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ISAAC NEWTON
VOLUME VI
1684-1691

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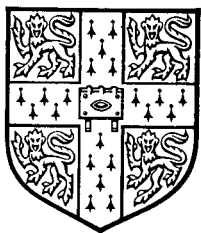
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THE
MATHEMATICAL PAPERS OF
ISAAC NEWTON
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EDITED BY
D. T. WHITESIDE

WITH THE ASSISTANCE IN PUBLICATION OF
M. A. HOSKIN AND A. PRAG



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*TO I. BERNARD COHEN
OUR LEARNED GUIDE TO THE BY-WAYS
OF THE 'PRINCIPIA'*

PREFACE

Even though Newton himself, half a century afterwards, preferred to look to the two ‘plague years’ of 1665 and 1666 as the ‘prime’ of his ‘age for invention’ when he ‘minded Mathematicks & Philosophy more then at any time since’,* the dozen and a half months from August 1684, when Edmond Halley first travelled to Cambridge to seek his opinion on the currently vexing question of how dynamically to determine the closed orbits of the planets round the sun, have (so it seems to us) an overriding claim to be regarded as the most deeply fruitful *annus mirabilis* of Newton’s life. Over that year and a half he was encouraged to expand his immediate confident answer to Halley, that the planetary ovals are exact ellipses having the sun at a focus, into a developed treatise ‘On the motion of bodies’ and thereafter further to augment its scope to be the magisterial *Philosophiæ Naturalis Principia Mathematica* which made its public bow in 1687. While it manifestly transcends the purpose of the present edition systematically to review the full content of this *chef d’œuvre* of classical theoretical physics, it would be equally intolerable for us here to ignore this supreme exhibition of Newton’s scientific genius. It is, of course, a truism that his expert ability to give appropriate and effective formal expression whether geometrical or (more rarely) algebraic to the various astronomical and physical problems which he therein came to confront, and the power and adequacy of his available mathematical techniques exactly or approximately to resolve them once thus precisely defined, were prime limiting factors on the success with which he was able to fulfil his central aim of reducing individual empirical phenomena to be deducible instances of a handful of guiding kinematical and dynamical ‘principles of natural philosophy’. In imitation of the phrase taken up by the present Master of Newton’s Cambridge college—and not wholly to depart from its context of human frailty—we might with good reason call the *Principia* the art of what was then possible. Merely to affirm as much *tout court* is in no way to effect the detailed enodation, point by point and proposition by proposition, of its arguments by which alone its mathematical substructure may be probed and analysed, and which in selected portions of its content we attempt in the present volume. Since here we run parallel to that crowded avenue of present-day scholarly research which seeks to establish and evaluate

* See ULC. Add. 3968.41: 85^r; a more connected quotation of this familiar excerpt from a draft letter of Newton’s to Pierre Desmaizeaux in about the summer of 1718 has already been made on 1: 152. The justice of his not everywhere documentable claims to priority of discovery in mathematics, mechanics and optics is examined more fully in D. T. Whiteside, ‘Newton’s Marvellous Year: 1666 and all that’, *Notes and Records of the Royal Society of London*, 21, 1966: 32–41.

its verbal niceties and physical connotations, and more narrowly because the variants in its published and publicly circulated versions are now accurately recorded in the ‘variorum’ edition of its text by Professors Alexandre Koyré and I. B. Cohen (which appeared just too late, unfortunately, to permit detailed reference here to its pages), our own attention is fixed squarely on the extant preliminary manuscripts, beginning with Newton’s initial tract ‘De motu Corporum’ of autumn 1684, and culminating with the preserved portion of the refined ‘De motu Corporum Liber primus’ which went to make up the *Principia*’s first book and the opening propositions of its second; the printed text itself we employ only minimally as an auxiliary to fill in small gaps in their deductive sequence and, on rare occasion, to complement their content. The opportunity has also been taken to deal more fully with some individual topics—namely, the resistance of surfaces of revolution to uniform translation along their axes, the construction of parabolic cometary paths from terrestrial sightings, and the dynamical derivation of the mean annual advance of lunar apogee—which are given summary or considerably diverse discussion in the published *Principia*. And a concluding section explores a number of provisional revisions of the opening pages of its first book, which Newton had it in mind, in or shortly after 1690, to incorporate in the *editio princeps*. As ever, the reader will judge best how far our severely mathematical commentary serves to penetrate below the formal façade which Newton presents.

For permitting reproduction of the manuscripts in their care I am once more chiefly indebted to the Librarian and Syndics of the University Library, Cambridge, but also, in the case of short individual items, to their equivalents in Trinity College and Kings College, Cambridge, the Bodleian and Corpus Christi College, Oxford, and the Royal Society, London; and in one instance I owe thanks to a private owner whose *carte blanche* to publish documents in his possession we have freely taken advantage of in earlier volumes. Let me also express my appreciation of the courtesy and efficiency of the staff of what, with the demise of the old Anderson Room there, has now divided to be the Manuscript and Rare Book Rooms at the University Library. The financial support without which the several years of preliminary research encapsulated in this volume could never have been possible has continued to be furnished by the Sloan Foundation, the Leverhulme Trust and the Master and Fellows of Trinity College, Cambridge. In this present period of economic stringency their generously renewed subsidy is especially welcome. On a sad note, that grand old man of British science, Sir Harold Hartley, is no longer with us. Let me add one more tribute to his memory by stressing that the quality and strength of the present edition (such as it may be) reflect the professional advice, worldly wisdom and affectionate (if sometimes stern) encouragement which he offered unstintingly over the dozen years before his death.

Preface

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To my *confrères*, Michael Hoskin and Adolf Prag, a very personal note of thanks for again helping me to guide a complex volume through the press. As before, Dr Hoskin has helped in preparing the text-figures, while the eagle eye of Mr Prag has spotted many a slip in my editorial phrasings, and to him also is due the credit for the terminal index of names.

Lastly, but never least, to the Syndics of Cambridge University Press and to the University Printer must go my collective appreciation of the efforts of all those—editors, draughtsmen, printers and other production staff—who have jointly laboured (amid the exigency and despair ensuing upon a three-day working week) to create out of a clumsily penned submitted manuscript a most elegant example of the typographer’s art.

D.T.W.
Easter Day, 1974

EDITORIAL NOTE

A brief reminder that this volume narrowly follows the style and conventions of previous volumes (on which see especially pages x–xiv of the first), and that faithfulness to the spirit of the manuscript originals here reproduced is our chief concern, in regard not merely to their verbal text but also its illustrative figures. Where, however, the text is not autograph but Humphrey Newton’s secretarial transcript, we have minimally standardised its accidence and punctuation to Newton’s preferred norm; and in reproducing Newton’s own sprawling numerical calculations we have taken some small liberties in recasting their spatial arrangement to fit the confines of the printed page and to point their sense. (Comparison of the photo-facsimiles in Plates I and III with our accompanying editorial selection and rearrangement of their detail will, we trust, allay any fear on the reader’s part that we have there gone too far in deviating from the strict letter of the original text as it is preserved.) We should add a caution to the unwary reader that many of the figures once appended to, or intended to be set with, the manuscript are now there lacking, and that these are perforce here restored: every such instance is specified in a pertinent footnote. As before, we have been free in smoothing out trivial slips of Newton’s pen and in rounding out the sparseness of his original where we have deemed this necessary (usually in the case of groups of computations devoid of verbal interlinking) by appropriate editorial correction and interpolation within square brackets. Within the bounds of fluency and modern idiom we have deliberately kept our English translations (facing the Latin text or, in footnotes, following it in parenthesis) closely literal, designing them to be an auxiliary tool and not an independent voice of Newton’s meaning. Once more, for forwards and backwards reference within the present volume we employ the formula typified in ‘3, §2, Appendix 2. 3’ (thereby understanding ‘[Section] 3, [Subsection] 2, Appendix 2. [Division] 3’); reference to preceding volumes is by the similar code ‘III: 244–54’ (understand ‘[Volume] III: [pages] 244–54’). Throughout, two thick vertical rules in the left-hand margin denote that the text alongside has been cancelled by Newton in the original. Finally, a number of non-standard mathematical symbols used by him in the manuscript are explained in footnote at the relevant places and also (where pertinent) keyed to analogous occurrences in previous volumes.

GENERAL INTRODUCTION

In our two previous volumes we took leave of Newton in the summer of 1684, hard at work in Cambridge, on a variety of mathematical—and also, indeed, chemical—projects. Edmond Halley’s visit to him in August, we need scarcely repeat, abruptly changed the focus of those researches. The open challenge from Christopher Wren theoretically to define the paths of the solar planets which Halley then communicated to Newton re-kindled in him a devouring flame of interest in the general dynamical motion of bodies which had more than five years before flickered fitfully into life during his correspondence with Robert Hooke and since lain dormant. Before, however, we enter upon the mathematical detail of the several successive and increasingly bulky treatises ‘De motu Corporum’ which Newton drafted over the next two years and ultimately subsumed into his *Principia* in 1687 we may, as in previous volumes, momentarily turn aside to consider the man himself as he entered the early forties of his life.

While in the privacy of his Trinity rooms he laboured untiringly to compose what was to prove his scientific masterpiece, the outward pattern of Newton’s day-to-day life changed but little. Except for two short absences (away in his native Lincolnshire the first time) at Easter and in mid-June of 1685⁽¹⁾ he passed the whole thirty months up to March 1687 in Cambridge, carrying out his minimal professorial duty of lecturing weekly—on the substance of his new findings in celestial mechanics, it would appear⁽²⁾—to the few who bothered to

(1) In his list of Newton’s exits and redits from and back to Trinity College (*Correspondence of Sir Isaac Newton and Professor Cotes*, London, 1850: lxxxv) Joseph Edleston records absences from Cambridge between 27 March and 11 April and again from 11 to 20 June; on the preceding 23 February Newton had written to Francis Aston that ‘now I am to goe into Lincolnshire for a Month or six weeks’ (*Correspondence of Isaac Newton*, 2, 1960: 415). No other departures occur till an exit on 25 March 1687; though no corresponding redit is found, Newton’s weekly buttery bills of the period (which Edleston lists on his ensuing p. lxxxvii) establish that, except for a few days in the last week in April, he was not to return to college—from promoting the University’s statutory rights before Judge Jeffreys in London, as we shall see—till late May. In 1688 he was briefly away from Cambridge, in Lincolnshire we assume, between 30 March and 25 April and again from 22 June to 17 July. After his election to be one of the University Members of Parliament in January 1689, his buttery bills confirm that he was from then on continuously out of residence, other than for a couple of days in the middle of the following October, till a redit on 4 February 1690 marked his return after the Convention Parliament was prorogued. Further brief absences in 1690 from 10 March to 12 April and from 22 June to 2 July—once more in Lincolnshire, we may guess—preluded a year’s unbroken return to academic routine. A similar pattern of long stays in Cambridge, punctuated only rarely by short excursions to London or his native Lincolnshire, prevailed during his remaining five year’s residence there.

(2) See I. B. Cohen, *Introduction to Newton’s ‘Principia’* (Cambridge, 1971): 84, 302, 307–9; and compare 1, §2: note (1) below.

come to listen to him at the university schools,⁽³⁾ and fulfilling such minor college responsibilities as voting in elections, but above all keeping a paternal eye on the new Trinity library which was then being built to Wren's design.⁽⁴⁾ Humphrey Newton, his secretary and constant attendant during the five years from spring 1684 till Newton himself went off to London in January 1689 on a prolonged stay,⁽⁵⁾ long afterwards set down on paper for John Conduitt his none too coherent reminiscences of that time, seeking (as he put it) to 'return as perfect & as faithful [an] Account of [his] Transactions as possibly

(3) We have already (II: xix) quoted Humphrey Newton's observations to John Conduitt in 1728 that when Newton at this time 'read in y^e Schools, as being Lucasianus Professor, . . . so few went to hear Him, & fewer y^t understood him, y^t oftines he did in a manner, for want of Hearers, read to y^e Walls' and that he then 'usually staid about half an hour, [though] when he had no Audit^{rs} he comonly return'd in a 4th part of that time or less' (King's College, Cambridge. Keynes MS 135; on which see note (6) following). As a comment on the accuracy of this testimony we may remark that the walking distance from Newton's rooms in the Great Court of Trinity to the University—now the 'Old'—Schools is only about one-third of a mile. Whiston's still later statement that he had in his student days heard Newton read 'one or two of [his *Principia*] lectures in the publick Schools, though I understood them not at all at that time' (*Memoirs of the Life and Writings of Mr. William Whiston. . . Written by himself* (London, 1749): 36) must refer to no earlier than 'the Middle of the Year 1686' when he was 'admitted of *Clare-Hall* . . . where I earnestly pursued my Studies, and particularly the Mathematicks, eight Hours in a Day' (*ibid.*: 19); subsequently, after his ordination at Lichfield in September 1693, Whiston returned to Clare College 'and went on with my own Studies there, particularly the Mathematicks, and the *Cartesian* Philosophy; which was alone in Vogue with us at that Time. But it was not long before I, with immense Pains, but no Assistance, set myself with the utmost Zeal to the Study of Sir *Isaac Newton's* wonderful Discoveries in his *Philosophiæ Naturalis Principia Mathematica* . . . Being indeed greatly excited thereto by a Paper of Dr. [David] Gregory's when he was Professor in *Scotland*; wherein he had given the most prodigious Commendations to that Work, . . . and had already caused several of his Scholars to keep *Acts*, as we call them, upon several Branches of the *Newtonian* Philosophy; while we at *Cambridge*, poor Wretches, were ignominiously studying the fictitious Hypotheses of the *Cartesian*, which Sir *Isaac Newton* had also himself done formerly, as I have heard him say' (*ibid.*: 35–6). (While after his election to the Savilian Chair of Astronomy at Oxford in late 1691 Gregory did lecture extensively on the *Principia*—see P. D. Lawrence and A. G. Molland, 'David Gregory's Inaugural Lecture [21 April 1692] at Oxford', *Notes and Records of the Royal Society*, 25, 1970: 143–78, especially 157–8 where the yet unpublished manuscript (Aberdeen, University Library 2206/8) of his professional lectures during 1692–7 is briefly described—, we would add that those of his earlier Edinburgh theses and lectures on mechanics and astronomy which survive are wholly elementary in character. It would be an understandable slip for Whiston, writing of an event more than half a century old, to have mistakenly put 'Scotland' for 'Oxford'.) Alternatively, his reference may have confusedly been to Davids's brother, James, who at Edinburgh in 1690 published a list of M.A. theses mostly Newtonian in theme.

(4) Edleston (*Correspondence* (note (1)): lviii, note (90)) prints an entry in the College Account Book of the building of the library: 'May 28, 1687. P^d . . . for erecting a scaffold for M^r Newton to measure the fret work of the staircase: 4^s 6^d.'

(5) 'In y^e last year of K. Charles 2^d', he told Conduitt, 'S^r Isaac was pleas'd through y^e Mediation of D^r Walker, (then Schoolmaster at Grantham) to send for me up to Cambridge, of Whom I had the Opportunity as well Hon^r to wait of, for about 5 years.'

does at this Time come to my Memory'.⁽⁶⁾ These half-forgotten impressions of a man whose intellectual depth was far beyond his own meagre capacity to comprehend it do yet, for all their essential superficiality and excess of maudlin eulogy, have a freshness and immediacy undiluted by stale interpretation which will, with some slight rearrangement of their sequence into a more logical order, bear extensive quotation:

His Carriage then was very meek, sedate & comely; I cannot say, I ever saw him laugh, but once⁽⁷⁾. . . I never knew him take any Recreation or Pastime, either in Riding out to take y^e Air, Walking, Bowling or any other Exercise whatever, thinking all Hours lost, y^t was not spent in his Study, to w^{ch} he kept so close. . . so intent, so serious upon [them], y^t he eat very sparingly, nay, oft times he has forgot to eat at all, so y^t going into his Chamber I have found his Mess untouch'd, of w^{ch} when I have reminded him, [he] would reply, Have I; & then making to y^e Table, would eat a bit or two standing, for I cannot say, I ever saw Him sit at Table by himself. . .

I cannot say I ever saw him drink, either Wine Ale or Bear, excepting [at] Meals, & then but very sparingly. He very rarely went to Dine in y^e Hall unless upon some Publick Dayes, & then, if He has not been minded, would go very carelessly, wth Shooes down at Heels, Stockins unty'd, Surplice on, & his Head scarcely comb'd. . . At some seldom Times when he design'd to dine in y^e Hall [he] would turn to y^e left hand,⁽⁸⁾ & go out into y^e street, where making a Stop, when he found his mistake, [he] would hastily turn back & then sometimes instead of going into y^e Hall, would return to his Chamber again. . .

In his Chamber he walk'd. . . very much. . . He very seldom sat by y^e Fire. . ., excepting y^e long frosty winter, w^{ch} made him creep to it against his will. I can't say I

(6) This account, delivered by Humphrey in two letters to Conduitt on 17 January and 14 February 1727/8 now in King's College, Cambridge (Keynes MS 135), was first printed—in an extensively modernised transcription which we do not here follow—by David Brewster in his *Memoirs of the Life, Writings and Discoveries of Sir Isaac Newton* (Edinburgh, 1855), 2: 91–8. To this, his opening sentence, Humphrey adjoined apologetically that 'Had I had y^e least Thought of gratifying after this Manner, S^r Is.'s Friend, I should have taken a much stricter view of his Life & Actions.'

(7) As William Stukeley had six months earlier recounted the event to Richard Mead in his letter of 15 July 1727 (now King's College, Cambridge. Keynes MS 136B) 'Twas upon occasion of asking a friend to whom he had lent Euclid to read, what progress he had made in that author, & how he liked him? he answered by desiring to know what use & benefit in life that study would be to him? upon which S^r Isaac was very merry.' For what it is worth, this put Humphrey 'in mind of y^e Ephesian Phylosopher, who laugh'd only once in his Life Time, to see an Ass eating Thistles, when Plenty of Grass was by.' At any rate Newton clearly did not encourage him to play the buffoon.

(8) Having gone down the staircase from his rooms, that is, and instead of then going virtually straight ahead across the Great Court at Trinity. The often reproduced general view of Trinity College at this time which David Loggan published as one of the engravings in his *Cantabrigia Illustrata* (London, 1690) will clarify the topography; see also Brewster's *Memoirs* (note (6)), 2: 85–6, and more generally G. N. Watson's essay on 'Trinity College in the time of Newton' (in (ed.) W. J. Greenstreet, *Isaac Newton 1642–1727*, London, 1927: 144–7).

ever saw him wear a Night-Gown, but his wearing Cloathes, that he put off at Night, at Night, do I say, yea rather towards y^e Morning,⁽⁹⁾ he put on again at his Rising. He never slept in y^e Day time, y^t I ever perceiv'd. I believe he grudg'd y^t short Time he spent in eating & sleeping. . . . In a Morning he seem'd to be as much refresh'd with his few hours Sleep, as though he had taken a whole Night's Rest. He kept neither Dog nor Cat⁽¹⁰⁾ in his Chamber, w^{ch} made well for y^e old Woman, his Bedmaker, . . . for in a Morning she has sometimes found both Dinner & Supper scarcely tasted of, w^{ch} [she] has very pleasantly & mumpingly gone away with. . . . In Winter Time he was a Lover of Apples, and sometimes at Night would eat a smal roasted Quince. . . . He was only once disorder'd with Pains at y^e Stomach, w^{ch} confin'd Him for Days to his Bed, w^{ch} he bare with a great deal of Patience & Magnanimity, seemingly indifferent either to live or dye. . . . He has given y^e Porter many a Shilling, not for leting him at y^e Gates at unseasonable Hours, for y^t he abhor'd, [I] never knowing him out of his Chamber at such Times. . . .

Near y^e [east end of y^e Chappel] was his Garden, w^{ch} was kept in Order by a Gardiner. I scarcely ever saw him do anything (as pruning &c) at it himself. . . . in his Garden, w^{ch} was never out of Order, . . . he would, at some seldom Times, take a short Walk or two, not enduring to see a Weed in it. . . . When he has some Times taken a turn or two [he] has made a sudden Stand, turn'd himself about, run up y^e Stairs⁽¹¹⁾ [&] like another A[r]chimides, with an *εὐρηκα* fall to write on his Desk standing, without giving himself the Leasure to draw a Chair to sit down on. . . .

On y^e left end⁽¹²⁾ of the Garden was his Elaboratory. . . , where he. . . employ'd himself in, with a great deal of Satisfaction & Delight. . . . [It] was well furnished with chymical Materials, as Bodyes, Receivers, Heads, Crucibles &c, w^{ch} was made very little use of, y^e Crucibles excepted, in w^{ch} he fused his Metals. . . . His Brick Furnaces *p[ro] re natá*, he made & alter'd himself, wthout troubling a Brick-layer. . . . At Spring or Fall of y^e Leaf. . . he used to imploy about 6 weeks in his Elaboratory, the Fire scarcely going out either Night or Day, he siting up one Night, as I did another, till he had

(9) Humphrey elsewhere observed more precisely that 'He very rarely went to Bed till 2 or 3 of y^e Clock, sometimes not till 5 or 6, lying about 4 or 5 hours, especially at Spring or Fall of y^e Leaf. . . '.

(10) It was not always so. In an unpublished jotting in one of his small green notebooks John Conduitt recorded at about this time that 'His cat at the University grew very fat' (King's College, Cambridge. Keynes MS 130.6²: 3^v).

(11) Those (demolished in the early nineteenth century) which led up from his garden to a wooden verandah outside his bedroom; again see David Loggan's contemporary view of Trinity (note (8)), in whose bottom right corner it is plainly visible. Humphrey elsewhere remarks that 'His Telescope [his reflector, presumably], w^{ch} was at y^t Time, as near as I could guess, . . . near a foot long. . . he plac'd at y^e head of y^e Stairs, going down into y^e Garden, buting towards y^e East. What Observations he might make, I know not'.

(12) Looking eastwards from Newton's room, that is, and so to the north. David Loggan shows the 'Elaboratory' as a small shed tucked away right up against the Chapel end, only yards—and the thickness of a brick wall—away from its altar. Newton's smoking chimney must have been in clear view of all who attended the Chapel services, and his chemical experiments common gossip in the College!

General Introduction

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finished his chymical Experiments, in y^e Performances of w^{ch} he was y^e most accurate, strict [&] exact...⁽¹³⁾

As for his private Prayers I can say nothing of them, I am apt to believe his intense Studyes depriv'd him of y^e better Part.... He very seldom went to y^e [college] Chappel, y^t being y^e Time he chiefly took his Repose; And as for y^e Afternoons, his earnest & indefatigable Studyes retain'd Him, so y^t he scarcely knew y^e House of Prayers. Very frequently on Sundays he went to [Great] S^t Mary's Church, especially in y^e fore Noons....

S^r Is. at y^t Time had no Pupils, nor any Chamber Fellow, for y^t, I presume to think, would not in y^e least have been agreeable to his Studies....

This pen-picture of a quirky, retiring scholar as neurotically precise in his private manner as he was utterly careless of his outward appearance, so deeply engrossed in elaborating the intricacies of his thought and in exploring the fiery secrets of his alchemist's furnace as to be all but oblivious of the bustle of life around him, is of course both overdrawn and incomplete. Newton's occasional petulances, flashing angers and outbursts of puritanical sternness are doubtless here unconsciously suppressed in favour of his more endearing and graphic eccentricities.⁽¹⁴⁾ His social and intellectual isolation at this period is certainly overstated. While Humphrey goes on to report that Newton 'always kept...to his Studyes...so close, y^t he seldom left his Chamber, unless at Term Time, when he read in y^e Schools', he also adds that 'When invited to a Treat,...[he] used to return it very handsomely, freely, & wth much Satisfaction to Himself.... At [which] seldom Entertainm^{ts} y^e Masters of Colledges were chiefly his Guests', and also that 'Foreigners [from outside Cambridge] He received wth a great deal of Freedom, Candour & Respect'. Of the '2 or 3 Persons' who were regular visitors to Newton's Trinity rooms Humphrey identifies 'Mr Ellis of Keys [Caius], Mr Laughton of Trinity, &

(13) Humphrey added: 'Nothing extraordinary, as I can remember, happen'd in making his Experiments, w^{ch} if there did, He was of so sedate & even Temper, y^t I could not in y^e least discern it... He would sometimes, tho' very seldom, look into an old mouldy Book, w^{ch} lay in his Elaboratory, I think it was titled, *Agricola de metallis*, the transmuting of metals being his chief Design, for w^{ch} purpose Antimony was a great ingredient'. Since Newton's library copy (now Trinity NQ.11.4) of the 1621 edition of Agricola's *De Re Metallica* is unstained, this is clearly not the volume which Humphrey saw Newton using. As we have already observed (v: xiv), since Newton's record of these chemical experiments in the notebook ULC. Add. 3975 pass immediately from an entry on 'Friday May 23 [1684]' (p. 149) to one overleaf on 'Apr. 26 1686' (p. 150) he would seem to have intermitted them during the crucial twenty months from August 1684 in which he was pre-occupied in elaborating the principles of dynamical motion under a central force and applying them to explain the physical system of the world.

(14) Such mentions by Humphrey as that of 'His Behaviour mild & meek, without Anger, Peevishness or Passion, so free from y^t, that you might take him for a Stoick' are, of course, so closely tailored to his hagiographic purpose of setting down for Conduitt 'things... worthy to be inserted into y^e Life of so great, so good, & so illustrious a Person as S^r Isaac' as to be meaningless.

Mr Vigani, a Chymist, in whose Company he took much Delight and Pleasure at an evening, when they come to wait upon Him'. John Ellis, now Master of Caius College, had a couple of years earlier been tutor to Henry Wharton, the only Cambridge student we know ever to have had access to Newton's private mathematical writings;⁽¹⁵⁾ John Laughton, 'ye Library Keeper of Trin. Coll.', in particular 'resorted much to his Chamber', Humphrey tells us; while John Francis Vigani, a private lecturer in chemistry, was an especial 'favourite' of Newton's 'till he told a loose story about a Nun & then S^r I. left off all confidence with him', as Catherine Conduitt later recorded.⁽¹⁶⁾ To this trio of Newton's close friends at Cambridge in the middle 1680's we should continue to add his Trinity colleague Humphrey Babington, whom Humphrey Newton elsewhere lists among 'others of his Acquaintance'.⁽¹⁷⁾ Who the other 'Masters of Colledges' were we may only guess, but it is significant that Newton's only contact at this time with undergraduates and others of less senior status in the University came from the height and distance of a professorial podium⁽¹⁸⁾ on those occasions when anyone at all turned up to hear him lecture. The visiting 'Foreigners' hospitably received by him included Halley, of course, on his several visits to Cambridge and also, by his own later testimony, the Scotsman John Craige.⁽¹⁹⁾

(15) See iv: 11, note (30) and 188–9, note (1).

(16) So her husband John initially wrote it down in one of his jotters (King's College, Cambridge. Keynes MS 130. 6²). In his later amplification of this anecdote (Keynes MS 130. 7, first printed by Brewster in his *Memoirs* (note (6)), 2: 92–3, note 3) Conduitt describes Newton as being 'very intimate' with Vigani and taking 'great pleasure in discoursing on Chymistry' with him. The little else known of Vigani's activities in Cambridge at this time is summarised by L. J. M. Coleby in *Annals of Science*, 8, 1952: 46–60, especially 46–8. Not till 1703 was he accorded the (honorary) title of University Professor of Chemistry.

(17) As we have seen (i: 8, note (21)) Babington was a close friend of Newton's mother and had perhaps been chiefly instrumental in securing him his sizar's place at Trinity in 1661: 'He was', William Stukeley wrote to Mead on 15 July 1727 (see note (7) above), 'own uncle to M^{rs} Vincent [Miss Storey], i.e. brother to her mother M^r Clarks wife where S^r Is: lodgd [as a day boy at Grantham Grammar School in the 1650's], & that seems to be the reason why he went to this college. The D^r is said to have had a particular kindness for him...'.
(18) By the additions to the statutes governing the Lucasian Professorship made under the royal seal on 18 January 1664, the holder of the Chair was (under penalty of deprivation) expressly forbidden to tutor any pupils other than fellow-commoners or to hold any teaching or administrative office, other than the Professorship and the College Fellowship tied to it, in the University. (See iii: xxvii, where the Latin text of the prohibition is given.) At this period only a 'M^r Robt. Sacheverell', who is otherwise unknown to us, entered (on 16 September 1687) fellow-commoner under Newton; see Edleston, *Correspondence* (note (1)): xlv, note (16).

(19) See his *De Calculo Fluentium Libri Duo* (London, 1718); Præfatio: [b2^r]. In the course of his few days' stay, in about the early summer of 1685, Newton showed Craige both early drafts of his *Principia* (compare Cohen's *Introduction* (note (2)): 204) and the manuscript of his 1671 tract on infinite series and fluxions (see iii: 354, note (1)). We will return to discuss the significance of this visit in the next volume when we reproduce the yet unpublished original text of Newton's treatise 'De quadratura Curvarum'.

We must be disappointed that Humphrey Newton says so little of his namesake's intellectual pursuits and academic activities as they must have, in some measure at least, been known to him. His sole qualification of the *Principia* 'w^{ch}...by his Order, I copied out before it went to y^e Press' as a 'stupendous Work' is as shallow and imperceptive as his more general observation that Newton's 'Thoughts were his Books, [&] tho' he had a large Study [he] seldom consulted with them' or his uncomprehending judgement on his 'chymical Experiments' that 'What his Aim might be, I was not able to penetrate into, but his Pains, his Diligence [therein]...made me think, he aim'd at something beyond y^e Reach of humane Art & Industry'. In filling out this disappointingly meagre reaction of his secretary to the genius and vigour of a man then at the height of his mathematical and scientific maturity Newton's own public writings at this time add not a great deal. Over the three years from August 1684 in which he was preoccupied with composing the several drafts of his *Principia* and then helping Halley to see its final version through the press he understandably wrote few letters not directly related to that end, and of these even fewer have survived: for two crucial periods during this interval, namely from late May to mid-September 1685 and again from mid-October 1685 to late May 1686, the record of his extant correspondence is completely blank. In brief exchanges during the month from 17 December 1684 and in a reprise over the three weeks from 19 September 1685⁽²⁰⁾ John Flamsteed put his Greenwich observations and astronomical expertise freely at Newton's disposal in furnishing him with a variety of requested information regarding corrected sightings of the comet of 1680-1⁽²¹⁾ and his computed orbital elements and periods of Jupiter and Saturn and their satellites; for his hard work (much of whose detail was incorporated in the *Principia*'s first edition) Flamsteed was rewarded with a curt acknowledgement from Newton that 'Your observations of y^e Comet, being so exact...will save me a great deale of pains. I shall have no need to give you further trouble at present, but after a while I believe I may have occasion to beg your further assistance'.⁽²²⁾ A long sequence of eighteen letters exchanged with Halley between 22 May 1686 and 5 July 1687 (together with at least two more which are no longer

(20) See *The Correspondence of Isaac Newton*, 2: 403-15/419-30.

(21) Compare v: 524-5, notes (1) and (3); and see also notes (1) and (3) on page 82 below.

(22) Newton to Flamsteed, 14 October 1685 (*Correspondence*, 2: 430). When eleven months afterwards Newton gave 'kind enterteinm^t' in Cambridge to Flamsteed's 'friend M^r Philips' even though, as the latter acknowledged on 9 September 1686, 'hee has beene with me but three weekes and has onely learnt some few propositions of Euclid with his plaine trigonometry in this time' (*ibid.*: 449), Newton refused to be drawn anew on the topics of 'Cassinis new Planets about Saturn' and the possible oblateness of Jupiter's sphere with which Flamsteed had primed Philips (see *ibid.*: 448) despite Flamsteed's further attempt (*ibid.*: 449-50) to provoke from him a 'better...concept' regarding the cause of Jupiter's apparent ellipticity.

extant) is narrowly concerned with resolving the various problems attendant on editing and printing so complicated and technical book as the *Principia*, and with responding to Robert Hooke's claim to prior discovery of the principle of universal inverse-square gravitation and also evaluating John Wallis' independent researches in the theory of resisted projectile motion. While the hot anger of Newton's verbal outburst against Hooke on 20 June 1686—'is not this very fine? Mathematicians that find out, settle & do all the business must content themselves with being nothing but dry calculators & drudges & another that does nothing but pretend & grasp at all things must carry away all the invention . . .'⁽²³⁾—says more about his inner passion and forcefulness than all Humphrey Newton's subsequent platitudes, and though his genuine anxiety lest Wallis pre-empt the credit for pioneering the study of resisted motion⁽²⁴⁾ is in a subtler way equally revealing of his deep-seated but long repressed desire for public recognition of his genius and originality, since pertinent quotation from these letters is made widely in editorial footnote in our main text we need not here linger to summarise their detail.⁽²⁵⁾

Somewhat surprisingly, Halley himself is, for all the closeness of their working relationship over many months, always addressed by Newton in a tone of distant formality which belies any overt personal affection for him, and this may explain why, once the *Principia* was published, their association ceased for several years thereafter. For Edward Paget, who flits as a (now) obscure and tantalising shadow though Newton's correspondence in the middle 1680's (and whom he had earlier on 3 April 1682 commended to Flamsteed⁽²⁶⁾ as a fit

(23) *Correspondence*, 2: 438. On Newton's reaction to Hooke's claim to 'ye duplicate proportion' of terrestrial—and indeed universal—gravitation see more generally note (46) on page 15 below, and compare note (61) on pages 20–1.

(24) Having learnt from Paget that 'Dr Wallis has sent up some things about projectiles pretty like those of mine in ye papers [*De motu Corporum*]', Newton made haste on 13 February 1686/7 to find out the truth of the matter from Halley (see *Correspondence*, 2: 464); the latter was able to reassure him eleven days later that Wallis' result was 'much the same with yours, and he had the hint from an account I gave him of what you had demonstrated' (*ibid.*: 469). The matter is discussed in detail in note (93) on pages 64–5 below.

(25) The text of the extant correspondence between Newton and Halley during this period is printed in *Correspondence*, 2: 431–47, 452–4, 464, 469–74, and 481–2. It is broadly treated by I. B. Cohen in his *Introduction* (note (2)): 131–42.

(26) *Correspondence*, 2: 373. In a testimonial of the same date to the Governors of Christ's Hospital Newton described Paget, 'Maister of Arts & Fellow of Trinity-Colledge in this University', as 'ye most promising person for this end I could think of; . . . of a temper very sober & industrious. . . . He understands ye several parts of Mathematicks, Arithmetick, Geometry, Algebra, Trigonometry, Geography, Astronomy, Navigation & w^{ch} is ye surest character of a true Mathematicall Genius, learned these of his owne inclination, & by his owne industry without a Teacher. . . .' (*ibid.*: 375). What a sad fulfilment is there implied of the high hopes of encouraging mathematical studies in the University with which Newton's Cambridge professorship had been established twenty years before! Since he already knew

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candidate for the post of Master of the Mathematical School at Christ's Hospital to which he was soon after appointed), Newton evidently held a warmer esteem. Paget it was who in November 1684 'when last with us', as Newton shortly afterwards informed the newly appointed Secretary of the Royal Society,⁽²⁷⁾ unsuccessfully 'pusht forward' the 'designe of a [regular] Philosophick Meeting here'⁽²⁸⁾ and who on his return journey from Cambridge to London carried with him the fair copy of Newton's first tract 'De motu Corporum' whose text was subsequently transcribed by Halley and also entered in the Royal Society's Register;⁽²⁹⁾ thereafter, he acted more than

Paget well enough to add that 'his hand is very steady & accurate, as well as his fancy & apprehension, good; as may be seen by his writing & drawing wth his Pencil very well', further noting his 'long acquaintance also wth variety of Learning here', perhaps Newton had indeed given him—like Henry Wharton a year later (see iv: 11, note (30))—some minimal mathematical instruction in the privacy of his room?

(27) His old friend and colleague at Trinity, Francis Aston, who ten years before—unlike Newton—had failed to obtain a royal dispensation to remain a Fellow without taking holy orders (see ii: xxiv, note (10)).

(28) Newton to Aston, 23 February 1684/5 (*Correspondence*, 2: 415). 'I concurred with him', he went on, 'and engaged D^r [Henry] More to be of it, and others were spoke to partly by me, partly by M^r Charles Montague but that which chiefly dasht the buisness was the want of persons willing to try experiments, he [Vigani?] whom we chiefly relyed on refusing to concern himself in that kind him self. And more what to add further about this buisness I know not, but only this that I should be very ready to concurre with any persons for promoting such a designe so far as I can doe it without engaging the loss of my own time in those thinges'.

(29) Two years afterwards, in clarifying for Newton (at the height of his squabble with Hooke over prior discovery of inverse-square planetary gravitation) the crowded sequence of events during 1684, Halley recalled in his letter to Cambridge on 29 June 1686 that in 'August... when I... learnt the good news that you had brought this demonstration to perfection, . . . you were pleased to promise me a copy thereof, which the November following I received with a great deal of satisfaction from M^r Paget' (*Correspondence*, 2: 442); compare Newton's subsequent reference to 'y^e papers M^r Paget first shewed you' in his later re-opening of correspondence with Halley on 13 February 1686/7 (*ibid.*: 464). At the Royal Society meeting on 10 December 1684 Newton's 'curious treatise *De Motu* . . ., upon Mr *Halley's* desire, was promised to be sent to the Society to be entered upon their register. Mr *Halley* was desired to put Mr *Newton* in mind of. . . securing his invention to himself till he could be at leisure to publish it. Mr *Paget* was desired to join with Mr *Halley*' (Thomas Birch, *History of the Royal Society of London*, 4 (London, 1757): 347). Flamsteed, who had also (in a lost letter from Newton passed on by Paget) been 'kindly offered y^e perusall of your papers' as he wrote back in acknowledgement on 27 December (*Correspondence*, 2: 403), was in fact—perhaps, as Flamsteed guessed a week later, because the prevailing 'hard weather' . . . prevented him as it did me from going to London' (*ibid.*: 410) but also because, as Newton replied on 12 January 1684/5, 'M^r Paget . . . has been laid up sick of an ague' (*ibid.*: 412)—not to gain a view of the 'De motu Corporum' for nearly another month when, he wrote to Newton on 27 January to thank him, 'a benifice haveing been bestowed upon me in the meane time I have not had leasure to peruse it yet' (*ibid.*: 414). The original 'papers' of Newton's communicated 'Notions about Motion', whose text had been registered at the Royal Society by mid-February following when Newton thanked Aston for entering them (*ibid.*: 415)—whether before or after they passed to Flamsteed is not clear—, thereafter disappear from the record. In 1, §1 below we

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once over the next couple of years as Newton's trusted informant in London⁽³⁰⁾ and during a further stay in Cambridge with him in autumn 1686 he 'mended' a number of verbal slips in the proof-sheets of the *Principia*'s first book,⁽³¹⁾ being duly rewarded the following summer with a gift copy of the published volume.⁽³²⁾ Here, one feels, is yet one more interesting minor acquaintance of Newton's of whom we should have liked to know more than his drunken downfall and ensuing death in exile.⁽³³⁾

For the rest, the dedicatory epistle which Newton contributed in April 1685 to the Latin version of William Briggs' *Theory of Vision*⁽³⁴⁾ and the similar certificate of approval which he set a few months later to George Mabbot's *Tables for renewing and purchasing of the leases of Cathedral Churches and Colleges*⁽³⁵⁾

reproduce them from the much corrected and overwritten draft retained by Newton, suitably filling out its minimal lacunas in line with Halley's (incompletely extant) transcript and the secretary copy in the Royal Society Register Book (see note (2) on pages 30–1 following).

(30) See note (24) above for Newton's enquiry in February 1687 of Paget as to 'how things were' in current 'differences in y^e R. Society' (*Correspondence*, 2: 464). Paget was also, we assume, Newton's unnamed informant in June 1686 regarding Hooke's 'great stir' at the Royal Society, 'pretending I had all from him' (see *ibid.*: 437).

(31) See Newton to Halley, 18 October 1686 (*Correspondence*, 2: 454).

(32) Along with 'the R. Society, Mr Boyle, . . . Mr Flamsteed and if there be any elce in town that you design to gratifie that way' as Halley wrote to Newton on 5 July 1687, having 'at length brought your Book to an end' (*Correspondence*, 2: 481).

(33) In the late 1690's Flamsteed added at the foot of Newton's letter introducing Paget to him in April 1682 (see note (26) above) a Latin note: 'Ebrietati deinde . . . nimium addictus immemor officij, pueros neglexit in Flandriam transiit deposuit munus in Indiam tandem navigavit. . . ' (*Correspondence*, 2: 373); while H. W. Turnbull in a following footnote quotes an extract from a letter from Roger Cotes to John Smith of 11 December 1703 (Trinity College, Cambridge, MS R. 4) observing that 'Pagett . . . died at Isp[h]ahan in his return from y^e great Mogul. . . Tis believed he had made severall valuable Observations in those parts. . . ' (*ibid.*: 375, note (6)). Before the black sheep showed his true colour Newton maintained an active relationship with Paget, arranging to visit Flamsteed with him on a trip to London in the summer of 1691 (see Newton's exploratory letter to the latter on 10 August 1691, printed in *Correspondence*, 3, 1961: 164) and writing to him in the late spring of 1694 regarding Paget's revised scheme of instruction for the Christ's Hospital Mathematical School (see Newton's unpublished letter to Paget of 25 May 1694, whose draft is now ULC. Add. 3965.12: 330^r, and the mass of related letters and alternative schemes of Newton's preserved in ULC. Add. 4005.16, only partially printed in *Correspondence*, 3, 357–68 from copies in the Christ's Hospital Record Book).

(34) This dedicatory letter is reprinted in Edleston's *Correspondence* (note (1)): 272–3, and also in *The Correspondence of Isaac Newton*, 2: 417–18. In it Newton took care to mention that Briggs' book 'in usum cedet juventuti Academicæ, & provectiores ad ulteriores in Philosophiâ progressus manuducet.'

(35) Newton's commendation states pithily: 'Methodus hujus Libri rectè se habet, numerique ut ex quibusdam ad calculum revocatis judico, satis exactè computantur. Is. Newton, Math. Prof. Luc.'. The author of these tables—directly attributed to Newton after their fifth edition (London, 1729)—was manciple (caterer) at King's College and, as far as we know, no personal friend of Newton's. (See Edleston, *Correspondence* (note (1)): lvi, note (78);

together remind us that Newton was never reluctant to lend his name (and professorial title) to promote the sale of others' books. Nor was he averse to allowing minor mathematical contributions of his own to appear in print, as the 'Directions' for constructing the volume of a paraboloid of revolution which appeared under his name in William Hunt's *Guager's Magazine* in 1687 illustrate.⁽³⁶⁾ Four letters from the retired Cambridge mathematics don Gilbert Clerke between September and November 1687 querying small points of mathematical style in the published *Principia*⁽³⁷⁾ are less memorable than Newton's extant reply to the first⁽³⁸⁾ where he displays an unusual tolerance of his aged correspondent's tiresome and near-trivial 'scruples' regarding the opening pages of his book. The stray draft, finally, of a letter of his⁽³⁹⁾ to the wayward tenant of his childhood home of Woolsthorpe in January 1688 both adds at one point the incidental information that he is now 'short-sighted' and, at a deeper level, underlines the continuity of his concern with his holdings of property in Lincolnshire.

The even tenor of Newton's existence at Cambridge, for so long undisturbed, was to begin to crack even while his *Principia* was still in press in London. The initial break came, as is well known, with the issue by James II in February 1687 of a *mandamus* to the University Vice-chancellor, Pechell, directing that a Benedictine monk, Father Francis, be admitted M.A. without taking the required oaths of supremacy and allegiance. Disturbed less by the political and religious implications of this assertion of the royal prerogative than by his

and also D. F. McKenzie, 'The Author of *Tables for purchasing Leases* attributed to Sir Isaac Newton', *Transactions of the Cambridge Bibliographical Society*, 3, 1960: 165–6.) A note by John Conduitt in 1729 (King's College, Cambridge. Keynes MS 130.5) adds the information that 'The leases for lives &c w^{ch} are printed in his name, a bookseller prevailed with him only to look over one page & so he writt what is there mentioned'.

(36) These 'Directions...necessary to be understood by every *Gager*...as I received [them] from the learned M. *Isaac Newton*, *Professor of the Mathematicks in Cambridge*' are conveniently reprinted from Hunt's *Magazine* (London, 1687: 272–4) in *The Correspondence of Isaac Newton*, 2: 478–80. In effect Newton obtains the segment of a paraboloid of revolution of height h and base radius r which is cut off at a distance z from the axis by a vertical parabolic section (of height $(h/r^2)(r^2 - z^2)$ and semi-base $y = \sqrt{[r^2 - z^2]}$) as

$$\begin{aligned} \frac{4}{3}h \int_r^z r^{-2}(r^2 - z^2)^{\frac{3}{2}}.dz &= \frac{4}{3}h \left[\int_r^z y . dz - \int_r^z r^{-2}yz^2 . dz \right] \\ &= \frac{4}{3}h \int_r^z y . dz - \frac{1}{6}h \int_{2r}^x v . dx, \end{aligned}$$

where $x = 2z^2/r$ and $v = \sqrt{[x(2r - x)]}$ are readily constructed as abscissa and ordinate in the base circle. (Compare III: 31, note (11).)

(37) *Correspondence*, 2: 485–6, 488–98.
(38) *Correspondence*, 2: 487.
(39) *Correspondence*, 2: 502–4, especially, 502.

outrage that statutory law had been cynically flouted,⁽⁴⁰⁾ Newton was a leading advocate of defying this mandate when the question was debated in the Senate in March, and a prominent member of the University delegation whose tentative appeal to legal precedent before the Ecclesiastical Commission in April was tersely dismissed by Judge Jeffreys from his President's chair with a typically stern rebuke.⁽⁴¹⁾ His reward came nearly two years later when, after the 'Glorious Revolution' which in December 1688 proclaimed William of Orange king in James' place, Newton was in January 1689 elected—not with the greatest share of the vote, we may add⁽⁴²⁾—one of the two University members of the Convention Parliament. If not conspicuously active therein, during the next thirteen months he lived an exciting existence on the fringe of political power at Westminster and Whitehall, actively involved at first in the preparation of the new declaration of allegiance to be sworn by members of the Universities⁽⁴³⁾ and thereafter with little to do but listen on his parliamentary back-bench. We may vicariously conceive the deep impact upon him of his sudden translation from a dowdy, monotonous scholarly retreat into the glittering, sophisticated world of a metropolis in which he had never before passed more than a few fleeting days. There he met not only influential politicians but the philosopher John Locke, with whom he was to maintain contact over the next ten years both by letter and by personal visit,⁽⁴⁴⁾ and the

(40) If we read aright the letter which he drafted on this issue to an unknown correspondent on 19 February 1686/7 (*Correspondence*, 2: 467–8).

(41) 'Go your way and sin no more, lest a worse thing come unto you' as Jeffreys quoted from scripture in reprimanding the delegates on 12 May (cited from Edleston, *Correspondence*: lviii, note (90) where it is added that 'Newton does not appear at all as a speaker during the proceedings'). A full account of the Father Francis affair is given by C. H. Cooper in his *Annals of Cambridge*, 3 (London, 1845): 614–33; see also Brewster's *Memoirs* (note (6)), 2: 104–9 and—though we need not wholly accept his individual interpretations of Newton's motives for making this unprecedented incursion into University politics—F. E. Manuel's *Portrait of Isaac Newton* (Cambridge, Massachusetts, 1968): 108–14.

(42) Newton attracted 122 votes to Sir Robert Sawyer's 125, while the third candidate in the election gained 117—a close result! (See *The Correspondence of Isaac Newton*, 3: 8, note (1).) 'In many of the voting papers [still preserved in the University archives] his name is preceded by the words "præclarum virum", in some the adjective is "doctissimum", "integerrimum", "venerabilem", "reverendum". Pulleyn, his old tutor [see 1: 10, note (26)], calls him "sumum virum"' (Edleston, *Correspondence*: lix, note (93)).

(43) See the thirteen extant letters written by him to the Cambridge Vice-Chancellor, John Covell, between 12 February 1688/9 and the following 15 May (*Correspondence*, 3: 10–23).

(44) The circumstances of Newton's first meeting with Locke are not recorded, but they were already well enough acquainted by 'Mar[ch 16]89/90' (as Brownover's secretarial inscription specifies) for Newton to send him a simplified 'Demonstration That the Planets by their gravity towards the Sun may move in Ellipses' (Locke's copy of which is now Bodleian MS. Locke.c.31: 101–4; on this see 3, §1.4: note (33) below). Of the letters which passed between them over the next decade and a half, many dealing with intricate points of biblical exegesis, eighteen are preserved (consult the indexes to *Correspondence*, 3/4, s.v. 'Locke'), and in

eminent Dutch scientist Christiaan Huygens⁽⁴⁵⁾ with whom (through Oldenburg) he had briefly corresponded sixteen years before; more unexpectedly, he became close friends with the émigré Swiss mystic and mathematician Nicolas Fatio de Duillier,⁽⁴⁶⁾ of whom we shall hear more in the next volume. And, to show that he had arrived at last in the public world, he for the first time in his life commissioned a portrait of himself.⁽⁴⁷⁾

Once implanted, the taste for political power and temporal prestige was never

the intervening periods Newton was a frequent visitor to Oates in Essex where Locke mostly lived from 1691 (till his death in October 1704) as the guest of Sir Francis Masham. In his last known letter to Locke on 15 May 1703 Newton wrote that ‘I had thoughts of going to Cambridge this summer & calling at Oates in my way [from London]. . .’ (*Correspondence*, 4, 1967: 406).

(45) During Huygens’ extended stay in London in 1689, namely. Their first meeting was at the Royal Society on 12 June when ‘M^r Hugen of Zulichen being present gave an account that he himself was now about publishing a Treatise concerning the Cause of Gravity, and another about Refractions giving amongst other things the reasons of the double refracting Island Chrystall’ while ‘M^r Newton considering a piece of the Island Chrystall’ erroneously asserted that the extraordinary ray ‘suffered no refraction, when [it] came parallel to the oblique sides of the parallelepiped. . .’ (Royal Society Journal Book, quoted from *Correspondence*, 3: 31–2). At another meeting two months later Newton gave Huygens two short papers on motion under resistance (see *ibid.*: 25–8, 33–4).

(46) The two had already met by late January 1690 when Fatio was the intermediary through whom Huygens arranged to send Newton a presentation copy (now Trinity College, Cambridge. NQ.16.186) of his newly published *Traité de la Lumière. . . Avec un Discours de la Cause de la Pesanteur* which (compare the previous note) had been the major topic of their first meeting the June before; see Huygens’ letter of 7 February 1690 (N.S.) to Fatio (*Œuvres complètes de Christiaan Huygens*, 9: 357–9), the pertinent extracts from which are given in *Correspondence*, 3: 67. Fatio wrote to Newton himself on 24 February 1689/90 that ‘I shall have I think to morrow an exemplar of M^r Hugen’s his book that he hath sent you’ (*Correspondence*, 3: 390) and an unpublished following letter two months later on 17 April evidently pursues an inquiry from Newton in assuring him that ‘M^r Hugen is not about y^e making of one of your telescopes, but onely some very large object glasses. . . which may, tho but short, bear a vast aperture for to discover more easily by them y^e satellits already known and their Eclipses, the fixed starrs and perhaps new Planets &c’ (Brotherton Library, Leeds. MS 248).

(47) The half-length study by Godfrey Kneller dated ‘1689’, now in possession of the Earl of Portsmouth, which is from time to time to be seen on public view in London—at the Royal Academy winter exhibition of 1960–1 on ‘The Age of Charles II’, for example, where it was catalogued as No. 217—and is often reproduced in print as uniquely depicting Newton in the prime of his intellectual maturity. The main features of the painting—but not its subdued near-monochrome—are accurately described by Manuel in his *Portrait* (note (41): 106–7 where he writes that at the age of forty-six ‘Newton. . . is presented with his own shoulder-length gray hair. A white shirt, open at the neck, is largely concealed by an academic gown from the right sleeve of which spidery fingers emerge. The face is angular, the sharp chin cleft, the mouth delicately shaped. A long, thin nose is elevated at the bridge. Beneath brows knit in concentration, blue, rather protuberant eyes are fixed in a gaze that is abstracted’. Why Newton commissioned his likeness from the most fashionable painter in London is not known but, without unduly stressing the psychological motives he may have had for so doing, we may treat it as a visible sign of his new-found concern with the outside world and the way in which others should look upon him. How true to reality this suggestive visualisation of his appearance was we have no precise way of telling.

to leave Newton, even though in February 1690 he was, on the dissolution of Parliament, seemingly happy enough to forego the bustle and sparkle of the capital for the dull provincial routine of the quiet Fenland university town where he had, till the last year, spent all his adult life.⁽⁴⁸⁾ While (as we shall see in the next volume) he continued over the next six years to make further significant advances in the fields of geometry and calculus and also attempted new systematised expositions of his earlier mathematical and scientific discoveries, he never really settled once more to the seclusion of dedicated scholarship. When in March 1696 he effectively severed his academic ties with Cambridge to pass his remaining thirty years of life in London, existing intellectually thereafter on the treasure of an achieved reputation in science, but careful to bolster it from time to time by publishing a carefully polished nugget or two from his private hoard of already mined scholarly gold, we cannot pessimistically sigh for all that he might have gone on to discover had the attractions of the London Mint not tempted him away. In the early 1690's Newton's mathematical and scientific powers passed their peak and entered on their decline, imperceptibly at first and with not a few momentary ascents to their former heights; after 1695 they began seriously to deteriorate, and we cannot believe that, had Newton continued to pursue his unhurried life of scholarship in Cambridge into his late middle age, he would have made any radically new discovery.

But in the context of the present volume this is all in the future. Here we may enjoy the full maturity of Newton's mathematical insight as he applied it in immediate preliminary—and partially in epilogue—to composing his *œuvre maîtresse*. We will delay the reader no longer from savouring its delights and illuminations in the field of central-force dynamics for himself.

(48) He may well of course, have had the ulterior motive of seeking academic preferment either within his own Cambridge college, Trinity, or elsewhere in the University. While in London the previous summer he had—evidently with some degree of support from others—tentatively allowed his name to go forward for consideration as the new Provost of King's. His appointment would, however, have required the variation of the college statutes which prescribed that the Provost should be both in holy orders and already a Fellow of King's; and when on 'Aug. 29, 1689. Before the King & Council was heard the matter of King's College about M^r Isaac Newton, why he or any other not of that foundation should be Provost, . . . after the reasons shewed & argued M^r Newton was laid aside' (Alderman Newton's Diary among the Bowtell MSS at Downing College, quoted from Edleston, *Correspondence*: lix, note (96); for a fuller history of the affair see King's College. Keynes MSS 117/117^A: 'An Account of King's College's Recovery of their Right to choose their own Provost', and John Saltmarsh's discussion in (ed. J. P. C. Roach) *The Victoria County History of Cambridgeshire*, 3 (London, 1959): 397–8). A full year afterwards the rejection still rankled with Newton and when, shortly after his return to Cambridge, moves were made to obtain for him the Mastership of Charterhouse—whose emoluments of '200^l^{ib} *per an* besides a Coach (w^{ch} I reccon not) & lodgings' he spurned—he justified his refusal in the draft of a letter to Locke in mid-December 1691 with the words 'the competition is hazzardous & I am loath to sing a new song to y^e tune of King's College' (*Correspondence*, 3: 184).

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