A BOOK OF
CURVES

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E. H. Lockwood
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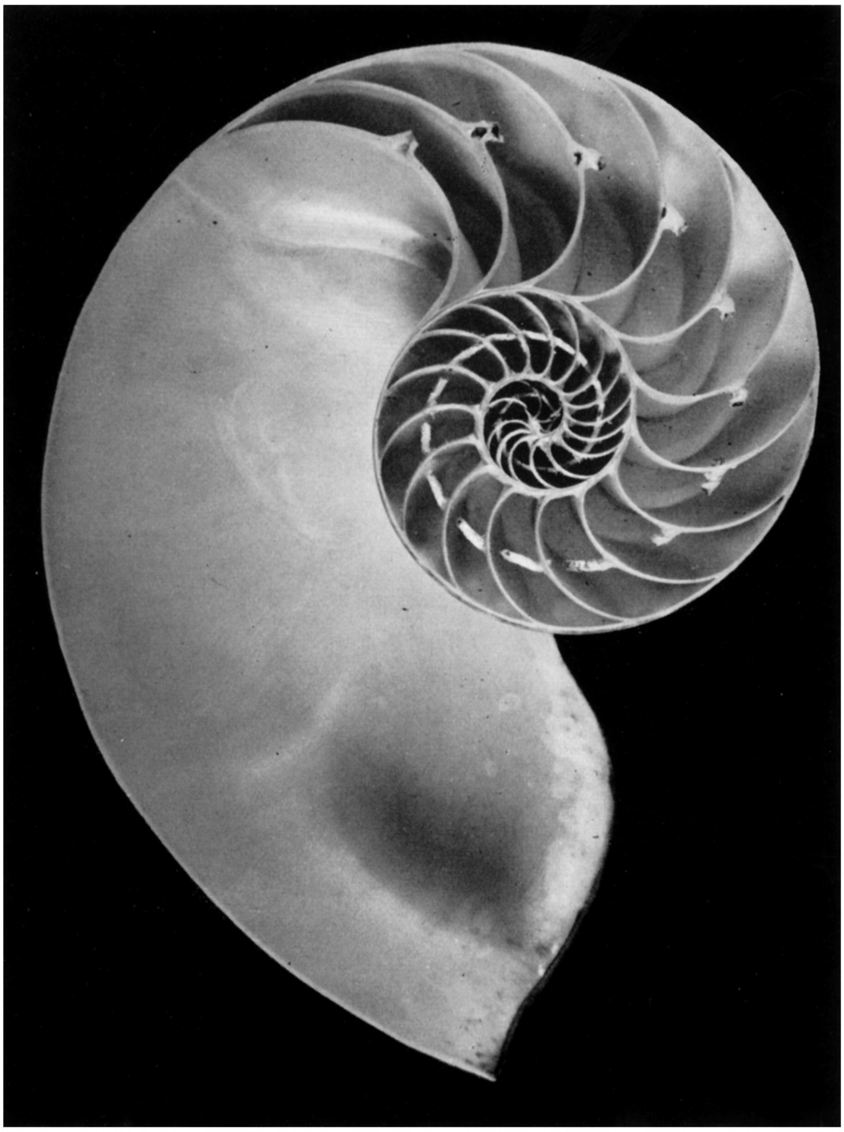

A BOOK OF CURVES

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Frontispiece. Section of the shell of a Pearly Nautilus fossil. (Reproduced by permission of Dr A. M. Winchester of Stetson University, Florida.)

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His Word accomplish'd the design

CHRISTOPHER SMART

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PREFACE

Plane curves offer a rich and to some extent unexplored field of study which may be approached from a quite elementary level. Anyone who can draw a circle with a given centre and a given radius can draw a cardioid or a limaçon. Anyone who can use a set square can draw a parabola or a strophoid. Anyone who knows a few of the simpler propositions of Euclid can deduce a number of properties of these beautiful and fascinating curves.

In school they may be used to instruct and entertain classes at all levels. In a class of mixed ability some will pursue the theory while others continue with the drawing.

Teachers may use the book in a variety of ways, but it has been written also for the individual reader. It is hoped that it will find a place in school libraries, and will be used too by sixth-form pupils, whether on the arts or the science side, who have time for some leisurely work off the line of their main studies, time perhaps to recapture some of the delight in mathematics for its own sake that nowadays so rarely survives the pressure of examination syllabuses and the demands of science and industry.

The approach is by pure geometry, starting in each case with methods of drawing the curve. In this way an appreciation of the shape of the curve is acquired and a foundation laid for a simple geometrical treatment. There may be some readers who will go no further, and even these will have done more than pass their time pleasantly; but others will find it interesting to pursue the geometrical development at least to the point at which one or other of the equations of the curve is established. Those who have a knowledge of the calculus and coordinate geometry may prefer to leave the text at this point and find their own way, using as a guide the summary of results which will be found at the end of each chapter of Part I and some chapters of Part II.

In Part II the reader is encouraged to explore further for himself, using whatever resources are available to him. While some individual curves are briefly discussed, this part of the book is mainly concerned with methods by which new curves can be found. Those whose delight is in the drawing will find much to occupy them here; but deduction can often contribute both to the shaping of a curve and to a discovery of its properties.

My particular thanks are due to Mr A. Prag, mathematics master and Librarian at Westminster School, who has written the Historical Introduction and most of the shorter historical notes, a scholarly contribution without which the book would have been sadly incomplete. I am grateful also to my colleague at Felsted Mr P. Gant, who has read the manuscript in detail and has offered many helpful criticisms and suggestions; to Mr Alan Breese, who made the drawings for the full-page diagrams; and to Dr A. M. Winchester, whose photograph of the Pearly Nautilus fossil appears as the frontispiece. Finally, I express my gratitude to the Syndics and Staff of the Cambridge University Press: how much the book owes to them in its planning and production the reader can now judge.

FELSTED
August 1960

E. H. L.

HISTORICAL INTRODUCTION

Men were fascinated by curves and curved shapes long before they regarded them as mathematical objects. For evidence one has only to look at the ornaments in the form of waves and spirals on prehistoric pottery, or the magnificent systems of folds in the drapery of Greek or Gothic statues. It was the Greek geometers who began to study geometrically defined curves as, for instance, the contour of the intersection of a plane with a cone, or the locus of points reached one by one through a geometrical construction. The straight line and the circle could be drawn with very primitive instruments in one continuous movement, and so they were distinguished as 'plane' loci from the 'solid' conic sections. All other curves were *loci lineares*, i.e. just 'lines', 'curves'. Some curves were generated by the movement of mechanical linkages, or at least were imagined to be so generated: the spirals of Archimedes were of that type. A classification into 'geometrical' and 'mechanical' curves (which does not quite correspond to the modern use of those terms) became fixed when analytical geometry, in the seventeenth century, made it possible to distinguish with precision what we should now (following Leibniz) call *algebraic* and *transcendental* curves.

In his search for the true shape of a planetary orbit Kepler tried a variety of curves before he found that the ellipse gave the best fit. In the old Ptolemaic system the planets were supposed to describe paths which could be constructed by means of *epicycles* (i.e. by circles carried on other circles or spheres). Kepler altogether enjoyed playing with curves and invented a great number of names (usually those of some sort of fruit) for the solids of revolution generated by curves rotating about various axes.

When Cavalieri tried to explain his method of integration (*Cavalieri's Principle*) he was careful to use a really general type of curve, but he lacked the analytical method of description; later in the seventeenth century, Gregory and Barrow gave the rules of the calculus (as we should call it) in geometrical form by referring to simple monotone arcs. Thus already the individual curves were beginning to be lost in more general theory.

A powerful device was the creation of a new curve by the transformation of another, as for instance, a curve formed by drawing ordinates equal to the lengths of the subtangents of a given one. A simpler example was the drawing of a *conchoid* $r = f(\theta)+c$ for a given curve $r = f(\theta)$: then, if the tangent to the given curve were known, the tangent to the conchoid could immediately be constructed. Problems in optics led to *caustics*, i.e. envelopes of pencils of rays. But the greatest influence in the study of curves was, of course, the invention of the calculus, which not only secured the solution of problems on gradients, areas, and lengths of arcs, but unified the whole field of research. A great variety of mechanical problems could then be precisely formulated, as, for instance, to find the curve of 'quickest descent', the *brachistochrone*.

But the interest had shifted from the geometrical origin of the concept 'curve' to the analytical aspect: it was as a diagram of a 'function' that the curve appeared in the text-book, and the individuality of many famous members of the family was lost.

NOTATION

The following notation will be used, more particularly in the summary of results at the end of each chapter:

(r, θ) are polar coordinates.

t is a parameter for the parametric equations of a curve.

ϕ = angle between radius vector and tangent.

ψ = angle between initial line (or x-axis) and tangent.

s = arc-length, measured usually from $\theta = 0$ or $t = 0$.

p = perpendicular distance from origin to tangent.

ρ = radius of curvature.

A = area enclosed by a curve.

L = total length of a closed curve.

The letters P, P' will be used to name points on the curve which is being drawn; Q, Q' for points on a subsidiary line or curve; and q for a point which will eventually move towards and coincide with Q.

For the drawing of the curves, suitable dimensions will be suggested for each of the following sizes of paper:

Size 1	9 in. by 7 in.	23 cm. by 18 cm.
Size 2	13 in. by 8 in.	33 cm. by 20 cm.
Size 3	15 in. by 11 in.	38 cm. by 28 cm.

When it is necessary to specify which way up the paper is to be used, the suffixes P, for the 'portrait' (i.e. upright) position, or L, for the 'landscape' position, will be added. Thus 'Paper 1_P' means 'Size 1, in the portrait position'.

Lines drawn across the paper from left to right will sometimes be referred to as 'horizontal' and lines drawn up the paper as 'vertical'.

* An asterisk will be used to mark the more difficult sections and exercises.

** A double asterisk will indicate work which, though not necessarily difficult, demands knowledge beyond the syllabuses of O-level 'Additional Mathematics'.