THE MATHEMATICAL PAPERS OF ISAAC NEWTON VOLUME II 1667-1670

Cambridge University Press
978-0-521-04596-4 - The Mathematical Papers of Isaac Newton: Volume II, 1667-1670
Edtied by D. T. Whiteside
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De Solutione Problematum per Motum.

Ne hujusmodi operationes obscuræ nimis evadant, Lemmata 6 sequentia brevitèr idcoqꝫ non demonstrata præmittam.

Lemma 1. Si corpus A in circumferentiâ circuli vel sphæræ ADCE movatur versus ejus centrum D: velocitas ejus ad unaquæqꝫ circumferentiæ puncta D, C, E, est ut cordæ AD, AC, AE, ductæ a Corpore A ad ista puncta D, C, E.

Lemma 2. Sit △ ADC similis △ AEC, etsi non sunt in eodem plano. Inquam, Si tria Corpora a puncto A, primum ad D, secundum ad E, tertium ad C, uniformiter et in æqualibus temporibus movantur: Motus tertii componetur ex motibus primi et secundi.

Notetur, quòd hic per corpus intelligitur ejus centrum gravitatis, vel aliquod ejus punctum.

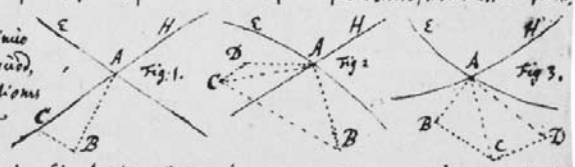
Lemma 3. Omnia puncta corporis parallelismum servantis æqualitèr moventur.

Lemma 4. Si corpus solo motu circulari circa axem quemvis rotetur; motus omnium ejus punctorum sunt ut distantiæ ab isto axe. Et hi duo motus simplices vocentur.

Lemma 5. Si motus corporis considerentur ut mixtus e motibus simplicibus: motus omnium ejus punctorum componetur ex motibus eorum simplicibus, eo modo quo motus ab A ad C, in Lemmate 2do componitur ex motibus ab A ad D et E.

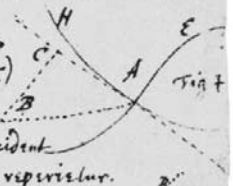
Nota, qd motus quilibet ad unum horum 3ⁱ casuum reduci potest. Et in casu tertio, linea quævis pro axe, (vel si linea aut plana superficies movetur in plano, quodvis punctum istius plani pro centro) motus assumi potest.

Lemma 6. Sint AE, AH, lineæ motæ et continuè secantes; Ducantur AB, AD, AC, BC, CD. Dico quòd, datis quæ requiruntur ad proportiones et positiones harum quinque linearum AB, AD, AC, CB, CD, determinandas; illæ designent proportiones et positiones horum quinqꝫ motuum, viz: puncti A in linêâ AE fixi et versus B mobilis, puncti A in linêâ AH fixi et versus D mobilis, Puncti A intersectionis A in plano ABCD mobilis versus C, (lineæ enim ista semper sunt in eodem plano), Puncti intersectionis A in linêâ AE mobilis secundum ordinem literarum C, B, et Puncti intersectionis A in linêâ AH mobilis secundum ordinem C, D.



Nota, Quod linea recta tangens curvam motu descriptam (ut AB, AD, AC) vel istæ tangenti parallela (ut CB, CD), dicitur designare positionem istius motûs in puncto contactûs.

Nota Etiam qd linêâ AH quiescente (ut in Fig 1, et 4), punctum D et A coincident et punctum C in linêâ AH, modò sit rectâ (Fig 1,), aliter in ejus tangente AC (fig 4) reperietur.

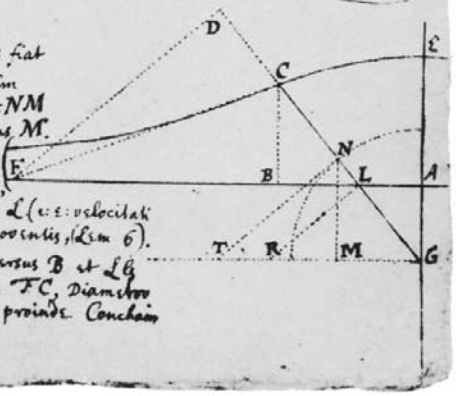


Prop 1. Ducere Tangentem ad Ellipsin.

Sit ACB filum per quod ellipsis describi solet, et CE Tangens. Cùm filum AC augetur eâdem velocitate quâ BC diminuitur, i.e: C habet eandem velocitatem versus D et B; et fit ∠DCA=ECB. per Lem 1. Idem de reliquis Conicis intelligatur.

Prop 2. Ducere Tangentem ad Conchoiden.

Sint GLC, ALF, GAE, regulæ quibus concha describi solet: fiat GT ∥ AF ⊥ CB = ∥ MN: et NG = CL ⊥ TN ∥ RL. Et, cùm æqualitas proportionalitate simplicior est, ponatur lineam CB = NM esse æqualem velocitati puncti C versus B, vel puncti N versus M. Erit NT = motui circulari puncti N circa G rotantis versus T, (Lem 1): et LR = Motui circulari puncti L in linêâ GL fixi, circa G (Lem 4): et LG = motui puncti intersectionis L (i.e: velocitati puncti C) a puncto G, sive versus punctum D in linêâ CG mobilis, (Lem 6). Jam cum habetur velocitas puncti C viz CB versus B et LG versus D, fiat TD ⊥ DC = LG: Et motus puncti C erit in linêâ FC, Diametro nimpe circuli transeuntis per puncta CDBF, (Lem 1): Quæ proinde Conchoen tangit in puncto C.



The 'solution of problems by motion' (**2**, 2, §2).

THE MATHEMATICAL PAPERS OF ISAAC NEWTON

VOLUME II

1667-1670

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WITH THE ASSISTANCE IN PUBLICATION OF
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CAMBRIDGE
AT THE UNIVERSITY PRESS
1968

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521058186

First published 1968
This digitally printed version 2008

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 65–11203

ISBN 978-0-521-05818-6 hardback
ISBN 978-0-521-04596-4 paperback
ISBN 978-0-521-72054-0 paperback set (8 volumes)

TO HAROLD HARTLEY

FOR SO MUCH

PREFACE

This volume, the second both in sequence and to appear of eight projected, continues the chronological reproduction of all Newton's mathematical papers now known still to exist, together with appropriate editorial commentary. To what was said in preface to the first volume regarding the aims underlying the present edition I have nothing to add, but I might perhaps remark, that with the exception of the 'De Analysi' (here edited from the original autograph manuscript retained by him), all of Newton's papers now reproduced effectively make their first appearance in print.

For permission to reproduce documents in their custody my collective gratitude must be expressed to the Librarians and Syndics of the Bodleian, Oxford; the Royal Society, London; the Niedersächsische Landesbibliothek, Hanover; and above all the University Library, Cambridge. The efficiency and courtesy of their staffs I again gratefully acknowledge, while to Professor J. E. Hofmann and especially Dr Theo Gerardy personal acknowledgement is due for their effort in making available a transcript and photocopy of Leibniz' notes on the 'De Analysi'. To a private owner my continuing thanks for permitting publication of his Newtonian papers. For financial assistance during the period of preparation of this volume I am largely indebted to the good graces of what is now the Science Research Council, while certain incidental expenses continue to be met by the Royal Society. Let me note in anticipation, however, that the Sloan Foundation, the Leverhulme Trust and the Master and Fellows of Trinity College, Cambridge, have each in their way pledged themselves to be future benefactors. To Sir Harold Hartley my inadequate thanks for his omnipresent support in ways of which only he can know the full detail.

At a technical level my acknowledgement is twofold. To Mr A. Prag, who has both proof-read and indexed this volume, rectifying much that was faulty in the editorial commentary, and to Dr M. A. Hoskin, who has been all—and more—that a helper can be, I cannot begin to express my gratitude. I myself, of course, remain uniquely responsible for the deficiencies, omissions, vagaries and other imperfections of the present volume.

As before, my final word of appreciation goes to the Syndics of the Cambridge University Press for the unsparing efforts of their staff in bestowing logical consistency and typographical beauty on the longhand manuscript which was initially submitted to them.

D.T.W.

1 December 1966

EDITORIAL NOTE

For the principal printing conventions here used and their underlying *raison d'être* the reader is referred to pages x–xiv of the first volume. The Newtonian text reproduced is strictly faithful to the autograph manuscript used, in that contractions and suffixes appear unchanged and grammatical inconsistencies are left unaltered or their editorial correction is indicated. For clarity's sake, however, a very few silent liberties have been taken in italicizing textual sub-heads and in inserting endpoints to sentences. Otherwise, all insertion in Newton's text is bracketed off in square parentheses and should be accepted as extraneous, though in each instance the reader is invited to consider why insertion was made. In English translation of Latin text we have not 'trans-literated' existing English phrases but merely repeat them within square parentheses (as 'editorial' insertion in that translation). As in the first volume, two thick vertical bars in the left-hand margin alongside a piece of text denote that the section so marked off has been cancelled by Newton (being here reprinted for its intrinsic interest). This convention should not be confused with the two faint vertical parallels '||' used within the text (here in **3**, 1, §§1/2) to indicate a page division in the manuscript reproduced: in all cases, in the left-hand margin immediately opposite is inserted a complementary '||[...]', where the square brackets enclose the arabic number of the new page which begins at that point. A few *ad hoc* conventions used in reproducing particular texts are explained in an opening footnote to the piece in question. Universally, the convention 'I, **3**, **4**, §3: note (5)' refers back to note (5) of [Volume] I, [Part] **3**, [Section] 4, [Subsection] 3: specifically, to note (5) on page 544 of the first volume. For brevity of quotation portions of the notation are frequently omitted from the left when reference is made to the same volume, part, section and so on: for example in note (121) on page 500 below the full reference is [II,] **1**, 3, §2, while in note (24) on page 9 it should be understood to be [II, **1**,] 3, §1, 2/3.

GENERAL INTRODUCTION

This second volume of Newton's mathematical papers reproduces that portion which, in our estimate, was composed during the years 1667 to 1670. The reason for choosing these chronological bounds is largely one of editorial convenience. On the one hand the October 1666 fluxional tract effectively terminates the thirty-month period of Newton's first creative mathematical researches, and indeed represents his conscious attempt at that time to gather the offshoots of his thoughts on calculus into a collective unity. On the other, the lengthy 1671 tract on fluxions and infinite series (which will appear in the next volume) opens a new cycle of analytical investigation which endured, if somewhat fitfully, till the early 1680's. The dearth of accurate documentary information relating to this period of his development, surely the least known of all the Newtonian dark ages, has not made the task of editing easy. The background to the *De Analysi* is now reasonably well established in consequence of the resurgence of interest in it at the time, forty years after its first circulation, of Newton's dispute with the Leibnizians over calculus priority: even so, its date of composition can only somewhat vaguely be bounded by the appearance of Mercator's *Logarithmotechnia* (in September 1668) and its communication by Barrow to Collins (in early July 1669). On the printing history of Newton's 'Observations' on Kinckhuysen's Dutch *Algebra* we are remarkably well informed, largely thanks to Collins' preservation of his correspondence with Newton and others on the topic. A glimmer of light is shed on his researches into the organic construction of curves and the geometrical construction of equations by Newton's later letters to Collins, particularly that of 20 August 1672. But for those papers which in time of composition precede mid-1669, when Newton's extant correspondence with his contemporaries opens,(1) editorial commentary must inevitably in large part reduce to essentially unsupported circumstantial argument.

Of Newton's life in general during this four-year period we know very little. A century ago Joseph Edleston(2) afforded some insight into his daily routine

(1) The first letter of Newton's now known, the abrupt note to an unknown correspondent reproduced (from Collins' copy for want of an autograph original) in *The Correspondence of Isaac Newton*, **1**, 1959: 3–4, is dated late February 1669. We should mention that the often quoted 'Newtonian' advice of '*c.* 1661' to a drink-sodden friend, which exists in two forms in the Pierpont Morgan notebook in New York, is in fact signed by Newton with the initials 'T.N.': in other words, the letter is one form or other of a copybook exercise. Compare the photocopy published by D. E. Smith in W. J. Greenstreet's bicentenary compendium, *Isaac Newton: 1642–1727* (London, 1927): 26.

(2) In notes which he added to his 'Synoptical View of Newton's Life', in his *Correspondence of Sir Isaac Newton and Professor Cotes* (London, 1850): xxi–lxxxi, especially xlii–xliv, with appendices (lxxxii–lxxxv) analysing both Newton's fellow's dividends and his exits and redits.

at Trinity College. This, together with a little additional information to be gleaned from Newton's current items of expenditure as listed in one of his pocket-books[3] and a few off-hand remarks made by him a few years later relating to the sequence of his optical discoveries,[4] is the slender documentary basis on which any reasoned account of his immediate post-graduate years in Cambridge has to be constructed. Enough remains, however, to show up inadequacies in the conventional picture of a nervous, wholly diffident, badly dressed young don interested solely in the flights of his intellect. If only to stress the normality of Newton's social behaviour, at this period at least, and so extirpate any lingering temptation to relate his exceptional intellectual growth to a hypothetical (in fact, non-existent) physical immaturity, we may briefly touch upon some biographical points.

Geographically, at the opening of the year 1667 Newton was in Lincolnshire awaiting the reconvening of his Cambridge college. (Its fellows and students had, we will remember, been 'dismissed' the previous summer because of a renewed outbreak of plague in the town.) With the ending of winter the university sprang back to life and Newton travelled to Cambridge on 22 April. There he remained without break till April 1671 apart from a brief visit 'into ye countrey'—no doubt to visit his mother—over Christmas 1667[5] and two short trips to London in the summer of 1668 and the autumn of 1669,[6] during the latter of which John Collins met him for the first time 'somewhat late upon a Saturday night at his Inne'.[7] Having satisfied his B.A. examiners in January 1665—though he was not, in fact, to pay for his 'Bachelors Act' till some days after his return to Cambridge in April 1667[8]—Newton was now

(3) That now in the Fitzwilliam Museum, Cambridge. The expenses, listed line by line on six and a half unpaginated 16° pages, cover the period 23 May 1665 to 'April' 1669 but are evidently incomplete. The most significant double page is reproduced in photocopy (facing page 52) in the illustrated version of John Taylor's *Catalogue* of the Portsmouth papers auctioned at Sotheby's on 13/14 July 1936. Some additional material relevant to these listed expenses is contained in a stray sheet (sold as part of Lot 201 at the 1936 sale) of about 1667, the present whereabouts of which is not known. All unidentified financial entries in this introduction are taken from the Fitzwilliam pocket-book.

(4) Particularly in his letter to Oldenburg on 5 February 1671/2. Compare note (15) below.

(5) The Fitzwilliam notebook lists his departure from Cambridge on 4 December 1667 and return on the following 12 February, together with an item of five shillings 'For keeping Christmas'.

(6) Specifically, with regard to the first Newton wrote in the Fitzwilliam notebook that 'I went to London on Wednesday Aug 5^t^ & returned to Cambridge on Munday Sept 28, 1668' (and, to be sure, he signed his redit at Trinity the following day) together with 'Spent in my Journey...5. 10. 0. As also 4^li^ 5^s^ more wch my Mother gave mee in ye Country'. His college exit and redit for the latter journey indicate that it was made between 26 November and 8 December 1669.

(7) Collins to James Gregory, 24 December 1670. See **3**, Introduction: note (9) below.

(8) This may mean that Newton did not officially achieve B.A. status till April 1667. Evidently the termination of his undergraduate days was celebrated in traditional fashion for,

in a position to enjoy the first pleasures and privileges of academic position. Certainly, his advancement in college status was rapid enough for any young don determined to make his mark: a minor fellow of Trinity from October 1667, he was quickly promoted to major fellow on 16 March 1668 (duly, and no doubt proudly, paying his shilling for his 'Fellows Key' and being allotted his fellow's quarters).[9] His necessary elevation at university level from pupil status followed speedily in the following July when he was created M.A. (then, as now, largely a matter of satisfying a residential qualification and paying out the requisite sum of money).[10] His election, finally, to the Lucasian Professorship on 29 October 1669, upon Barrow's retirement, gave him security, academic independence and a useful stipend at the expense of only a moderate portion of his time.[11] Socially, it is evident that Newton during his first years as a senior member of Cambridge university made a determined effort to live up to his position. During the two years 1667 and 1668, academic necessities apart, he spent over twenty pounds with his tailor 'Mr Jeffreys' on new clothes[12] and footwear, while for his newly decorated dining room he bought a table and chairs, paying out sixteen shillings for 'A Table cloth' and 'Six Napkins'—for the use of invited guests no doubt. Not a few of his evenings were spent 'wth Mr Lusmore, Hautrey, Salter' and 'other Acquaintance' or with his room-mate Wickins wining and gambling at the local tavern, playing cards—twice losing heavily—or coming out an equally expensive second-best on the college bowling green. The fifty pounds his mother contributed to supplement Newton's income during this period were presumably quickly spent, given away in tips to his college servant Caverly or loaned—at interest

in the Fitzwilliam notebook on the line immediately beneath that where he recorded payment of '0. 17. 6' for his 'Act', he listed a pound spent 'At ye Taverne severall other times &c' together with a further 12*s*. 6*d*. paid out 'on my Couz. Ayscough'.

(9) The fellow's room Newton received in early October 1667 was the celebrated 'Spirituall chamber' (Edleston's *Correspondence* (note (2)): xliii), but it would appear that he himself did not occupy it for in late 1668 he 'Received for Chamber rent 1. 11. 0'. In the spring of the previous year he had, in fact, laid out a considerable sum of money refurbishing the set of rooms he shared with Wickins, putting in new glass, making minor repairs to the fireplace, walls and woodwork, having it repainted, installing new furniture and carpets and, not least, contributing almost a pound for 'My part of a Couch'. He would naturally be reluctant to move out of such comfortable quarters after only six months' tenancy of them.

(10) In fact, 'For my degree to ye Colledg 5. 10. 0. To ye Proctor 2. 0. 0. Expences caused by my Degre 0. 15. 0. 18 yards of Tammy for my M^{r} of Arts Goune 1. 13. 0. Lining 0. 3.6. Making y^{t} & turning my Batchelors Goune 1. 0. 6. A Hood 1. 3. 6.' (Fitzwilliam pocket-book.)

(11) Newton's appointment to the Lucasian Professorship and the concomitant restrictions and responsibilities it imposed upon him raise several interesting questions but we will delay our commentary on these till the next volume. See also Edleston's *Correspondence* (note (2)): xliv.

(12) With his countryman's good sense Newton bought the best cloth, lined 'Woosted Prunella' and 'Stuffe'.

we may be sure—to 'Dr Wickins' or other acquaintances (such as Perkins, Boucheret or Wadsley) whose names are listed by Newton for loans of between five shillings and two pounds together with a cross to signify repayment. To ensure that his garments were well washed and his rooms clean, warm and well lit he made regular payments to his 'Laundresse' and 'Goodwif Powell', buying 'New Feathers', and a 'Ticken' for his bed and calling frequently on the local chandler for 'coales & sedge'. Altogether the picture we have is of a young man spending his income to the full in the mild pursuit of luxury and pleasure, an acceptably mature version of the cautious, money-wise young puritan who had entered Trinity half a dozen years earlier.

Enough of the worldly face which Newton presented to his fellows at this time. What of the intellectual giant within?

There can be no doubt that Newton's scientific researches proceeded apace during these years, notably in optics but also in chemistry and to some lesser degree—as the extant manuscripts now reproduced themselves attest—in mathematics. The impressive corpus of optical theory and experiment which (in continuation of Isaac Barrow's professorial investigations of the 'genuinæ rationes' of the phenomena of white light(13)) he began to present to his Cambridge audiences from January 1670 was evidently neither conceived nor systematized in one single, brief creative spell, while we have Newton's own confirmation that as early as mid-1668 he was hard at work constructing a 'small prospective', a first version of the portable reflecting telescope of his own design and fabrication which he gave to the Royal Society in 1672.(14) His pocket-book accounts, moreover, list purchases in summer 1668 of a 'Lath & Table' and 'Iron worke for it' together, with 'Drills, Gravers, a Hone & Hammer & a Mandrill' and 'files', all clearly destined for grinding lenses and mirrors. A few months later he listed purchases of 'Glass bubbles 0. 4. 0' and '3 Prismes 0. 3. 0', spending a total of twenty-nine shillings on 'Glasses' both in Cambridge and in London.(15) Newton's practical interest in chemistry,

(13) Published in 1669 (and reissued in 1670 with his *Lectiones Geometricæ*) as his *Lectiones XVIII, Cantabrigiæ in Scholis publicis habitæ; In quibus Opticorum Phænomenωn Genuinæ Rationes investigantur, ac exponuntur*. These eighteen lectures were given on unknown dates in 1667 and 1668 to the university at large.

(14) Newton's Lucasian lectures on optics, later to be published, from a secondary transcript of the version (ULC. Dd. 9.67) deposited in 1674 in the university archives, as his *Lectiones Opticæ, Annis, MDCLXIX, MDCLXX & MDCLXXI. In scholis publicis habitæ* (London, 1729), will be discussed in detail in the third part of the next volume. For Newton's first metal-reflector telescope see his *Correspondence*, **1** (1959): 3–4, 74–7.

(15) Presumably when he wrote to Oldenburg on 6 February 1671/2 (*Correspondence*, **1** (1959): 92–102, especially 95) that he had 'left off' his 'Glass-works' in 1666 he was reporting something less than the truth.

(16) Listed in the Musgrave library roll (R. de Villamil, *Newton the man*, London, 1931: 98) as '6 vols. . . . 8vo'. Curiously, these chemical entries occur a decade earlier than the first dated

inspired no doubt by his reading of Boyle, Hooke and other mechanical philosophers of his period, began about the time of his first visit to London in August 1668. Shortly afterwards he recorded the purchase of two pounds worth of 'Aqua Fortis, sublimate, oyle, perle, fine silver, Antimony, vinegar, Spirit of Wine, White lead, Allome, Niter, Tartar, Salt of Tartar, [Mercury]' (from London, no doubt, since he added 'Carriage of ye oyle 0. 2. 0') and a charge of fifteen shillings for the building of two furnaces (one for 'tin'), acquiring also Ashmole's encyclopedic 'Theatrum Chemicum [Britannicum]'.(16) But apart from a stray reference to his buying 'Gunters book & sector &c' from 'Dr Fox' for five shillings(17) Newton's expense lists are barren of mathematical entries, though, for example, it is certain that he bought the complete set of James Gregory's published works before late 1670.(18) The autograph texts of Newton's mathematical investigations are themselves, however, firm testimony to the volume and quality of his geometrical and fluxional researches during the years 1667 to 1670—if, that is, our present dating of these papers is accurate. Internal evidence convinces us that their posited chronological sequence is indeed correct: the *De Analysi* at one point, for example, borrows its terminology from preceding discussions of the nature of asymptotes,(19) while the concluding tract on the geometrical construction of equations leans heavily in its description of the Wallisian cubic on a prior knowledge of his innovations in the organic construction of curves.(20) For independent external support of our early dating of this group of mathematical papers we can call upon John Collins, who saw a representative selection of Newton's scientific manuscripts some time in the early 1670's and was allowed to make detailed transcripts (now in private possession) of those portions which interested him. Subsequently, towards the end of 1677, when the Oxford Savilian Professor, John Wallis, wrote to him enclosing for comment an early draft of what was to appear in 1685 as his *Treatise of Algebra, both Historical and Practical* and in which bare mention was made of Newton's researches in infinite series, Collins in reply(21)—after an initial rebuke 'in regard you lye under a censure from

passage, an entry on 10 December 1678, in Newton's chemical notebook (ULC. Add. 3975) and we may conjecture that the earliest manuscript record of his researches in that field has been lost. Compare A. R. and M. B. Hall, 'Newton's Chemical Experiments', *Archives internationales d'Histoire des Sciences*, **11** (1958): 113–52, especially 120. (17) I, **3**, 2, §1: note (14).

(18) Newton's library copy of Gregory's *Optica Promota* (London, 1663), sold at the partial auction of his books in the early 1920's, has now vanished but his *Vera Circuli et Hyperbolæ Quadratura* (Padua, 1668 reissue), *Geometriæ Pars Universalis* (Padua, 1668) and *Exercitationes Geometricæ* (London, 1668) are now in Trinity College, Cambridge (NQ.9.48, bound up with Newton's copy of Nicolaus Mercator's *Logarithmotechnia* (London, 1668) and John Wallis' review of it which appeared in the *Philosophical Transactions* in 1670).

(19) See **2**, 2: note (81) below. (20) Compare **3**, 2, §2: note (121).

(21) ULC. Add. 3977.13. published in the *Correspondence of Isaac Newton*, **2** (1960): 241–3, especially 242–3.

diverse for printing discourses that come to you in private Letters without permission or consent'—went to some trouble in elaborating for Wallis the content of some of the Newtonian papers he had earlier seen:

I...must take liberty to tell you some things concerning your intended Explanation of Mr Newtons Series. If I had been so minded I could about 9 yeares since namely at the beginning[!] of 1669 have imparted to you a full treatise of his(22) of that Argument....

In your narrative you say Mr Newton began to fall into these methods in 1669 or 1670, whereas in the larger Letter he tells you he seemed delighted *hisce* [*in*]*ventis* namely in Calculating Logarithmes and Van Ceulens Numbers in his retirem^t from the University in the Plague yeare in 1665,(23) and in 1666 he writt the treatise above mentioned.(24) All the account you can give out of these Letters is but very slender in relation to his performances. He intends a full treatise of Algebra consisting of these Parts according to the best of my app^s

1 an Introductory part from Kinckhuysē out of low Dutch turned by Mercator into Latin, which he bought(25) and is so excellent, that it comprehends many of Huddens reductions, and those mentioned by Dary at the end of his tract of Interest (26) & some others to which Mr Newton added much of his owne.(27)

2 A discourse about bringing Problemes to an Æquation with a Collection of diverse notable ones.(28)

3 A Treatise about the Construction of Problemes and Æquations which I have seen. All Solid Problems viz those of 4 and 3 Dimensions are solved by ayd of one

(22) The 'De Analysi per æquationes numero terminorum infinitas', reproduced as **2**, 2 below. Collins did not, in fact, see Newton's tract till August 1669.

(23) In his 'larger Letter' (the *epistola posterior* to Oldenburg for Leibniz, 24 October 1676) Newton had remarked of his 1665 retirement to Lincolnshire that 'tunc sanè nimis delectabar inventis hisce'. Collins had communicated the content of the *epistola prior* (of 13 June 1676) to Wallis in September 1676 (*Correspondence of Isaac Newton*, **2** (1920): 101) and his transcript of the second letter was no doubt communicated soon after Oldenburg sent his own copy off to Leibniz. In an omitted portion of the present, undated letter to Wallis, Collins noted that 'Mr Newton last yeare sent up these Letters, you have seene with particular leave upon my importunity to print the same...if I had not [imparted the first Letter to you] I believe you had not seen either to this day'. This is evidently Collins' quick rejoinder to Wallis' letter to him of 2 October 1677 with its assertion that 'I am stil of Opinion y^t Mr Newton should perfect his notions, & print them suddenly. These letters, if printed, wil need a little review by himself, for there be some slips in hasty writing them' (*Correspondence*, **2**: 238).

(24) Collins here confuses the 'De Analysi' with the October 1666 fluxional tract, which no doubt he had likewise seen some time before. The Wickins transcript (I, **2**, 7: note (1)) may indeed have already been in his possession.

(25) Compare **3**, Introduction: note (50).

(26) Michael Dary, *Interest Epitomized, both Compound and Simple....Whereunto is added, A Short Appendix For the Solutions of Adfected Equations in Numbers by Approachment: Performed by Logarithms* (London, 1677: Newton's copy is now Trinity College. NQ.16.62). The 'Short Appendix' (pages 32–8) is 'Laid down in Two Methods', the first of which is an adapted form of Stevin's numerical technique, the latter being Dary's own method of iteration applied to the equation $y^8 = 6y^3 + 200$. Kinckhuysen in his *Algebra* (see **3**, 2, §1: note (108)) 'compre-

Constant Circle (if so desired) supposed to be intersected by Conick Sections, the description whereof is avoyded by helpe of mooveable angles, that give the severall Points of Intersection sought. Other Equations betweene the 5 and 9 degree he performes by ayd of a Cubicall Parabola that being once described in like manner remaines constant, and is to be intersected by a Conick section the description whereof is avoyded as before &c. He hath also diverse tentative Constructions for Cubicks and Biquad^s^ from Plaine Geometry.(29)

4 A Discourse concerning the severall kinds of infinite Series considering which kinds are most convincing and fitt for Calculation, and which for Construction and Demonstration, of this Argum^t^ and of the whole buisinesse of Series he hath written a new and large treatise since that above mentioned,(30) and hath performed aboundantly more than is either mentioned or can be guessed from the Lettters above mentioned.(31)

5 A Treatise *de Locis*.(32)

6 The same(33) applyed to Dioptriques concerning the worth of both which Dr Barrow affirmed he was not only surprized but others would thinke it incredible.(34)

The first three—and the fifth also, if we identify its vague title correctly—of the unpublished Newtonian mathematical tracts here listed by Collins are reproduced for the first time in print in the present volume, while the fourth and portions of the sixth will form the centre-piece of that following. Without further prevarication let us hasten to the rich detail of the papers themselves and appreciate Newton's mathematical genius in its original dress.

hends' only the former. Writing to Newton about July 1675 Collins had already indicated a second point of resemblance between Dary's researches into equations and Kinckhuysen's 'Huddenian' reductions: 'Mr Dary by observing the Complication of the Coefficients hath well performed to this Purpose without the ayd of a Cubick æquation viz. Any Biquadratick æquation being proposed without a Resolvend, to break the same into two rationall quadratick æquations, whose Resolvends shall be what they will happen, and consequenty to give the series of all that shall rationally breake, and withall to breake with one or two Assayes as neare as may be rationally to any Resolvend offered' (*Correspondence*, **1**: 346–7).

(27) Mercator's Latin version of Kinckhuysen's *Algebra Ofte Stelkonst* (Haerlem, 1661) and Newton's 'Observationes' upon it are reproduced below in **3**, 1, §§1/2.

(28) In his letter to Collins on 27 September 1670 Newton spoke of 'having composed somthing pretty largely about reducing problems to an æquation' (*Correspondence*, **1**: 43) but seems merely to refer to his amplified opening to Kinckhuysen's 'Pars Tertia. Quomodo quæstio aliqua ad æquationem redigatur'. Compare **3**, 1, §2: note (110). In October 1676 Leibniz noted of this discourse that 'Collins has not yet seen it' (*Correspondence of Isaac Newton*, **2**: 236).

(29) These 'Problems for construing æquations' are reproduced in **3**, 2, §2 below.

(30) The 'De Analysi', that is: see note (22).

(31) Newton's 1671 tract on fluxions and infinite series (ULC. Add. 3960. 14/4), which will appear in the third volume.

(32) Perhaps the enumeration of cubics reproduced in **1**, 1, §3.

(33) Collins means the 1671 tract described in the fourth part: item 5 is a late insertion.

(34) A clear reference to the later draft (ULC. Dd. 9.67) of Newton's *Lectiones Opticæ* (note (14)), whose section on geometrical dioptrics (Book 1, Part 4) makes extensive use of limit-increment arguments and, on one occasion, a series expansion.

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