 0$, for which s is defined (p. 134).

SI (Savage independence) Let E and $-E$ be mutually exclusive and exhaustive events conditioning the various components of four gambles g_{13}, g_{23}, g_{14}, g_{24}, and let the schedule of consequences be as follows:

	E	$-E$
g_{13}	g_1	g_3
g_{23}	g_2	g_3
g_{14}	g_1	g_4
g_{24}	g_2	g_4

Then $g_{13}\ R\ g_{23}$ iff $g_{14}\ R\ g_{24}$ (p. 45).

SR (simple reduction) Let T be any decision tree with associated set of plans S such that each plan s in S requires for its implementation a single choice "up front" by the agent, and let G_S be the set of prospects associated with such plans. Then for any plan s in S and associated prospect g_s in G_S, s is in $D(S)$ iff g_s is in $D(G_S)$ (p. 113).

SUB (substitution) Let $g_{1x} = [\ldots g_1 \ldots]$ and $g_{2x} = [\ldots g_2 \ldots]$ be two complex gambles that are alike in every respect except that in one or more places where g_{1x} has g_1 as a component outcome, g_{2x} substitutes g_2. Then $g_1\ I\ g_2$ iff $g_{1x}\ I\ g_{2x}$ (p. 45).

TR (truncated plan reduction) Let n_i be any node in a decision tree T, and let $S(n_i)$ be the set of truncated plans that can be associated with $T(n_i)$. Then $s(n_i)$ is in $D(S(n_i))$ iff $g_{s(n_i)}$ is in $D(G_{S(n_i)})$ (p. 121).

VRPR (very restricted plan reduction) For any plan s in any tree T, such that s satisfies VSF, s is in $D(S)$ iff g_s is in $D(G_S)$ (p. 136).

VSF (very separable feasibility) If s and r are such that both satisfy the "only if" part of SF but there exists some n_i, $i \neq 0$, such that both s and r are defined at n_i, then neither s nor r itself is a feasible plan; what is feasible are (1) a modified version of s that is just like s except that at n_i it calls for choosing either $s(n_i)$ or $r(n_i)$ and (2) a modified version of r that is just like r except that at n_i it calls for choosing either $s(n_i)$ or $r(n_i)$ (p. 136).

WO (weak ordering) An agent's preference ordering R of X constitutes a weak ordering R of X just in case R is connected, is fully transitive, and satisfies CFO (p. 30).

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