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978-0-521-06345-6 - Numerical Simulation of Unsteady Flows and Transition to Turbulence

Edited by O. Pironneau, W. Rodi, I. L. Ryhming, A. M. Savill and T. V. Truong

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***Invited Lecture***

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## Developments in computational modelling of turbulent flows

J.C.R. Hunt – University of Cambridge.

### Summary

This paper begins with a review of some of the many different approaches to computational modelling of turbulent flows in which a distinction is drawn between methods that compute, more or less approximately, individual realisations of the flow and those that compute statistics of the turbulence, such as 1 and 2-point moments; also the different levels of model are discussed. By considering the time and length scales, and eddy structure, of turbulence in different flows it is possible to justify where the statistical structure has a general form determined by the local mean velocity gradients, so that it can be simply modelled in terms of local quantities (e.g. using the mixing length approach), they are contrasted with flows in which the turbulence depends on the development of the turbulence from its initial state and on the boundary conditions. This provides a systematic method for deducing when more advanced levels of model (e.g. 2-point) are required, even for computing 1-point statistics, and what should be the appropriate levels of modelling for different statistical quantities. The principles and various techniques involved in modelling the statistics of some of the most critical aspects of turbulence such as the exchange of energy between different components, inhomogeneity and the rate of dissipation are reviewed, and related to fundamental studies of these topics based on the use of direct numerical simulation, theoretical methods and experiments.

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## §1. Introduction.

### 1.1. Different problems and methods for CFD.

The purpose of most practical calculations of turbulent flows is to obtain ensemble average statistics such as the mean values of velocity  $U_i$ , pressure  $\bar{p}$  and temperature  $\bar{\theta}$  and moments  $\overline{u_i u_j}$ ,  $\overline{p^2}$  etc, given certain boundary and initial conditions. The central assumption of most turbulence models is that there are closed sets of equations governing these statistics. Although such models are improving and providing an increasing range of practical answers, questions still remain about the equations and boundary conditions of current models such as when and where they are valid, and whether their mathematical properties are well behaved, and what are the approximate techniques for their solution by numerical methods. Usually these questions are decided by comparison between model results and experimental data and sometimes direct numerical simulations (Launder 1989).

In this review it is shown how many of such empirical comparisons can be explained and sometimes can be more focussed by a theoretical analysis of the different elements of these models, and by considering the actual eddy structure of turbulence. As we shall show, a number of factors such as the form of the initial and boundary conditions of a turbulent flow, and whether the turbulence undergoes rotational or irrotational straining by the mean velocity field, largely determine the eddy structure and thence the statistical relations between the different components of the turbulent velocity fields. Therefore, since it is the assumption about the form of these relations that chiefly distinguishes different models, considering these factors often provides answers to our questions about models (such as those listed above) and generally helps suggest the most effective use of statistical models of turbulence.

Understanding turbulence structure is not optional but essential in the study of those practical problems which depend on the local fluctuating eddy motions and therefore the individual realisations of the random field, rather than its ensemble statistics. For the case

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of turbulent flow round bluff bodies different kinds of model for computing the turbulence statistics and individual realisations of the flow are shown in figs. 1a, 1b (from Murakami *et al* 1990 and Tamura *et al* 1990).

Another practical example where this approach is necessary is in calculating the instantaneous pattern of dispersion of pollutant using simulations of the trajectories of fluid elements.

It is also found that the individual flow fields need to be measured or computed to improve understanding and the statistical modelling of many chemical reactions in thin turbulent flows because it is not generally possible to relate the statistics of the reactions between species, e.g.  $A$  and  $B$ , in terms of low order statistics of the velocity and scalar field such as  $\overline{u_i u_j}$ ,  $\overline{u_i c_A}$  etc. [Broadwell, 1991, Leonard & Hill 1991]. Also such “realisation” modelling may be necessary for designing systems where there is real time control of flows. Various approximate methods are now available to simulate the random flow fields with an accuracy adequate to model many of the major features of the flow without having to compute the “fully resolved” 3-dimensional equations (see Fig.1b and sec. 3.1).

The main aim of this review is to discuss the principles underlying the methods for these two kinds of calculations for modelling the statistics and realisation of turbulent flows. This approach complements (and generally does not contradict) other reviews which more specifically describe and compare different models (notably that by Launder 1989).

## 1.2. Modelling at different computational levels of CFD.

Since there will always be limitations on computer capacity and speed, it will only be possible to calculate relatively idealised turbulent flows in complete detail; for more complex flows calculations must be performed with less detail and/or less accuracy. Of course as the speed and capacity of computers improve, our definition of “complex” also changes! (Hunt 1991). In fig. 2 schematic “graphs” are shown of contours of constant computing power, or time, for different types of calculation, ranging from

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- (i) direct numerical simulations (DNS) in which 100% of the flow field is defined, but these are flows confined by boundaries with simple shapes (e.g. channels) whose Reynolds numbers (for the turbulence) are less than about 200, to
- (ii) approximate general methods for “realisation” modelling by simulating random fields, such as large eddy simulation (LES), approximate numerical simulation (ANS), kinematic simulation (KS) (less accurate, but more detail), or statistical modelling such as the calculation of one-point moments of the turbulence, at the second or third order level of ‘closure’ (Launder, Reece & Rodi 1975, Lumley 1978) (sec. 3.2).
- (iii) methods that are specific to particular classes of flows, such as modelling Reynolds stress with an eddy viscosity defined by equations containing coefficients chosen for particular flows, e.g. models of boundary layers (Cebeci & Smith 1974) or integral models of jets and plumes (Turner 1973 Chap. 6), or random flight models for mean and variances of concentration fields in turbulence (Thomson 1987).

The latter models are more “expert” in that they contain ideas from complex modelling and/or experiments, but they are also available to users that are less expert in fluid mechanics and CFD! For many practical problems (e.g. from turbomachinery boundary layers to environmental flows) fast and specific methods are necessary, because they have to be incorporated into larger engineering or environmental codes. In fact there has to be a “cascade” of methods in CFD from DNS down to these specific codes!

In fig. 2 note that the contour lines for each method are fuzzy, because the methods are evolving and continually being extended. Inevitably the methods are being applied where they are not strictly applicable. As Popper explained, it is the errors in science that lead the way forward!

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## §2. Characterising turbulent flows.

### 2.1. Classification in terms of boundary and initial conditions.

The study and understanding of turbulence always has to be related to what kind of flow is being considered, because the structure of turbulence is not the same in all turbulent flows. Also, depending on the flow and its boundary conditions, the turbulence may be an intrinsic feature of the flow or may be largely determined by the turbulence on the boundary of the flow domain. Classification of turbulent flows has, surprisingly, seldom been attempted by turbulence researchers (perhaps because they are making an ambitious attempt to describe every kind of turbulence?); but we have found it to be useful in guiding the choice of appropriate turbulence models, and for relating complex practical problems in turbulence to simpler and better understood turbulent flows. This is related to the approach, advocated by Kline (1981), of analysing complex flows in terms of “flow zones”, in each of which the flow is best modelled by different methods.

We first classify turbulent flows according to three types of boundary conditions:

#### 2.1.1. *Closed domains* (fig. 3):

In this case the turbulent flow domain  $D$  is confined by surfaces on which the velocity is zero or is completely specified, such as flow in a heated or stirred tank, an electromagnetic induction furnace, or by a piston in a closed cylinder. The turbulence is generated by flow instabilities (e.g. shear, buoyancy, rotational etc.), which, along with the direct effects of the boundaries, determine the large scale structure of the turbulence. (c.f. Castaing *et al* 1989, Davidson, Hunt & Moros 1988).

#### 2.1.2. *Open domains* (fig. 4):

In this class, flow enters and leaves  $D$  through the bounding surface  $B$ , from or into the external domain  $E$ . The flow in  $D$  is affected by the velocity field on  $B$ , so there may

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be some significant correlation between the turbulence in **D** and **E**. Generally the form of these boundary conditions is specific to the particular flow problem.

- (a) No turbulence enters **D** from **E**: in this case the flow entering **D** from **E** is laminar, and turbulence is generated within **D**. But where the flow leaves **D** back into **E** the flow may well be turbulent. An example would be a turbulent boundary layer **D** with no free stream turbulence; the flow leaving the layer forms a turbulent wake in **E**. (The transition between these regions is described by a theoretical asymptotic analysis by Neish & Smith 1988 using a simple turbulence model, and by a numerical solution using a complex turbulence model by Launder (1989).
- (b) Turbulence is transported between **E** and **D**, so that turbulence in **D** is affected or even determined by the turbulence specified on **B**: there is an important physical and computational difference between flows where there is a strong mean velocity (relative to the turbulent eddy velocities), and those where there is no appreciable mean velocity across **B**, but turbulent energy is still being transported in or out of **D** by the action of turbulent eddies across **B**. Whether or not this transport of turbulence is significant within **D**, generally depends on the level of turbulent energy in **D**, and may also depend on the sensitivity of turbulence in **D** to external disturbances.

As shown in figure 4(b), a typical example of the former case is turbulence in a gas turbine approaching a row of turbine blades; outside the boundary layers on the blades the turbulence in **D** is largely determined by and highly correlated with that entering from **E**, (e.g. Goldstein & Durbin 1980), whereas the turbulence in the boundary layers depends on an interaction between the incoming turbulence and the local boundary layer instabilities (e.g. Goldstein 1981). A typical example of the latter case is turbulence near a density interface, see figure 4(c) (Carruthers & Hunt 1986).

These are both examples where the turbulence in **D** depends its structure at **B**, but there are other examples where this is not true such as fully turbulent flow in a long pipe



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which is independent of the structure of the turbulence at entry. The reasons are discussed in sec. 2.2.

### 2.1.3. Time dependent problems

Here the complete flow field  $u(\mathbf{x}, t)$  in  $\mathbf{D}$ , involving mean and random quantities, is specified at an initial moment in time, say  $t_I$ , and boundary conditions are also applied on  $\mathbf{B}$ . The problem is to calculate and understand how the flow changes with time. This may be very slow (in a statistical sense) if the initial form of the turbulent velocity field and the boundary conditions define a state that is close to equilibrium. On the other hand there may be a rapid rate of change if the initial turbulence does not satisfy the boundary conditions, so that the turbulence has to adjust. Examples of such flows which have been studied experimentally and theoretically include the change of isotropic turbulence under the actions of mean straining motions, or anisotropic fluctuating body forces.

2.1.4. Most basic turbulent flows, and many practical problems, fall into one or other of the categories described in 2.1.1 to 2.1.3. Time dependent flows have proved to be useful as model problems for studying the structure of distorted turbulence in well developed or even statistically stationary flows; as Lumley remarked “turbulence is a black box that needs shaking to find out what is inside”. The results of many distortions or “shakings”, such as those performed in their laboratories by Corrsin, Mathieu and Townsend and their colleagues using ingenious different wind tunnel experiments, helped form the basis of much current understanding and modelling (e.g. Launder, Reece & Rodi 1975, Lumley 1978). More recently a few laboratories have focussed on the equally important flows where the turbulence is not locally homogeneous or statistically stationary (e.g. Veeravalli & Warhaft, 1989).

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[More information](#)**2.2. The time scales of the turbulence.**

We have seen (in §2.1) that turbulence in a flow region  $\mathbf{D}$  at a given time may be determined by the turbulence  $u_{\mathbf{B}}$  or  $u_I$  defined by the conditions on the bounding surface  $\mathbf{B}$  (especially where turbulent energy is transported into  $\mathbf{D}$ ) and conditions at the initial time  $t_I$ , or by turbulence generated in the interior of  $\mathbf{D}$ . These different sources of turbulence in any given flow could only be identified by considering a number of different conditions on  $\mathbf{B}$  and at  $t_I$ .

In order to understand and model the turbulence  $\mathbf{u}(\mathbf{x}, t)$  in the interior of  $\mathbf{D}$  it is essential to know the extent to which it depends on the imposed turbulence, relative to the locally generated turbulence. For example is  $\mathbf{u}(\mathbf{x}, t)$  statistically correlated with  $u_{\mathbf{B}}$  or  $u_I$ , so that the structure of turbulence in  $\mathbf{D}$  depends sensitively on the structure of  $u_{\mathbf{B}}$  or  $u_I$ , or does the variance of  $\mathbf{u}$  ( $= u_0^2$ ) only depend on the variance of  $u_{\mathbf{B}}$  or  $u_I$ , and perhaps other factors such as the Reynolds number of the flow? An example of the former situation is the turbulence between the turbine blades which is correlated with the turbulence in the wakes of upstream blades Fig. 4b. The commonest example of the latter situation is the flow in a circular pipe when the Reynolds number is above a critical value of about 1,000 where turbulence only occurs when the inlet flow contains velocity fluctuations of finite amplitude.

As in other statistical problems in physics the criterion for the sensitivity to boundary or initial conditions must depend on whether the turbulence time scale  $T_L$ , (the intrinsic or “relaxation” time scale of the system) is of the same order as the imposed time scale  $T_{\mathbf{D}}$  in the domain.  $T_{\mathbf{D}}$  is the time over which the fluid elements are in the domain, where the turbulence may or may not be undergoing distortion.

One measure of the intrinsic time scale of the turbulence in the interior of  $\mathbf{D}$ ,  $T_L$ , is the “relaxation” time over which some disturbance to the turbulence structure decays. But any adjustment of a turbulence flow field always involves the transfer of energy between different scales, or “turning over” of eddies (e.g. Kellogg & Corsin 1980). Since this is