PART 1

THE FIRST TRACT 'DE QUADRATURA CURVARUM'

(early winter 1691–2)

INTRODUCTION

When Newton in 1685 made the Scots mathematician John Craige welcome during an extended visit to Cambridge, showing him in the privacy of his Trinity rooms a selection of his yet unpublished papers on calculus and infinite series, he could not have foreseen the tangled consequences which that innocent and generous act would have over the next half dozen years. Craige was then on the point of publishing a short tract wherein he expounded a systematic (if far from general) 'Method of determining the quadrature of figures comprehended by straight lines and curves,(1) there gathering a variety of techniques of rational algebraic quadrature and arc-rectification developed over the preceding quarter of a century by John Wallis, Nicolaus Mercator, Isaac Barrow, Leibniz and most recently David Gregory 'in his very fine treatise *On the Dimension of Figures*'.(2) Greatly dissatisfied with a newly appeared article by Walther von Tschirnhaus which sought—none too successfully—to lay down criteria for such rational quadrature in the case of conic, cubic and quartic curves,(3) and hearing that Wallis was about to make public a summary of Newton's method for squaring curves,(4) Craige had at some unknown previous date written to Cambridge for enlightenment and was now in consequence granted the rare privilege of a private view of Newton's papers as they related to his interests. Craige himself specifies that he was then shown 'manuscripts', doubtless including the 'De Analysi',(5) in which quadratures were attained by reducing

(1) Our English rendering of the title of his *Methodus Figurarum Lineis Rectis & Curvis comprehensis Quadraturas determinandi* (London, 1685).

(2) 'clarissimus Nostras D. *David Gregorius* in pulcherrimo suo Tractatu, [*Exercitatio Geometrica*] *de Dimensione Figurarum*' (*Methodus Figurarum*: 12). On Gregory's 1684 *Exercitatio* see IV: 414–16; for the other contemporary mathematicians named by Craige see his *Methodus*: 3, 12, 16, 26–7 and 30–1.

(3) 'Methodus datæ figuræ rectis lineis et curva geometrica terminatæ aut quadraturam aut impossibilitatem ejusdem quadraturæ determinandi' (*Acta Eruditorum* (October 1683): 433–7). Tschirnhaus' title is deliberately reflected, of course, in that of Craige's 1685 work, in whose appended 'Animadversio in Methodum Figuras dimetiendi, A clarissimo Quodam Germano editam in Actis Eruditorum Lipsiæ publicatis' (*Methodus Figurarum*: 38–43) he was deeply critical of Tschirnhaus' pretensions.

(4) Doubtless from the *Proposal about Printing a Treatise of Algebra, Historical and Practical*... which Wallis circulated in 1683 and where (see IV: 413, note (20)) mention is made of his plan to treat 'lastly, the method of infinite *Series*...invented by Mr. *Isaac Newton*,...which is of great use for the...squaring of *Curve-lined Figures*...'. The quadrature series which Newton had communicated to Leibniz in October 1676 did not in fact appear in the 1685 edition of Wallis' *Algebra*, and this omission occasioned the sequence of events which we summarize in sequel.

(5) See II: 206–46.

the 'root' of the integrand to an infinite series[(6)] and it was presumably on this occasion that he was permitted to make the abstract from the 1671 fluxional tract to which we have earlier referred;[(7)] but, though to Tschirnhaus' asserted criteria for quadrability he was given two counter-instances—notably that of the curve of ordinate $kx(a^2+x^2)^{\frac{1}{2}}$ which he himself was quick to publish[(8)]—and while Problem 9 of the 1671 tract[(9)] would have afforded him some insight into Newton's several techniques of exact quadrature, he was not, it would appear, allowed more than a glimpse of the 'prime' theorem for squaring curves of

(6) In speaking in his 1685 tract of his *desideratum* of a general 'Methodus...Figurarum Quadraturas determinandi [quæ] ad omnes figuras extendatur (exceptis iis quæ a Curvis transcendentibus terminantur, quas nulla hactenus vulgata Methodus comprehendit)' he remarked that 'cum figuram aliquam Quadrando necesse sit & Radicem ex æquatione affecta (& supra Quadraticas ascendente) extrahere,...unicum remedium mihi cognitum est radicem istius æquationis in seriem infinitam (juxta Methodum clarissimi viri D. Isaaci Newtoni Geometræ non minus quam Analystæ præstantissimi) resolvere, quam prælo commissam esse à clariss. Wallisio [see IV: 672, note (54)] audimus, quamque insignis ipse D. Newtonus mihi in Manuscriptis pro summâ suâ humanitate communicavit' (*Methodus Figurarum*: 26–7).

(7) See III: 354–5, note (1).

(8) In the 'Animadversio' on Tschirnhaus' 1683 'general' method of algebraic quadrature which he appended to his *Methodus Figurarum...Quadraturas determinandi* (see note (3) above); it is there (p. 43) cited—without notice of its Newtonian provenance—as 'Æquatio naturam curvæ...exprimens $z^2 = \dfrac{\overline{m^2+x^2}[\times]x^2}{p^2}$ in qua x denotat abscissas...& z ordinatas, m & p quantitates datas & [d]eterminatas'. More than thirty years afterwards Craige recalled in the 'Præfatio' to his *De Calculo Fluentium Libri Duo* (London, 1718) that 'Calcul[i] fluentium... prima Elementa, cum Juvenis essem, circa Annum 1685 excogitavi: Quo tempore *Cantabrigiæ* commoratus D. *Newtonum* rogavi, ut eadem, priusquam prælo committerentur, perlegere dignaretur: Quodque Ille pro summa sua humanitate fecit: Nec-non ut Objectiones in Schedulis meis contra D. *D*[*e*] *T*[*schirnhausio*] allatas corroboraret, duarum Figurarum Quadraturas sponte mihi obtulit; erant autem harum Curvarum Æquationes $m^2y^2 = x^4+a^2x^2$ & $m^2y^2 = x^3+ax^2$' (signature [b2^r]). In his article 'On the Early History of Infinitesimals in England' (*Philosophical Magazine* (4) **4**, No. 26 (November 1852): 321–30) Augustus De Morgan set forth (pp. 326–7) the curious argument that this passage relates to the amended 'Responsio ad Literas Domini D. T. *Lipsiam* missas Feb. 20. 1686' which Craige seven years later appended to his much reworked *Tractatus Mathematicus de Figurarum Curvilinearum Quadraturis et Locis Geometricis* (London, 1693): 55–61, and he was led consequently to speculate that 'it was this very tract of Craig's which immediately suggested to Newton the progress which the views of Leibnitz were [by then] making, and induced him to forward to Wallis [in August 1692!] the extracts from the *Quadr. Curv.* [which appeared in Wallis' *Opera*, **2**, 1693: 390–6]': both these unfounded suppositions were soon afterwards accurately demolished by H. Sloman in his augmented examination of *The Claim of Leibnitz to the Invention of the Differential Calculus* (Cambridge, 1860): 111–17.

(9) See III: 210–90, especially 236–64. We there remarked (*ibid.*: 237, note (540)) that the quadrature series which Newton communicated to Leibniz in 1676 (see next note) is but an easy generalization of the *Ordo secundus* of the 'Catalogus' of algebraic curves having exact quadrature which is there set out. No portion of the 1671 tract's 'Prob: 9' is, however, included in the 'Tractatus de Seriebus infinitis et Convergentibus' (reproduced in III: 354–72) as Gregory transcribed it from Craige's notes upon the tract after the latter's return to Scotland.

general ordinate $dz^\theta(e+fz^\eta)^\lambda$ which Newton had communicated to Leibniz in his *epistola posterior* nine years before(10) and which he now described to Craige as 'able to exhibit innumerable quadratures of this sort by means of an infinite series which shall in given circumstances, breaking off, determine the geometrical quadrature of a propounded figure'.(11)

Upon his return to Scotland shortly afterwards, Craige soon became close friends at Edinburgh with both David Gregory and the physician (and learned amateur scientist) Archibald Pitcairne, informing them of the quality of Newton's quadrature series 'which both confessed to be completely unknown to them'(12) and citing the two instances of it which he had earlier been given in Cambridge. By building upon these and accurately divining their sequence Gregory found little difficulty in recovering Newton's general theorem for himself and thereupon, careless of first inventor's rights, immediately assumed ownership of the algorithm, telling Pitcairne of his 'discovery' and writing to a common friend Colin Campbell on 2 October 1686 that 'As for my Methode of quadratur I resolve godwilling to print it shortly'.(13) When Pitcairne in turn informed Craige of this intention, the latter was at first uncertain whether Gregory's generalization was identical with the original from whose instances it had been drawn and in the summer of 1688 he wrote to Cambridge requesting Newton to send along his full theorem so that comparison between the two might be made. Newton did so in a lost letter to Craige on 19 September, and when the two expansions were set down side by side their identity, superficial

(10) See *The Correspondence of Isaac Newton*, **2**, 1960: 110–29, especially 115–17.

(11) So Craige afterwards expressed it when he later wrote in the 'Præfatio' to his *De Calculo Fluentium* (see note (8)) that Newton, having given him particular counter-instances to disprove Tschirnhaus' method of quadratures, 'me interim certiorem fecit se posse hujusmodi [Curvas] innumeras exhibere per *Seriem Infinitam*, quæ in datis conditionibus abrumpens Figuræ propositæ Quadraturam Geometricam determinaret' (signature [b2^r]).

(12) 'In Patriam postea redeunti magna mihi intercedebat familiaritas cum Eruditissimo Medico D. *Pitcairnio* & D. *D. Gregorio*; quibus significavi qualem pro Quadraturis Seriem haberet D. *Newtonus*, quam penitùs ipsis ignotam uterꝗ fatebatur' (Craige, *De Calculo Fluentium*; Præfatio: [b2^r]),

(13) *Correspondence*, **2**: 451. Having 'in y^e meane time' urged Campbell to take as 'instances' of his quadrature method both the Newtonian curves, of ordinates $\sqrt{[(m^2+x^2)\ x^2/p^2]}$ and $\sqrt{[(ax^2+x^3)/n]}$, which (see note (8)) Craige had earlier passed onto him, Gregory added that 'I could give you millions of such but I choose y^e most easy and simple. You see these at y^e first brew seem not quadrable but by a Series but my present methode does them universally.' We may well wonder if Gregory would have been half as confident without Newton's indirect tutoring: when some time before May 1694 his attempt to duplicate Newton's result in Corollary 2 to Proposition XCI of Book 1 of the *Principia* (London, $_1$1687: 220–1, reproduced on VI : 225–6) led him to seek the integral of the 'formula' $ex(b^3x-c^2x^2-d^4)^{-\frac{1}{2}}$ he was at a loss to do other than weakly jot down in a memorandum (ULE. Gregory C 60) 'Sed hæc...cum proprie ad trinomium revocetur nequit per nostram methodum quadrari. tentandum an per methodum Newtoni.'

discrepancies in notation apart, was evident(14)—and so Craige no doubt hastened to tell Gregory. But it was too late to prevent Pitcairne publishing(15) the quadrature series as Gregory's invention without mention, to Craige's barely concealed disgust,(16) of Newton's prior discovery and private circulation of it a dozen years before. Newton himself, it would seem, was not informed at once of the plagiarism—if he had been, we may imagine the anger which the incident would have roused in him so soon after his upset at the appearance of Gregory's *Exercitatio* four years earlier(17)—and the onrush of political event in England carried him off almost at once into a momentarily crowded existence, far from academic squabble, as a University member of the Convention Parliament.(18)

The affair was all but forgotten, certainly, when more than two years afterwards in the summer of 1691 the Savilian Professorship of Astronomy at

(14) 'Post aliquot verò menses narrabat mihi D. *Pitcairnius* D. *Gregorium* Seriem similiter abrumpentem invenisse. Ego nullus dubitans, quin eandem ex duabus prædictis Quadraturis ipsi à me communicatis deduxerit, per Literas D. *Newtonum* rogavi, ut Seriem suam mihi transmittere vellet, ut an eadem esset cum *Gregoriana* perspicerem: Rogatui meo annuit Vir illustrissimus per Literas 19 Sept. 1688 datas: Nec mirum si parva esset inter utramque Seriem discrepantia, cum *Gregorius*, ex duobus illis Exemplis & indicatâ a me Seriei Newtonianæ indole, suam facile deducere potuisset' (*De Calculo Fluentium* (note (8)): Præfatio: [62r/62v]).

(15) In his *Solutio Problematis de Historicis seu Inventoribus* (Edinburgh, 1688). No copy of this rare tract today exists in any of the university and college libraries in Cambridge and it would appear highly unlikely that Newton saw a copy till at least three years later when he met Gregory for the first time on a visit to London (see below).

(16) Certainly, in sending to Colin Campbell on 30 January 1688/9 a 'general Method for finding the Curvature of any given curve...copied out of Mr Newton's manuscript'—that is, Problem 5 of the 1671 tract (see III: 354, note (1))—and promising 'for your further satisfaction...Mr Newton's Series for Quadratures, which he was pleased to send me, in a letter [*viz.* of the previous 19 September; see note (14) above] not long since' (*Correspondence*, **3**, 1961: 8), he was quick to add in parenthesis to the latter the confidence that 'I must tell you b[y] the by that I saw this Series at Cambridge & acquainted Dr Pit. & Mr Greg: with it, & told them the cheife propertie of it, *sc*: that it breaks of when the figur's Quadrable. At which time they were altogether ignorant of such a series as I can let you see by letters of the Dr: written to me at Cambridge; which astonished me to find no mention made of Mr Newton by the dr: but keep this to your selfe' (*ibid.*: 8–9). In 1718, ten years after Gregory's death, Craige was prepared to speak out publicly in giving his 'historiola' of the episode when it could matter only to him and to Newton: 'Hanc...Lectoribus impertire æquum videbatur, ut soli *Newtono* Seriem illam tribuendam esse cognoscerent. Satius quidem multo fuisset, si ipse (dum vivus esset) *Gregorius* eandem Orbi Mathematico communicasset, quodqꝫ se facturum promisit per Literas dat. *Londini* 10, Oct. 1691. Me interim in iis hortatus est, ut, si quid haberem ad Memoriam ejus in hoc negotio juvandam, id ego quam citissime ad illum transmitterem; quod sine mora a me rem omnem fideliter ab initio narrante factum erat. Opus enim erat mihi facillimum, utpote qui omnes ejus & *Pitcairnii* Literas hanc materiam spectantes tum apud me habuerim, & adhuc habeo' (*De Calculo Fluentium* (note (8)): Præfatio: [b2v]).

(17) See IV: 413–17. As we observe below, when the full implications of Gregory's 'independent' discovery of the quadrature series were brought home to Newton in November 1691 his reaction then followed a similar pattern to that which it had seven years earlier.

(18) See VI: xxii–xxiv.

Oxford fell vacant and Gregory—then, as it chanced, in London on business for his Scots university[19]—applied to Newton to support his candidature for it. Though thereupon, 'having known him by his printed Mathematical performances & by discoursing wth travellers from Scotland & of late by conversing with him', Newton was quick to commend him to the electors to the Chair as 'prudent sober industrious modest & judicious & in Mathematiques a great Artist...very well skilled in Analysis & Geometry both new & old',[20] as the autumn of 1691 drew on Gregory came more and more to despair of his prospect of gaining the Professorship against the strong competition of his rivals Edmond Halley and, even more, John Caswell.[21] By early October, with still 'nothing done in the affair of Oxford', he was reluctantly making ready to journey back to Edinburgh to resume his professorial duties there when, evidently in a late move to promote his sagging chances of the Savilian Chair, he decided to

(19) Gregory wrote to Newton on 7 November 1691 to excuse himself for having 'been diverted long from writing to you as ye allowed me, by some affairs of our Coledge [at Edinburgh] that I was oblidged to manage with our Scots secretaries here' (*Correspondence*, **3**: 170). The 'affairs' had no doubt in part to do with Gregory's refusal the year before to take the newly required oath of allegiance to the English—rather than, as previously, the Scottish—throne (see *ibid.*: 171, note (1)); while his act of abjuration did not immediately cost him his professorship at Edinburgh, it left him with an uncertain prospect of continuing permanently in office there, and he was very happy to escape to the academic peace of an English university. When he first met Gregory in London in late July 1691 Newton evidently showed considerable interest in the institutions and methods of teaching in Scotland's 'Coledges', for—if only as a token of appreciation for supporting his candidature at Oxford—Gregory shortly afterwards wrote to Newton a long letter, dated '8 Agust', on the subject (reproduced in *Correspondence*, **3**: 157–62.)

(20) Or so Newton wrote from 'London. July y^{e} 27th 1691' in the draft (ULC. Add. 4005.5: 16^{r}, given in *Correspondence*, **3**: 154–5) which he retained of his recommendation; the undated letter he sent to Arthur Charlett, secretary to the Professorship Electors, presumably on the same day (now in Bodleian MS Ballard 24, and first printed by John Nichols in his *Illustrations of the Literary History of the Eighteenth Century. Consisting of Authentic Memoirs and Original Letters of Eminent Persons*, **4** (London, 1822): 49–50) only trivially emends and amplifies this. In continuation he further asserted that Gregory 'has been conversant in the best writers about Astronomy & understands that science very well. He is not only acquainted wth books but his invention in mathematical things is also good....I take him to be an ornament to his Country...'. So much praise after only a few hours' meeting with the man! At this first encounter the topic of Gregory's purportedly independent discovery of Newton's quadrature series would seem to have been broached, since in his letter on 10 October following (quoted below) Gregory assumes that its prime author is broadly familiar with 'my series for quadratures', at least in the bare form in which Pitcairne had published it three years earlier (see note (15) above).

(21) On 27 August Gregory wrote to Newton that 'Ther hath been no further discourse about the Savilian profession of Astronomie, and I beleeve that without more noise Mr Casswell will gett in ther' (*Correspondence*, **3**: 166). Three months later on 26 November, only days before his election to the Savilian Chair, he added in the same vein that 'the electors for the Astronomy professor have not yet met, but so farr as I can perceive Mr Casswell will defeat Mr Hally and me in that affair' (*ibid.*: 181).

publish a revised version of his quadrature series. On the 10th, accordingly, he wrote to Newton

to give you account that M^r Hally and others have told me that it will be fitt to illustrate my series for quadratures with some examples and publish it in the [*Philosophical*] *transactions*. I beg then Sir that ye will give me your opinion of it and allow that I transmitt it to you to be revised, and published in form of a letter to you. And since, by what you have told me, I know that ye have such a series long agoe I entreat ye'l tell me so much of the historie of it as ye think fitt I should know and publish in this paper. Sir since I am resolved to doe in this according to your opinion, I hope ye will freely allow me it on the whole matter.(22)

When Gregory's augmented account of the quadrature series, formally addressed 'to the finest of [natural] philosophers, Isaac Newton',(23) duly arrived in Cambridge a month later accompanied by his further plea that 'besides your answer to it which ye [will] allow to be publick..., ye will be pleased to advertise me if ther be any mistake in it, for I wrote it when I was not much at leisure',(24) Newton obligingly began at once to draft an equally measured response directed 'to the very celebrated David Gregory, Professor of Mathematics at Edinburgh University':(25)

I have read through your letter, and I compliment you on the series set out in it and likewise on your plan to publish it afresh,(26) illustrated with examples. The method whereby you fell upon it is very elegant and neat, and will beyond doubt prove agreeable to your readers. But when with your usual politeness you request my own series from me in return, it is necessary first to point out several things in its regard. For instance, when the eminent Mr G. W. Leibniz exchanged correspondence with me fifteen years ago through the agency of Mr Oldenburg and I took the occasion to disclose my method of infinite series, I described the present expansion in the second, dated 24 October 1676, of my letters—the two, that is, out of which the eminent geometer Dr John Wallis expounded my method of infinite series in his *Algebra* [in 1685], there testifying that he

(22) *Correspondence*, **3**: 170. In his lost reply to Gregory's plea Newton evidently acquiesced, since in a following letter on 26 November Gregory had cause to remind him that 'you wrote that you would allow [yours] to be published with [my Series which Dr pitcairn published]' (*ibid.*: 181).

(23) 'Philosophorum optimo Isaaco Neutono apud Cantabrigenses matheseos professori...'. Gregory's submitted account of his series (now ULC. Add. 3980.6) is reproduced by H. W. Turnbull in *Correspondence*, **3**: 172–6 with pertinent notes thereon.

(24) *Correspondence*, **3**: 171.

(25) 'Viro Clarissimo Davido Gregorio in Academia Edinburgensi Matheseos Prof.' In sequel we render Newton's Latin draft (ULC. Add. 3980.11) into modern English; to stress the manner in which Newton at once reshaped it to be the opening paragraph of his ensuing tract 'De quadratura Curvarum', we reproduce its original text (first printed in *Correspondence*, **3**: 181–2) in appendix to this introduction.

(26) Pitcairne had, of course, published its bare enunciation in 1688; see note (15) above.

(27) In Wallis' own words 'Mr. *Isaac Newton*, the worthy Professor of Mathematicks in *Cambridge*,... about the Year 1664, or 1665,...did with great sagacity apply himself to that

had passed over much of what I had related in them.(27) Again, when your countryman Craige was with us six years ago on an extended stay, he perused my manuscripts (as he himself owns in the book of his he then published(28)) and at that time transmitted to you my quadrature of the curve(29) about which dispute had arisen in correspondence; it was then that you attacked the quadrature of curves afresh and hit upon your series. You are, of course, aware that after his subsequent return to Scotland Craige confirmed that he had seen my series; at that period I myself had no inkling either that he was stirring up contention about the matter or that you had found the series, nor had you, as I believe, previously learnt that I had hit on a closely similar series. But, that I may now satisfy your demand, I need to quote the text of my letter. And, seeing that what I shall perhaps communicate to you at some future date on the quadrature of curves depends on a certain analytical method which I there touched upon somewhat obscurely, I shall describe also what regards this method.

After a curt and none too well informed summary of its historical origins—'Fermat devised the elements of this method and Sluse advanced them'—Newton then began to transcribe his quadrature method as he had set it down for Leibniz in October 1676, stressing that it was taken over from 'a tract on infinite series and the method of fluent quantities' which he had composed five years earlier still.(30) But almost at once he broke off his formal epistle, never to complete it. What answer he returned to Gregory's anxious letter three weeks later on 26 November inquiring if his previous one had miscarried(31) we do not

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Speculation [of Infinite Series]. This I find by Two Letters of his (which I have seen,) written to Mr. *Oldenburg*, on that Subject, (dated *June* 13, and *Octob*. 24. 1676,) full of very ingenious discoveries, and well deserving to be made more publick....He doth therein, not only give us many such Approximations fitted to particular cases; but lays down general Rules and Methods, easily applicable to cases innumerable; from whence such Infinite Series or Progressions may be deduced at pleasure; and those in great varieties for the same particular case....' (*A Treatise of Algebra, Both Historical and Practical,* London, 1685: 330). He concludes his ensuing excerpts from Newton's *epistolæ prior et posterior* for Leibniz (see *ibid.*: 330–46) with the observation that 'There is a great deal more (in these Papers) of like nature'.

(28) That is, Craige's 1685 *Methodus Figurarum*... *Quadraturas determinandi*; see note (1) above.

(29) Namely, that of Cartesian equation $pz = x\sqrt{[m^2+x^2]}$, whose exact 'square' was a crushing counter-instance to the would-be universal validity of Tschirnhaus' 1693 criterion for such quadrature; see notes (3) and (8) above.

(30) Slightly to paraphrase Newton's own remark 'De quodam...Tractatu quem annis abhinc viginti de seriebus infinitis et methodo fluentium quantitatum conscripseram.' His reference is, we need scarcely say, to his 1671 treatise on infinite series and fluxions; see III: 32–328, especially 237, note (540). In his *epistola posterior* itself he had spoken more vaguely of a 'Tractatu[s] de his seriebus [quem] conscripsi '(*Correspondence*, 2: 114).

(31) 'About three weeks agoe I wrote to you a letter wherin I gave some examples of Curves quadrable by my Series [and] entreated ye would let me know yours which was not unlike to it.... I am affraid this letter be lost and hath not come to your hands. But if it hath, I beg ye'l give me your opinion of it freely, and whither it ought to come abroad' (*Correspondence*, 3: 181). H. W. Turnbull's affirmation (*ibid.*) that Newton responded with the unfinished draft letter from which we have just now heavily quoted cannot be right, nor is there any reason for thinking that Gregory received a written reply of any sort.

know—perhaps it was given only by word of mouth when Newton met him in London at the end of December.(32) By then Gregory had been elected to the Savilian Professorship and the pressure upon him to make rapid English publication of his variant of the quadrature series was removed; it appeared only a year and a half later in the Latin edition of Wallis' *Algebra*, and without mention of Gregory's earlier move to have it printed along with Newton's equivalent prior expansion.(33)

For Newton himself that was far from the end of it. As his hitherto unpublished papers of the period reveal, the pattern of his private reaction to this new stimulus from Gregory to review his earlier researches into the quadrature of curves almost exactly parallels that of his earlier response in June 1684 to

(32) See the extracts from Gregory's 'Varia Astronomica et Philosophica 1691 Londini. 28 Dec[r]' (ULE. Gregory C85) reproduced in *Correspondence*, **3**: 191, where for instance is cited Newton's 'opinion' on 'a good design of a publick speech' for him to give as the newly appointed Savilian Professor of Astronomy at Oxford: namely one 'to shew that the most simple laws of nature are observed in the structure of a great part of the Universe, that the philosophy ought ther to begin, and that Cosmical Qualities are as much easier as they are more Universall than particular ones'. (Gregory did indeed take heed of Newton's advice in the inaugural which he delivered 'in Auditorio Astronomico Oxoniæ' on the following 21 April; see P. D. Lawrence and A. G. Molland, 'David Gregory's Inaugural Lecture at Oxford', *Notes and Records of the Royal Society of London*, **25**, 1970: 143–78, particularly 159–65). It is true that Newton recorded his exit from Trinity College in December 1691 as being not till New Year's Eve (see Joseph Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes* (London, 1850): lxxxv), but this is a readily explained slip.

(33) See John Wallis, *Opera Mathematica*, **2** (Oxford, 1693): 377–80. Gregory's revised and illustrated account of 'Methodus mea Quadraturarum' was there, much as before, presented as a formal letter, dated 'Oxoniæ 21 Julii 1692', to his fellow Savilian Professor and introduced with the non-committal observation (*ibid.*: 377–8) that 'Anno 1688 edidit *Archibaldus Pitcarnius* M.D.... Schedulam quam *Solutionem Problematis de Inventoribus* dixit, in qua methodum tradit à me inventam, cujus ope series deteguntur, quibus infinitæ numero Curvæ & spatia iis & rectis contenta nullius methodi hactenus cognitæ legibus subjecta mensurantur'. Wallis himself was well aware that Newton's *epistola posterior* gave the lie to this implied assertion of priority, and speedily wrote to Cambridge asking permission of the expansion's prime discoverer to publish the quadrature series as he had communicated it to Leibniz in October 1676; in lost letters to Wallis of 27 August and 17 September Newton not only gave his approval to such publication but also sent along, extracted from his newly written treatise 'De quadratura Curvarum', the generalization which duly appeared in print with his 1676 series the next year (*ibid.*: 390–1, reproduced in §2, Appendix 3 below). Meanwhile David Gregory during a visit to Holland in May/June 1693 (see *Correspondence*, **3**: 274, note (1)) had met Huygens and discussed with him his own variant 'Methodus...prout a D: Pitcarnio tibi tradita erat'; upon his subsequent return to Oxford he wrote to Huygens in mid-August 1693, enclosing a further summary of his method 'omnia prout a Wallisio in *Algebræ* suæ nova editione brevi in lucem mittendâ explicantur' (*Correspondence*, **3**: 275–6) and also an 'extrait'—'une grande feuille d'écriture', Huygens described it to L'Hospital on 16 June 1694 (*Œuvres*, **10**, 1905: 622) —taken from the companion 'Methodu[s] D. Newtoni qualem...illam descripsit Wallisius' which Huygens in turn eventually passed on to Leibniz in late August 1694 'puisque vous le souhaitez' (see *ibid.*: 669). The reaction of neither Huygens nor Leibniz to this attempt by Gregory to defend the independence of his discovery of the quadrature series is recorded.

Gregory's transmission of a copy of his *Exercitatio Geometrica*:[34] once provoked, his interest in the matter passed almost immediately from specifying and clarifying the historical context of their discovery to refining and reshaping their verbal exposition and then to developing their content and enlarging their applications. Straightaway, Newton in November 1691 minimally reworked the unfinished draft of his intended reply to Gregory's missive to be the opening of a short tract 'De quadratura Curvarum' wherein an expansion of the 'first Theorem' on general algebraic quadrature which he had communicated to Leibniz in 1676 was joined to a revision of his researches at the same period into the 'Quadrature of all curves whose æquations consist of but three termes'[35] to form a preliminary scheme[36] for determining the 'simplest figures with which exhibited curves can be compared'. And within days this was enlarged to be a developed treatise[37] on the same broad theme of the quadrature of algebraic curves, but now further embracing general methods for resolving fluxional equations into equivalent fluent ones either by exact techniques or, in default, by expanding their roots as infinite series, and thereafter applying these methods to solving problems in the curvilinear motion of geometrical points and in measuring the central forces induced in given dynamical orbits.

By late December Newton's new tract was already far enough advanced to completion for him to tell Gregory at the beginning of a three-week visit he made to London that he planned to publish it as the second of 'two parts of Geometry, the first the Geometry of the Ancients, the second of Quadratures'.[38] In the course of the same visit he likewise informed Fatio de Duillier more vaguely that his 'treatise of curves would soon see the light'.[39] A month afterwards, having

(34) Again see IV: 413–17, and compare note (17) above.

(35) See III: 373–82; compare §1: note (33) below.

(36) Reproduced in §1 following.

(37) The revised 'De quadratura Curvarum' given in §2 below.

(38) As Gregory noted down on 28 December among his 'Varia Astronomica et Philosophica' (see *Correspondence*, **3**: 191 and compare note (32) above.) The surviving manuscript drafts in which Newton afterwards variously implemented this plan for a dual-purpose 'Geometriæ Libri Duo' are reproduced in Part 2 following; see especially **2**, 3, §§1/2.

(39) Or so Huygens repeated it back to Fatio in his letter of 29 February 1692 (N.S.) with the words 'Son Traité des Lignes a ce que mon frere [Constantijn] me mande (qui le tenoit de vous...) devoit bientost voir le jour' (*Notes and Records of the Royal Society*, **6**, 1949: 159 = *Correspondence*, **3**: 196) and commenting thereon that 'j[e l]'attens avec impatience, esperant d'y apprendre toutes ces belles choses dont vous faites mention.... J'ay depuis peu...songé à cette affaire des quadratures,...et j'ay encore remarqué quelques regles et tentatives pour trouver la quadrature quand l'Equation d'une ligne quadrable est donnée, mais cela ne va pas bien loin lors que les Equations sont un peu composées, et je voudrois bien sçavoir si Monsieur Newton a des regles generales pour cela, et s'il peut connoitre quand la quadrature est impossible, ou quand elle depend de celles de l'Hyperbole ou du Cercle....Au reste je n'entens pas ce que signifie la fluxion de la fluxion...' (*ibid.*). How serious Newton's own intention of publishing his 'De quadratura' was at this time is difficult to determine, and any