CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

Subject index

Absolutely maximal essential extension 39f
Absolutely maximal pure essential extension 393f
Abelian category
  see category
Adjoint associativity 342f, 342–344, 363
A.C.C. 45f, 121
Adjunction
  back 326f
  front 326f
Adjunction of identity 9
Algebra
  cyclic 267f
  degree of an element 417
  exterior 74f, 74–85, 412–415, 422–426
  free 72–74
  Γ-graded 416
  homogeneous subspace 417
  module 61f, 104
  tensor 72f, 420, 423
  symmetric 420–422
  subalgebra 61
Annihilator 4
Artinian module 213, 214, 218
Ascending chain condition 45f, 121, See A.C.C.
Associator 241f
Axiom of Choice 400
Basis 20
  free 20–28
  projective basis 170
Baer criterion 30, 33f, 34, 35, 46, 50, 362, 370
Bilinear 56
  -R 56
Bi-module 2
Biproduct 316
Birkhoff theorem 141
Boolean space 282
Brauer theorem 193
Cardinal 400f
  cardinality 108, 401f, 404
Category 298f
  $\mathscr{A}b$ 301
  abelian 320
  balanced 318
  conormal 318
  cogenerator 307f, 332, 337, 338, 361
  equivalence 305
  exact 318
  full 300
  isomorphism 305
  generator 305, 307f, 336
  $\mathscr{M}od_R$ 301f
  morphism 299f, 300
  normal 318
  object 299f
  opposite 303
  preadditive 316
  product 308
  quotient 333
  Ring 301f
  $Ring^{(1)}$ 301f

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

Sets 301f
skeletal 306
skeleton 306
small 333
subcategory 300f
Center of nonassociative ring 241f
Central simple 246f
Centralizer 246f
double 263
double centralizer theorem 264
Centroid 243f
Character module 359
Chinese remainder theorem 147
Coimage (categorical) 315
Cokernel 3, 314
Column finite 108, 368
Complement to decomposition 221
Completely reducible 186f
Complex 198
map of 198
Composition series 180f
length 180f
Coproduct 316
c.r. 186f
Cyclic vector 87
D.C.C. 121f
Descending chain condition 121f, 121–124, 191–196
Decomposition 221
complements direct summands 221
equivalent 221
Dense ring 97
Density theorem 98, 99–104
Derivation 255f
inner 255f
θ-derivation 47f
Determinant 82, 413
LaPlace expansion of 414
Dimension over simple subring 261
Direct limit 348f, 350–352, 355, 356
short exact sequences 380
Direct product 5
external 5
components 5
coordinates 5
universal property 6
Direct root 357
Direct sum 5
external 5
internal 7
conditions 8
universal property 6
Direct system 347f
short exact sequences 380
map of direct systems 355f
Division algorithm 48
Division ring 50, 92, 250–252, 267
maximal subfield 251, 252
Domain 34f
Dual basis lemma 197
Duality 6, 166, 209
Equationally compact 388f
$\aleph^<$-equationally compact 388f
Essential
absolute maximal 43, 44
extension 39f
extension, proper 39
monomorphism 334
submodule 39f
Exact sequence 10, 296, 297, 318, 319
short exact 10, 319
complexes 198
Exact subcategory 318
Ext, $\mathrm{Ext}^n_R(\cdot,\cdot)$ 198
Field
maximal subfield 251
Filter 410, 411
hereditary 353
prefilter 353
Finite rank element 106, 107
Finitely presented 371f, 372
Fitting lemma 218
Five lemma 296
Flat 362f
Free subset 16
Fully invariant 232f
Functor 302f
adjoint 326
additive 316
bifunctor 308
category 334
constant 357f
contravariant 302
covariant 302
exact 319
faithful 308
forgetful 302
hom 306f

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

Functor (*Continued*)
idempotent 353
identity 302
inclusion 302
left adjoint 326
left exact 319
right exact 319
subfunctor 334
subfunctor of the identity 352
Generated
submodule by 7, 8
generators 7
Goldie dimension 51
Group ring 139
Hereditary
filter 353
ring 269
module 281
semihereditary 269, 277–281
Hilbert Basis theorem 354
Hilbert Nullstellensatz 161
Hom functor 338
Homogeneous algebra ideal 417
Homogeneous component 82, 210, 417
Homogeneous element 82
Homology group 198
Homomorphism of modules 2
cokernel 3
epimorphism 2
image 3
isomorphism 2
kernel 3
monomorphism 2
natural projection 3
Homomorphism of rings 406
kernel 406
image 406
one to one 406
onto 406
isomorphism 406
nonassociative 406
Ideal 406
nil 116f
nilpotent 116f, 122
prime 149, 152–156, 233
Idealizer 90
Idempotent 116f
primitive 180
orthogonal 180
Image (categorical) 315
Idempotent lifting lemma 192–193
Identity
left modulo L 87
relative left 87
ring 405
Image 3
Independent set 16, 17
Injective
hull 44f, 45
minimal submodule 44f
module 30f, 34, 38, 39, 42–46, 165
property relative to 371
pure injective hull 395f, 391–395
Inverse limit 349f, 355, 336
inverse system 348f
Inverse root 357, 358f
Invertible ideal 171f, 172
Isomorphism Theorems 407–408
Jacobson radical 111–139, 182–185, 201
Kernel 3
Krull-Schmidt theorem 228
Krull-Remak-Schmidt theorem 230
Kulikov's theorem 396
Large
extension 39
submodule 39
Linearly compact 357
Linearly disjoint 63, 249
Linear topology 356f, 357
Local ring, see ring
Matrix ring 12, 28, 29, 98, 99, 123, 124, 129, 159, 179, 183
Maschke theorem 139
Modular law 8
Module 1f
algebra module 61f, 104
Artinian 213, 214, 217, 218
bimodule 2
basis 20
completely reducible 186
cyclic 87
divisible 34
endomorphism 2
essential 39f, 40, 42–44, 209
faithful 4
finitely presented 371
free 21
free R-module on T 21

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

generalized divisible 34f, 35
hereditary 281
indecomposable 217
injective 30f, 34, 38, 39, 42–46, 165
large, see essential
left 2
Noetherian 45f, 213, 214, 217, 218
presentation of 372
projective 165f, 165–177
proper 2
quasi-injective 99f, 100–102, 231
right 2
semihereditary 281
simple 91f
small, see superfluous
superfluous 117f, 209, 211
torsion 36
torsion free 36
uniserial 212
unital 2, 9
Monoid 405
Morphism, category 298
equivalent 312, 313
epimorphism 300, 333
inverse 300
isomorphism 300
monomorphism 300
m-system 150
m-sequence 152
Multiplication algebra 242f
Multiplication centralizer 243f
$\mathcal{M}\text{od}_R$ 301
Nakayama's lemma 117
Natural transformation 304f
kernel of 334
Nil 116
Nilpotent 116, 155–158
strongly 153f, 154
Nil radical 157f
Nine lemma 296
Nonassociative F-algebra 242f
Nonassociative ring 48, 406f
Noether-Skolem theorem 253f
Nucleus 241f
Nullstellensatz
see Hilbert Nullstellensatz
Object, category 300
zero 300
initial 300
terminal 300
Opposite ring 246
Ore
condition 49f, 50
quotient ring 49, 50f
Ordinal 108, 229, 400
induction 158, 270, 295, 390
initial 401
predecessor 400
successor 400
p-adic integers J_p 17, 18
p-adic numbers R_p 17, 18
Partially ordered set 400
poset 400
Polynomial ring 130, 131
skew 47, 48
twisted skew 48, 49
Power series ring 132
Prefilter 353
Prime radical 152f, 153–156
Primitive 113
Principal right ideal
domain 34f
ring 34f
prid 34
pri ring 34
Projective property 371
Projective 163–177
dimension 173
module 165f, 165–177
resolution 173f, 176
Projective Basis Theorem 197
Pullback 284, 297
Pure exact sequence 371
Pure injective 368, 371f, 387, 388
hull 395f
Pure projective 371f
Pushout 297
Quasi-order 399f
upper directed 399f
Quasi-
composition 112
regular, right 112
regular, left 112
inverse 112
regular right ideal 114f, 115
Quasi-injective, see module
Quaternions 14f, 16, 267
Radical of module 209–211

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

rad R 148, 152
Reject 210
Ring 405f
  dense 97
  ideal of 406
  fractions 134
  local 133–135, 229, 232, 233
  nonassociative 406f
  n-transitive 97
  prime 149
  radical ring 137, 138
  opposite 246, 263–265
  primitive 97
  quotient ring 49, 134, 407
  regular 106f, 107
  right ideal of 406
  semi-primitive 125
  Noetherian 45f, 46, 214–217
  semiprime 152
  right ore 49f
  simple 105f
  subring 406
  von Neumann regular 106
Right ideal
  annihilator 96
  basic lemma 89
  left identity of 87
  maximal 87
  minimal 106, 107
  quasi-regular 114f, 115
  regular 87f
R-map 2f
Row finite 108, 368
Schanuel's Lemma 174
Schröder-Bernstein theorem 401, 409
Schur's lemma 92
Semigroup 405
  monoid 405
  ring 15
Semihereditary 269
Semi-endomorphism 99
Semiprime 152
Semisimple 165
Sequence
  exact 318
  right exact 318, 319
  splitting 318
  short exact 319
Singular
  module 231
  right ideal 231
  submodule 231
  second submodule 353
Skolem-Noether theorem 253f
Skew polynomial ring 47
Socle 206
Strongly nilpotent 126
Subdirect product 141
  dense 145, 146
  subdirectly irreducible 142
  subdirectly reducible 142
Submodule
  complement 40–42
  essential 39f, 40, 42–44, 209
  invertible 171
  large, see essential
  maximal 88, 210, 211
  minimal 88
  proper 2f, 88
  R-submodule 2
  small, see superfluous
  superfluous 117f, 209
  uniserial 212
System of equations 368, 369
  compatible 368
  equivalent 368
  finitely solvable 368
Tensor product
  algebras 64f
  functor 339
  modules 58f
Torsion
  submodule 36f, 232
  free 36f, 232
Torsion preradical 353
Torsion radical 353
Trace 201
Transitive 97f
  n-transitive 97
  two-transitive 103
Universal
  mapping theorem 65
  property 283
Weakly independent 247f
Wedderburn, J. L. M.
  see Wedderburn theorem
Wedderburn-Artin theorem 123

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

Wedderburn theorem 122, 123f, 124, 183f, 184–191, 196
Weyl algebra 50
Zero divisor 34
$Z(p^\infty)$ 17f, 357
Zorn's Lemma 400f

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-06348-7 - Modules and Rings
John Dauns
Index
More information

Author index

Amitsur, S. 130
Andrunakievitch, V. 138
Bergman, G. M. 138
Birkhoff, G. 141, 143
Braver, R. 193
Bumby, R. T. 220
Burgess, W. D. and Stewart, P. N. 333
Bitzer, C. W. 14
Clifford, A. H. and Preston, G. B. 48
Cohn, P. M. see Sasiada, E. and Cohn, P. M.
Cozzens, J. and Faith, C. 50
Dauns, J. 50, 267, 268
Eckstein, F. 357
Eklof, P. C. and Mekler, A. H. 396
Faith, C. see Cozzens, J. and Faith, C.
Freyd, P. 320
Fuchs, L. 234, 235, 395
Golan, J. 20
Goodearl, K. 50, 220, 235
Hilbert, D. 161
Hopkins, C. 204, 216
Jacobson, N. 98, 113
Jones, B. 172
Kaplansky, L. 281
Lambek, J. 212
Lambek, J. and Michler, G. 233
Levitzki, J. 204, 215, 216
MacLane, S. 308
Michler, G. see Lambek, J. and Michler, G.
Passman, D. 139
Preston, G. B. *see* Clifford, A. H. and Preston, G. B.
Ribenboim, P. 110, 165
Rinehart, G. 267
Rosenberg, A. and Zelinsky, D. 354
Rowen, L. 235
Sasiada, E. and Cohn, P. M. 138
Small, L. 281
Spivak, M. 84
Stewart, P. N. see Burgess, W. D. and Stewart, P. N.
Storrer, H. 232, 233
Zelinsky, D. see Rosenberg, A. and Zelinsky, D.

© Cambridge University Press www.cambridge.org