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Cellular structures in topology

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To our wives Nair Piccinini and Gerda Fritsch
for their continuous and steadfast support

Contents

Preface

Felix, qui potuit rerum
cognoscere causas!

P. Vergilius Maro, Georgica 2,490

Cellular structures play an essential role in topology, analysis and geometry; they appear in the form of CW-complexes, simplicial sets and so on. The idea of this book is to give a unified treatment of their fundamental geometric and topological (in the sense of general topology) properties. As a common basis for their representation we have chosen the CW-complexes.

CW-complexes were formally introduced in the literature in 1949 by the great English mathematician John H.C. Whitehead. To appreciate better the depth and perception of Whitehead's ideas, it is worth looking back into the development of algebraic topology; on this trip through history we take Solomon Lefschetz as our Virgil. In his beautiful history of the early development of algebraic topology (see Lefschetz, 1970), Lefschetz shows us how homology was defined by Henri Poincaré – whom he calls the 'Founder' of algebraic topology – using spaces with a combinatorial structure; Lefschetz then points out the next stage in the development of the subject, namely the definition of homology for topological spaces and the introduction of the homotopy groups of spaces. What Whitehead did was to impose again a combinatorial structure on the spaces and to show how this leads to a much deeper insight into their homotopy groups. This and other particularly interesting properties of CW-complexes explain why their presence is felt throughout many branches of mathematics. The first two chapters of this book are devoted to the theory of CW-complexes.

Chapters 3 and 4 deal with the theory of simplicial complexes and simplicial sets; we feel that the existence of a very large body of research in that area and the importance of combinatorial structures in topology amply justify the relatively large size of these two chapters.

In the fifth chapter we study the category of spaces having the homotopy type of CW-complexes. We end the book with an appendix containing the results of homotopy theory, topology and dimension theory necessary to the development of the book. Normally we do not prove the results presented in the appendix but we indicate where the proofs can be found. The appendix should be read using the index, as sometimes the definitions are not written in order but, rather, following the flow of each section.

Because we emphasize geometric and combinatorial structures (and the arguments related to them), the material we borrowed from algebraic topology is mostly related to the theory of homotopy groups with only a minimal contribution from homology; in our minds we view homotopy groups as more intimately related to our geometric intuition than homology groups. As a consequence, the results about cellular structures that are heavily dependent on homology theory (e.g., cellular homology, obstruction theory, Wall obstruction to finiteness, classifying spaces, etc.) are not discussed in the book. However, we lay down the ground work needed for the development of these areas.

Although most of the exercises can be worked out easily using the material in the text, there are some exercises which require the reader to consult the references given in each case; the problems of this latter type have been inserted in the book in order to draw the reader's attention to interesting results which, however, could not be incorporated in the text without enlarging it to unmanageable dimensions. We apologise to their authors for presenting their work as exercises, possibly giving the impression that we do not consider it as important enough to be in the text; indeed, the contrary is true: in spite of the obvious lack of space, we did not just pass by and overlook these results!

With regard to the historical notes we wish to say that we have not done specific research to trace back carefully all the definitions and results presented in the book. We just give hints to our sources and apologise to all concerned if we have unintentionally given incorrect credits.

The reader is assumed to be familiar with the standard facts of general topology and category theory; as basic sources of information on these areas one can take, respectively, the classical books by John L. Kelley (1956) and Saunders MacLane (1971).

A few remarks about the notation used in the book: with the exception of Section A.1, the symbol *Top* denotes the category of weak Hausdorff k-spaces and continuous functions, explained just in that section. The word *map* always indicates continuity; a non-necessary continuous assignment between points of spaces of simply called a *function*. Finally, the symbol $\times$ between k-spaces always denotes the product in the category of k-spaces.

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