PART I

General equilibrium

Arrow, along with Debreu, initiated modern general equilibrium theory in their landmark 1954 paper. The importance of rigorous analysis to economics is nowhere so highlighted as in this subject. On the one hand, economists had long been used to haphazard reasoning ranging from a naive belief that any economic system of importance had to have a solution to the slightly more methodical approach of the counting of equations and unknowns. On the other hand, a major benefit of Arrow and Debreu's work was the realization that the conditions for existence were strikingly restrictive. In particular, the importance of convexity to the existence of equilibrium was first recognized by them. For example, convexity is rarely true of the individual firm's technology set, and so the importance of large numbers of agents as a precondition for the workability of competition is now better understood.

A third major use of the Arrow–Debreu model is in applications. Once the way was paved in the abstract setting, countless papers utilized the basic framework and methods to extend the analysis to a variety of more concrete settings. Chichilnisky's contribution is a good example. She examines a general equilibrium model of international trade intended to represent an advanced industrial nation trading with a less developed country rich in resources. Under more general assumptions than heretofore considered, she shows that it is possible that trade will lower the real wages and the welfare of workers in the labor-abundant country.

Bergstrom studies a general equilibrium model of indivisible occupational choice. He examines conditions under which simple lotteries with purely monetary prizes can improve welfare. In addition, he shows that a simple lottery equilibrium exists where an Arrow–Debreu equilibrium might not.

Stokey considers a model of industry structure in which there is "industrywide learning by doing," namely, average cost declines as a function of industry output. She shows that price always falls over time, as it should, and that a unique symmetric Nash equilibrium exists. She also shows that increasing the number of firms can lower welfare, because free-rider incentives grow larger with the number of firms and can overwhelm the loss of monopoly power.

Starr examines the problem of how transactions are accomplished in general equilibrium in the absence of a central clearing mechanism that accepts all goods in trade. Even if prices are set at equilibrium levels, without the double coincidence of wants, traders must exchange money or IOUs. If there is a lack of trust and of money, Starr shows that a simple bank credit arrangement will suffice. Moreover, other arrangements, such as commodity money or multiple credit instruments are shown to create coordination problems.

Shell, in collaboration with Balasko, extends their earlier joint work on the overlapping-generations model. They show that, in the infinite-horizon case, a perfect foresight competitive equilibrium with a positive price of money exists if the public debt is forever zero after some date. Counterexamples are given to show that asymptotically zero public indebtedness will not suffice for existence of such equilibria, nor will the requirement that public debt be zero infinitely often.

Heller considers a simple general equilibrium oligopoly model with a full set of markets. Consumers earn income in industries that produce output they do not desire. Conditions are given under which there are multiple Pareto-ranked equilibria. The situation is akin to pecuniary externalities. It is also similar to effective demand failures, but with perfectly flexible prices and perfect foresight. Policy remedies are examined.

CHAPTER 1

A general equilibrium theory of North–South trade

Graciela Chichilnisky

This chapter presents an application of competitive general equilibrium theory of markets in the spirit of Walras, formalized in the 1950s by K. Arrow and by G. Debreu. In using general equilibrium theory to generate insights into current policy issues, it follows a tradition established by Arrow in his work on welfare economics of medical care (1963), on the organization of economic activity (1969), on the evaluation of public investment (Arrow and Lind, 1970), and in urban economic development (1970).

The intention is to use formalized general equilibrium theory to derive general statements about the economic behavior and interrelations between two groups of countries: industrial and developing countries. The first group is represented by a cluster of competitive market economies called the *North,* and the second by a similar cluster of competitive economies called the *South:* thus the name North–South trade. The goal is to obtain simple and general results, and for this purpose we consider a stylized model with the minimum of characteristics needed for the task: two regions, two produced goods, and two factors of production. Within this simple model, we explore issues of current import, such as export-led policies and the transmission of economic activity between regions. The underlying theme is that general equilibrium analysis is indeed useful for disclosing patterns of economic behavior and for suggesting policies, a point of view that guided classical international economics.

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1 Policy issues and main results

The classical trade models developed by Heckscher, Ohlin, Lerner, and Samuelson were concerned with gains from trade between countries having similar preferences and technologies, but with different endowments of factors of production. These models explained why trade takes place between similar countries. In the earliest part of this century, trade among similar countries, indeed industrial countries, was the most important segment of international trade. However, North–South trade, which now accounts for 40 percent by value of OECD trade, takes place between countries of very different characteristics. They differ not only in factor endowments but also in preferences and technologies. To understand this important and growing component of world trade, we need a framework that can incorporate explicitly the diversity of technologies and demand patterns as well as the more traditional differences in factor endowments and that can relate this diversity to the welfare effects of trade.

This chapter develops a rigorous general equilibrium analysis of trade between two competitive market economies with significant differences both in technologies and in endowments. Within this framework, we use general equilibrium comparative statistics analysis to study the welfare effects of changes in the volume of trade across free trade equilibria. We study changes in the market equilibrium in response to changes in parameters that are exogenous to the model. We also examine the comparative statics effects that an expansion in the North has on the South. These are two current topics: Export policies of the developing countries are at the forefront of discussions on the international debt, and the issue of whether or not an economic expansion in the industrial countries is transmitted to the developing countries underlies many policy prescriptions. Many oil-exporting countries and exporters of other raw materials show disappointing records after a decade of concentration on production for exports.

The aim of this essay is to explain why, in the words of Arthur Lewis, the international market works at times to concentrate rather than to diffuse the gains from trade (Lewis 1983). I hope to explore in some detail how the international market transmits economic activity from one region to the other. The explanations that I seek are in terms of the primitive structural characteristics of the domestic economies of the trading regions, such as technologies factor supplies and demand structures, and not in terms of the derived parameters of international markets, such as elasticities of international demand at a market equilibrium.

For many developing countries, the degree of involvement in the international economy is a major policy decision, with export-oriented domestic production having been strongly recommended by international agencies for many years. However, as Samuelson has noted, even under the assumptions of classical trade theory we cannot, in general, claim that a country will benefit from orienting its production toward international trade.[1] The policy issue facing most countries is not one of choosing between free trade and autarky. It is one of choosing between policies that would result in more or less emphasis on an international sector of the economy. The classical theorems on the welfare effects of trade provide little information about such choices. This is the issue we study here.

Initial results on these problems were obtained in Chichilnisky (1981), who dealt with trade between an industrial country and a labor-abundant developing country and showed how the structural differences between the countries (and also between sectors within a country) play an important role in determining the welfare impact of changes in the volume of trade and in the transmission of economic activity through international markets.

A feature of the 1981 results that seemed counterintuitive and attracted attention is that an increase in the exports of a labor-intensive product could lower real wages and terms of trade in the labor-abundant South. Labor abundance was described by the responsiveness of labor supply to real wages, and it was proved that even in cases where the labor supply is abundant, an expansion of labor-intensive exports could have these negative effects. Of course, these same effects occur when labor is *not* abundant, and in this chapter we give necessary and sufficient conditions for such results. We consider here cases where the South's demand for basics derives both from capital and from wage income.

Another feature of the results that attracted attention was that an expansion in the North parameterized by an increase in its demand for industrial goods could lead to an expansion in the South's exports and simultaneously could lower terms of trade and real wages in the South. This result led to several comments that centered on the question: How is it possible that in a stable market economy such as that of the North–South model, one can have simultaneously an increase in the volume of exports from the South demanded by the North and a drop in their market prices? This chapter deals with this question. It shows how under conditions similar to those discussed above, a move toward an equilibrium with a higher industrial demand in the North leads indeed to a higher volume of exports of labor-intensive goods from the South but to lower terms of trade and lower real wages in the South, within a stable market economy. The results are traced to simultaneous supply and demand responses, which are not readily perceived within a partial equilibrium framework but appear quite naturally in general equilibrium models. Further results are obtained here showing that in such cases the North may actually consume more of both goods and the South less, following the industrial expansion in the North and the increase in exports from the South.

Of course, precisely the opposite effects can also happen. This essay also examines sufficient conditions for *positive* outcomes of an export expansion: As exports increase, terms of trade and real wages improve. It also examines conditions for a positive transmission of an expansion in the North to the South: As the North increases its industrial demand, its imports increase, leading to better terms of trade and real wages in the South. The purpose of our general equilibrium analysis is to provide a rigorous framework to analyze which case is likely to occur and under which specifications of the economies. We also discuss some empirical aspects on the basis of recent econometric implementations of the model for the case of trade between Sri Lanka and the United Kingdom in Chichilnisky, Heal, and Podivinsky (1983) and for the case of Argentina and the United States in Chichilnisky and McLeod (1984).

2 The North–South model and its solutions

This section summarizes the general equilibrium model in Chichilnisky (1981, 1984a, b). A version of this model is a special case of an Arrow–Debreu general equilibrium model: This is shown in Section f of the appendix.

There are two regions, North and South. The North represents the industrial countries, the South the developing countries. Each region produces and consumes two goods: basics (B) and industrial goods (I). There are two inputs to production: capital (K) and labor (L). The two regions trade with each other.

Consider first the economy of the South. It produces basics and industrial goods using labor and capital, as described by the Leontief production functions

$$B^S = \min(L^B/a_1, K^B/c_1),$$

$$I^S = \min(L^I/a_2, K^I/c_2),$$

where the superscripts B and I denote the sector in which inputs are used, and the superscript S denotes supply. Basics are labor-intensive and in-

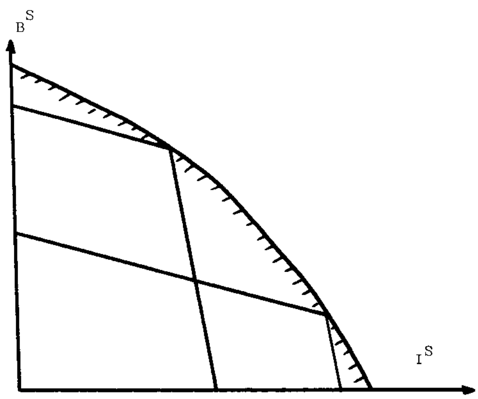

Figure 1. The overall production possibility frontier across equilibria when $D \neq 0$. For each set of prices, the production possibility frontier is piecewise linear; as prices change, endowments of factors vary and a new piecewise linear production set arises. The overall frontier is smooth.

dustrial goods capital-intensive, that is, $D = a_1c_2 - a_2c_1 > 0$. These production functions were chosen for the sake of analytic tractability. More general production functions can be utilized with no significant changes in the results (see, e.g., Benhabib and Chichilnisky 1984). In any case, across equilibria, this economy exhibits substitution between the total amount of labor and capital employed; this is discussed below.

We can now write the equations that specify equilibrium of the model. Competitive behavior on the part of the firms ensures zero profits, so that

$$p_B = a_1w + c_1r, \tag{2.1}$$

$$p_I = a_2w + c_2r, \tag{2.2}$$

where p_B and p_I are the prices of B and I; w and r are the wages and the rate of return on capital.[2]

Labor and capital supplied are increasing functions of their rewards:

$$L^S = \alpha(w/p_B) + \bar{L} \quad (\alpha > 0), \tag{2.3}$$

$$K^S = \beta r + \bar{K} \quad (\beta > 0). \tag{2.4}$$

Since factor supplies vary with factor prices by equations (2.3) and (2.4), the model exhibits substitution in the total use of capital and labor across equilibria when $D = a_1c_2 - a_2c_1 \neq 0$. Figure 1 illustrates the production possibility frontier: Across equilibria, commodity prices change and factor prices change so that factor endowments change too, by (2.3) and (2.4).

The diagram exhibits a piecewise linear production possibility frontier for each price vector of prices and indicates the overall frontier as the envelope of the piecewise linear frontier across different prices. The market clearing conditions (superscript S denotes supply and D denotes demand) are

$$L^S = L^D, \tag{2.5}$$

$$K^S = K^D, \tag{2.6}$$

$$L^D = L^B + L^I = B^S a_1 + I^S a_2, \tag{2.7}$$

$$K^D = K^B + K^I = B^S c_1 + I^S c_2, \tag{2.8}$$

$$B^S = B^D + X_B^S, \tag{2.9}$$

where X_B^S denotes exports of B,

$$I^D = X_I^D + I^S, \tag{2.10}$$

where X_I^D denotes imports of I, and

$$p_B X_B^S = p_I X_I^D, \tag{2.11}$$

that is, the value of exports equals the value of imports.[3]

The North is specified by a set of equations similar to (2.1)–(2.11), with possibly different technology and factor supply parameters. In a world equilibrium, the prices of traded goods are equal across regions (factors K and L are not traded) and exports match imports:

$$p_I(\mathrm{S}) = p_I(\mathrm{N}), \tag{2.12}$$

$$p_B(\mathrm{S}) = p_B(\mathrm{N}), \tag{2.13}$$

$$X_B^S(\mathrm{S}) = X_B^D(\mathrm{N}), \tag{2.14}$$

$$X_I^S(\mathrm{N}) = X_I^D(\mathrm{S}), \tag{2.15}$$

where (S) and (N) denote South and North, respectively.

In each region, there are eight exogenous parameters: a_1, a_2, c_1, c_2, α, $\bar{L}$, β, and $\bar{K}$, making a total of *sixteen exogenous parameters* for the North–South model. When we add the price-normalizing condition,[4]

$$p_I = 1, \tag{2.16}$$

we have a *total of twenty-six independent equations:* (2.1)–(2.11) for North; (2.1)–(2.11) for South, (2.12)–(2.14), and (2.16).[5] There are in total twenty-eight *endogenous variables,* fourteen for each region: p_B, p_I, w, r, L^S, L^D, K^S, K^D, B^S, B^D, X_B^S, I^S, I^D, and X_I^D. Therefore, the system is undetermined so far up to two variables.[6] Thus, we now specify two more variables exogenously, industrial demand in the South, $I^D(\mathrm{S})$, and in the North, $I^D(\mathrm{N})$, adding two more equations:

$$I^D(\mathrm{N}) = \bar{I}^D(\mathrm{N}), \tag{2.17}$$

$$I^D(\mathrm{S}) = \bar{I}^D(\mathrm{S}). \tag{2.18}$$

Obviously, we could have solved the model by specifying exogenously other variables or else by postulating demand equations; this will be done in the following sections. The demand specifications of the model are chosen to meet two criteria: analytical tractability and empirical plausibility.

The North–South model is, therefore, a system of twenty-eight equations in twenty-eight variables, depending on eighteen exogenous parameters: a_1, a_2, c_1, c_2, α, $\bar{L}$, β, $\bar{K}$, and $\bar{I}$ for each region.

The economies of the North and of the South are identical except possibly for the values of their exogenous parameters. Differences in the structural characteristics of the two regions are described by differences in their exogenous parameters. For instance, in the North the two sectors (B and I) use approximately the same technology, that is, the economy is technologically homogeneous. This means that $a_1/c_1 \sim a_2/c_2$ so that the determinant $D(\mathrm{N})$ of the matrix of technical coefficients

$$\begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix}$$

is close to zero in the North. In the South, instead, technologies are dualistic: The two sectors use factors very differently, and $D(\mathrm{S})$ is therefore large. In both regions, $D(\mathrm{N})$ and $D(\mathrm{S})$ are positive, which indicates that the B sector uses labor more intensively than the I sector. Another difference arises in factor markets. In the North, labor is relatively more scarce, that is, less responsive to increases in the real wage w/p_B. This means $\alpha(\mathrm{N})$ is small. In the South the opposite is true, $\alpha(\mathrm{S})$ is large. The reciprocal relations hold in capital markets: $\beta(\mathrm{N})$ is large and $\beta(\mathrm{S})$ is small. These parameter specifications can be presented so as to be independent of the units of measurements.

It is worth noting that whereas most equations are linear in the variables, some are not [e.g., (2.3) is nonlinear]. The solutions also display nonlinearities, as we shall see in the following.

Proposition 1. The North–South model has at most one equilibrium.[7] This equilibrium can be computed explicitly by solving one equation that depends on all exogenous parameters of the model.

Proof: From

$$X_I^D(\mathrm{S}) = X_I^S(\mathrm{N}),$$

we have

$$\bar{I}^D(\mathrm{S}) - I^S(\mathrm{S}) = I^S(\mathrm{N}) - \bar{I}^D(\mathrm{N}). \tag{2.19}$$

Inverting (2.7) and (2.8), we obtain

$$B^S = \frac{c_2 L - a_2 K}{D}, \qquad I^S = \frac{a_1 K - c_1 L}{D}, \tag{2.20}$$

and inverting (2.1) and (2.2),

$$w = \frac{p_B c_2 - c_1}{D}, \qquad r = \frac{a_1 - p_B a_2}{D}. \tag{2.21}$$

Using (2.3), (2.4), (2.20), and (2.21), we can rewrite equation (2.19) as a function of one variable only, p_B (which is the *terms of trade* of the South, since $p_I = 1$) and obtain

$$p_B^2(A + A(\mathrm{N})) + p_B[C + C(\mathrm{N}) + \bar{I}^D(\mathrm{S}) + \bar{I}^D(\mathrm{N})] - (V + V(\mathrm{N})) = 0, \tag{2.22}$$

where

$$A = \frac{\beta a_1 a_2}{D^2}, \qquad V = \frac{\alpha c_1^2}{D^2},$$

and

$$C = \frac{1}{D}\left(c_1 \bar{L} - a_1 \bar{K} + \frac{\alpha c_1 c_2 - \beta a_1 a_2}{D}\right),$$

and where expressions A, V, and C contain parameter values for the South, and $A(\mathrm{N})$, $V(\mathrm{N})$, and $C(\mathrm{N})$ for the North. Solving equation (2.22) yields an equilibrium level of terms of trade p_B^* as a function of all eighteen exogenous parameters of the system.

It is easy to check that equation (2.22) has at most one positive root p_B^* because the constant term is negative and the quadratic term is positive. From this and (2.21), one obtains the equilibrium values of w^* and r^* for each region; from (2.3) and (2.4), L^* and K^* for each region; from (2.20), $(B^S)^*$ and $(I^S)^*$ for each region. From (2.9), (2.10), and (2.11) we then obtain $(B^D)^*$, $(X_B^S)^*$, and $(X_I^D)^*$ for each region. All endogenous variables have been computed, and the solution is complete. ■

In the following, we shall consider different specifications of demand. Equation (2.18) in the South is substituted by one of three different specifications of equilibrium levels of demand: