

Contents

PREFACE	xiii
NOTE TO THE READER	xix
CHAPTER 1 MODULES	1
Introduction	1
1-1 Definitions	1
1-2 Direct Products and Sums	5
1-3 Adjunction of 1 to R	8
1-4 Sequences of Modules	10
1-5 Exercises	12
CHAPTER 2 FREE MODULES	19
Introduction	19
2-1 Definition of Free Modules	20
2-2 Bases of Free Modules	24
2-3 Exercises	29

vi **Contents****CHAPTER 3 INJECTIVE MODULES** 30

Introduction 30

3-1 Properties of Injectives 30

3-2 Divisibility 33

3-3 Embeddings in Injectives 36

3-4 Injective Hulls 39

3-5 Noetherian Rings 45

3-6 Examples 47

3-7 Exercises 51

CHAPTER 4 TENSOR PRODUCTS 53

Introduction 53

4-1 Tensor Products of Modules 53

4-2 Definitions for Algebras 60

4-3 Tensor Products of Algebras 63

4-4 Exercises 69

CHAPTER 5 CERTAIN IMPORTANT ALGEBRAS 71

Introduction 71

5-1 Free and Tensor Algebras 72

5-2 Exterior Algebras 74

5-3 Exercises 83

**CHAPTER 6 SIMPLE MODULES
AND PRIMITIVE RINGS** 86

Introduction 86

6-1 Preliminaries 87

6-2 Cyclic Modules 90

6-3 Simple Modules 91

6-4 Examples 94

6-5 Density 95

Contents	vii
6-6 More on Density and Simples	99
6-7 Examples	105
6-8 Exercises	109
CHAPTER 7 THE JACOBSON RADICAL	111
Introduction	111
7-1 Characterizations	112
7-2 Radicals of Related Rings	125
7-3 Local Rings	132
7-4 Examples	135
7-5 Exercises	138
CHAPTER 8 SUBDIRECT PRODUCT DECOMPOSITIONS	140
Introduction	140
8-1 Subdirect Products	141
8-2 Dense Subdirect Products	145
8-3 Exercises	147
CHAPTER 9 PRIMES AND SEMIPRIMES	148
Introduction	148
9-1 Prime Ideals	149
9-2 Semiprime Ideals and the Prime Radical	151
9-3 Nil Radicals	156
9-4 Primes and Semiprimes in Derived Rings	158
9-5 Exercises	162
CHAPTER 10 PROJECTIVE MODULES AND MORE ON WEDDERBURN THEOREMS	163
Introduction	163
10-1 Projective Modules	165

viii	Contents	
10-2	Projective Dimension	172
10-3	Minimal Right Ideals	177
10-4	Main Theorems	180
10-5	Direct Proofs	184
10-6	Uniqueness	190
10-7	Rings with D.C.C. and Idempotents	191
10-8	Exercises	196
 CHAPTER 11 DIRECT SUM DECOMPOSITIONS		204
	Introduction	204
11-1	Completely Reducible Modules	206
11-2	Radical of a Module	209
11-3	Artinian and Noetherian Modules	213
11-4	Direct Sums of Indecomposables	221
11-5	Singular Submodule	231
11-6	Exercises	234
 CHAPTER 12 SIMPLE ALGEBRAS		239
	Introduction	239
12-1	Algebra Modules	240
12-2	Multiplication Algebra	241
12-3	Tensor Products of Simple Rings	246
12-4	Centralizers	250
12-5	Double Centralizers	259
12-6	Exercises	266
 CHAPTER 13 HEREDITARY RINGS, FREE AND PROJECTIVE MODULES		269
	Introduction	269
13-1	Hereditary Rings	269
13-2	Injectivity and Projectivity	272
13-3	Finitely Generated Modules	275

Contents	ix
13-4 Examples	279
13-5 Exercises	281
CHAPTER 14 MODULE CONSTRUCTIONS	283
Introduction	283
14-1 Pullbacks	284
14-2 Pushouts	289
14-3 Pushout Application	293
14-4 Exercises	295
CHAPTER 15 CATEGORIES AND FUNCTORS	298
Introduction	298
15-1 Basics of Categories	298
15-2 Objects	312
15-3 Pre-additive Categories	315
15-4 Adjoint Functors	325
15-5 Exercises	333
CHAPTER 16 MODULE CATEGORIES	335
Introduction	335
16-1 Generators and Cogenerators	335
16-2 Hom Functor	338
16-3 Tensor Product Functor	339
16-4 Adjoint Associativity	342
16-5 Elements of Tensor Products	345
16-6 Direct and Inverse Limits	347
16-7 Exercises	352
16-8 Exercises on direct and inverse limits	355
CHAPTER 17 FLAT MODULES	359
Introduction	359

x	Contents	
	17-1 Character Modules	359
	17-2 Flat Module Basics	362
	17-3 Exercises	365
	CHAPTER 18 PURITY	367
	Introduction	367
	18-1 Systems of Equations in Modules	368
	18-2 Pure Projectives and Pure Exact Sequences	370
	18-3 Direct Limits	379
	18-4 Pure Injectives	383
	18-5 Pure Injective Hull	391
	18-6 Exercises	396
	APPENDIX A BASICS	398
	Introduction	398
	A-1 Sets, Symbols, and Functions	398
	A-2 Background Review	405
	A-3 Exercises	408
	APPENDIX B CERTAIN IMPORTANT ALGEBRAS	412
	Introduction	412
	B-2 Exterior Algebras	412
	B-3 A Unified Approach	415
	LIST OF SYMBOLS AND NOTATION	427
	BIBLIOGRAPHY	432
	SUBJECT INDEX	436
	AUTHOR INDEX	442