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Observational basis

1.1 Historical development

The study of stellar rotation began at the turn of the seventeenth century, when sunspots were observed for the first time through a refracting telescope. Measurements of the westward motion of these spots across the solar disk were originally made by Johannes Fabricius, Galileo Galilei, Thomas Harriot, and Christopher Scheiner. The first public announcement of an observation came from Fabricius (1587–c. 1617), a 24-year old native of East Friesland, Germany. His pamphlet, *De maculis in Sole observatis et apparente earum cum Sole conversione*, bore the date of dedication June 13, 1611 and appeared in the *Narratio* in the fall of that year. Fabricius perceived that the changes in the motions of the spots across the solar disk might be the result of foreshortening, with the spots being situated on the surface of the rotating Sun. Unfortunately, from fear of adverse criticism, Fabricius expressed himself very timidly. His views opposed those of Scheiner, who suggested that the sunspots might be small planets revolving around an immaculate, nonrotating Sun. Galileo made public his own observations in *Istoria e Dimostrazioni intorno alle Macchie Solari e loro Accidenti*. In these three letters, written in 1612 and published in the following year, he presented a powerful case that sunspots must be dark markings on the surface of a rotating Sun. Foreshortening, he argued, caused these spots to appear to broaden and accelerate as they moved from the eastern side toward the disk center. The same effect made the sunspots seem to get thinner and slower as they moved toward the western side of the disk. Galileo also noticed that all spots moved across the solar disk at the same rate, making a crossing in about fourteen days, and that they all followed parallel paths. Obviously, these features would be highly improbable given the planetary hypothesis, which is also incompatible with the observed changes in the size and shape of sunspots.

The planetary hypothesis, championed by Scheiner among others, was thus convincingly refuted by Galileo. Eventually, Scheiner's own observations led him to realize that the Sun rotates with an apparent period of about 27 days. To him also belongs the credit of determining with considerably more accuracy than Galileo the position of the Sun's equatorial plane and the duration of its rotation. In particular, he showed that different sunspots gave different periods of rotation and, furthermore, that the spots farther from the Sun's equator moved with a slower velocity. Scheiner published his collected observations in 1630 in a volume entitled *Rosa Ursina sive Sol*, dedicated to the Duke of Orsini, who sponsored the work. (The title of the book derives from the badge of the Orsini family, which was a rose and a bear.) This was truly the first monograph on solar physics.

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It is not until 1667 that any further significant discussion of stellar rotation was made. In that year the French astronomer Ismaël Boulliaud (1605–1694) suggested that the variability in light of some stars (such as Mira Ceti) might be a direct consequence of axial rotation, with the rotating star showing alternately its bright (unspotted) and dark (spotted) hemispheres to the observer. This idea was popularized in Fontenelle’s *Entretiens sur la pluralité des mondes* – a highly successful introduction to astronomy that went through many revised editions during the period 1686–1742. To be specific, he noted “. . . that these fixed stars which have disappeared aren’t extinguished, that these are really only half-suns. In other words they have one half dark and the other lighted, and since they turn on themselves, they sometimes show us the luminous half and then we see them sometimes half dark, and then we don’t see them at all.”* Although this explanation for the variable stars did not withstand the passage of time, it is nevertheless worth mentioning because it shows the interest that stellar rotation has aroused since its inception. As a matter of fact, nearly three centuries were to elapse before Boulliaud’s original idea was fully recognized as a useful method of measuring the axial rotation of certain classes of stars, that is, stars that exhibit a detectable rotational modulation of their light output due to starspots or stellar plagues.

For more than two centuries the problem of solar rotation was practically ignored, and it is not until the 1850s that any significant advance was made. Then, a long series of observations of the apparent motion of sunspots was undertaken by Richard Carrington and Gustav Spörer. They confirmed, independently, that the outer visible envelope of the Sun does not rotate like a solid body; rather, its period of rotation varies as a function of heliocentric latitude. From his own observations made during the period 1853–1861, Carrington derived the following expression for the Sun’s rotation rate:

$$\Omega(\text{deg/day}) = 14^\circ 42' - 2^\circ 75' \sin^{7/4} \phi, \quad (1.1)$$

where ϕ is the heliocentric latitude. Somewhat later, Hervé Faye found that the formula

$$\Omega(\text{deg/day}) = 14^\circ 37' - 3^\circ 10' \sin^2 \phi \quad (1.2)$$

more satisfactorily represented the dependence of angular velocity on heliocentric latitude. Parenthetically, note that Carrington also found evidence for a mean meridional motion of sunspots. Convincing evidence was not found until 1942, however, when Jaakko Tuominen positively established the existence of an equatorward migration of sunspots at heliocentric latitudes lower than about 20° and a poleward migration at higher latitudes.

The spectroscope was the instrument that marked the beginning of the modern era of stellar studies. As early as 1871 Hermann Vogel showed that the Sun’s rotation rate can be detected from the relative Doppler shift of the spectral lines at opposite edges of the solar disk, one of which is approaching and the other receding. Extensive measurements were made visually by Nils Dunér and Jakob Halm during the period 1887–1906. They showed a rotation rate and equatorial acceleration that were quite similar to those obtained from the apparent motion of sunspots. They concluded that Faye’s empirical law

* Bernard le Bovier de Fontenelle, *Conversations on the Plurality of Worlds*, translation of the 1686 edition by H. A. Hargreaves, p. 70, Berkeley: University of California Press, 1990.

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adequately represented the spectroscopic observations also, but their coverage of latitude was double that of the sunspot measurements. The first spectrographic determinations of solar rotation were undertaken at the turn of the twentieth century by Walter S. Adams at Mount Wilson Solar Observatory, California.

William de Wiveleslie Abney was the first scientist to express the idea that the axial rotation of single stars could be determined from measurements of the widths of spectral lines. In 1877, he suggested that the effect of a star's rotation on its spectrum would be to broaden all of the lines and that "... other conditions being known, the mean velocity of rotation might be calculated."^{*} In 1893, while doubts were still being expressed with regard to measurable rotational motions in single stars, J. R. Holt suggested that axial rotation might be detected from small distortions in the radial velocity curve of an eclipsing binary. Thus, he argued,

... in the case of variable stars, like Algol, where the diminution of light is supposed to be due to the interposition of a dark companion, it seems to me that there ought to be a spectroscopic difference between the light at the commencement of the minimum phase, and that of the end, inasmuch as different portions of the edge would be obscured. In fact, during the progress of the partial eclipse, there should be a shift in position of the lines; and although this shift is probably very small, it ought to be detected by a powerful instrument.[†]

Confirmation of this effect was obtained by Frank Schlesinger in 1909, who presented convincing evidence of axial rotation in the brightest star of the system δ Librae. However, twenty more years were to elapse before Abney's original idea resulted in actual measurements of projected equatorial velocities in single stars. This notable achievement was due to the efforts of Otto Struve and his collaborators during the period 1929–1934 at Yerkes Observatory, Wisconsin.

A graphical method was originally developed by Grigori Shajn and Otto Struve. The measurements were made by fitting the observed contour of a spectral line to a computed contour obtained by applying different amounts of Doppler broadening to an intrinsically narrow line-contour having the same equivalent width as the observed line. Comparison with an observed line profile gave the projected equatorial velocity $v \sin i$ along the line of sight. These early measurements indicated that the values of $v \sin i$ fell into the range 0–250 km s⁻¹ and may occasionally be as large as 400 km s⁻¹ or even more. As early as 1930 it was found that the most obvious correlation between $v \sin i$ and other physical parameters is with spectral type, with rapid rotation being peculiar to the earliest spectral classes. This was originally recognized by Struve and later confirmed by statistical studies of line widths in early-type stars by Christian T. Elvey and Christine Westgate. The O-, B-, A-, and early F-type stars frequently have large rotational velocities, while in late F-type and later types rapid rotation occurs only in close spectroscopic binaries. A study of rotational line broadening in early-type close binaries was also made by Egbert Adriaan Kreiken. From his work it is apparent that the components of these binaries have their rotational velocities significantly diminished with respect to single, main-sequence stars of the same spectral type. The following year, 1936, Pol Swings

* *Mon. Not. R. Astron. Soc.*, **37** (1877), p. 278.

† *Astronomy and Astro-Physics*, **12** (1893), p. 646.

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properly established that in close binaries of short periods axial rotation tends to be either perfectly or approximately synchronized with the orbital motion.

At this juncture the problem was quietly abandoned for almost fifteen years. Interest in the measurements of axial rotation in stars was revived in 1949 by Arne Slettebak. Extensive measurements of rotational velocities were made during the 1950s and 1960s by Helmut A. Abt, Robert P. Kraft, Slettebak, and others. However, because the only observational technique available was to determine line widths in stars from photographic spectra, these studies were limited almost entirely to stars more massive than the sun ($M \gtrsim 1.5M_{\odot}$) and to main-sequence or post-main-sequence stars. Since appreciable rotation disappears in the middle F-type stars, higher-resolution spectra are therefore required to measure rotational broadening in the late-type stars. In 1967, Kraft pushed the photographic technique to its limit to measure $v \sin i$ as low as 6 km s^{-1} in solar-type stars. Now, as early as 1933, John A. Carroll had suggested the application of Fourier analysis to spectral line profiles for rotational velocity determinations. In 1973, the problem was reconsidered by David F. Gray, who showed that high-resolution data make it possible to distinguish between the Fourier transform profile arising from rotation versus those arising from other broadening mechanisms. Since the late 1970s systematic studies of very slow rotators have been made by Gray, Myron A. Smith, David R. Soderblom, and others. Current techniques limit the measurement accuracy of projected rotational velocities to 2 km s^{-1} in most stars.

Periodic variations in the light output due to dark or bright areas on some rotating stars have also been used to determine the rotation periods of these stars. Although the principle of rotational modulation was suggested as early as 1667 by Ismaël Boulliaud, convincing detection of this effect was not made until 1947, when Gerald E. Kron found evidence in the light curve of the eclipsing binary AR Lacertae for surface inhomogeneities in its G5 component. The principle was therefore well established when in 1949 Horace W. Babcock proposed the so-called oblique-rotator model for the magnetic and spectrum variations of the periodic Ap stars. Kron's result was forgotten till 1966, when interest in the principle of rotational modulation was independently revived by Pavel Chugainov. A large body of literature has developed since the late 1960s. This work generally divides according to the method used to estimate the rotation periods, with the two types being (i) photometric monitoring of light variations produced by large starspot groups or bright surface areas and (ii) measurements of the periodic variation in strength of some emission lines that are enhanced in localized active regions in the chromosphere. These techniques have the advantage that a rotation period can be determined to much higher precision than $v \sin i$ and are free of the $\sin i$ projection factor inherent to the spectrographic method. Moreover, very accurate rotation periods can be derived even for quite slowly rotating stars at rates that would be impossible to see as a Doppler broadening of their spectral lines.

A different line of inquiry was initiated by the discovery of the so-called five-minute oscillations in the solar photosphere. The first evidence for ubiquitous oscillatory motions was obtained in the early 1960s by Robert B. Leighton, Robert W. Noyes, and George W. Simon. However, it is not until 1968 that Edward N. Frazier suggested that "... the well known 5 min oscillations are primarily standing resonant acoustic waves."*

* *Zeit. Astrophys.*, **68** (1968), p. 345.

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Two years later, Roger K. Ulrich presented a detailed theoretical description of the phenomenon, showing that standing acoustic waves may be trapped in a layer beneath the solar photosphere. This model was independently proposed in 1971 by John W. Leibacher and Robert F. Stein. In 1975, Franz-Ludwig Deubner obtained the first observational evidence for these trapped acoustic modes. Soon afterward, it was realized that a detailed analysis of the frequencies of these many oscillatory modes could provide a probe of the solar *internal* rotation. Indeed, because axial rotation breaks the Sun's spherical symmetry, it splits the degeneracy of the nonradial modes with respect to the azimuthal angular dependence. A technique for measuring the solar internal rotation from these frequency splittings was originally devised by Edward J. Rhodes, Jr., Deubner, and Ulrich in 1979. Since 1984, following the initial work of Thomas L. Duvall, John W. Harvey, and others,* diverse methods have been used to determine the Sun's internal angular velocity.

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In Section 1.1 we briefly discussed the early measurements of the axial rotation of the Sun. With the advent of more sensitive instruments, however, Doppler and tracer measurements have shown that the solar atmosphere exhibits motions on widely different scales. Besides the large-scale axisymmetric motions corresponding to differential rotation and meridional circulation, velocity fields associated with turbulent convection and also with oscillatory motions at about a five-minute period have been observed. Considerable attention has focused on analysis of these oscillations since, for the very first time, they make it possible to probe the Sun's *internal* rotation.

1.2.1 Large-scale motions in the atmosphere

The solar surface rotation rate may be obtained from measurements of the longitudinal motions of semipermanent features across the solar disk (such as sunspots, faculae, magnetic field patterns, dark filaments, or even coronal activity centers), or from spectrographic observations of Doppler displacements of selected spectral lines near the solar limb. Each of the two methods for deriving surface rotation rates has its own limitations, although few of these limitations are common to both. Actually, the determination of solar rotation from tracers requires that these semipermanent features be both randomly distributed throughout the fluid and undergo no appreciable proper motion with respect to the medium in which they are embedded. In practice, no tracers have been shown to possess both characteristics; moreover, most of them tend to occur in a limited range of heliocentric latitudes. By the spectrographic method, rotation rates can be found over a wider range of latitudes. But then, the accuracy is limited by the presence of inhomogeneities of the photospheric velocity field and by macroscopic motions within coronal and chromospheric features, so that the scatter between repeated measurements is large.

Figure 1.1 assembles sidereal rotation rates obtained from photospheric Doppler and tracer measurements. The observations refer to the sunspots and sunspot groups, magnetic field patterns, and Doppler shifts. In all cases the relationships shown in Figure 1.1 are

* *Nature*, **310** (1984), pp. 19 and 22.

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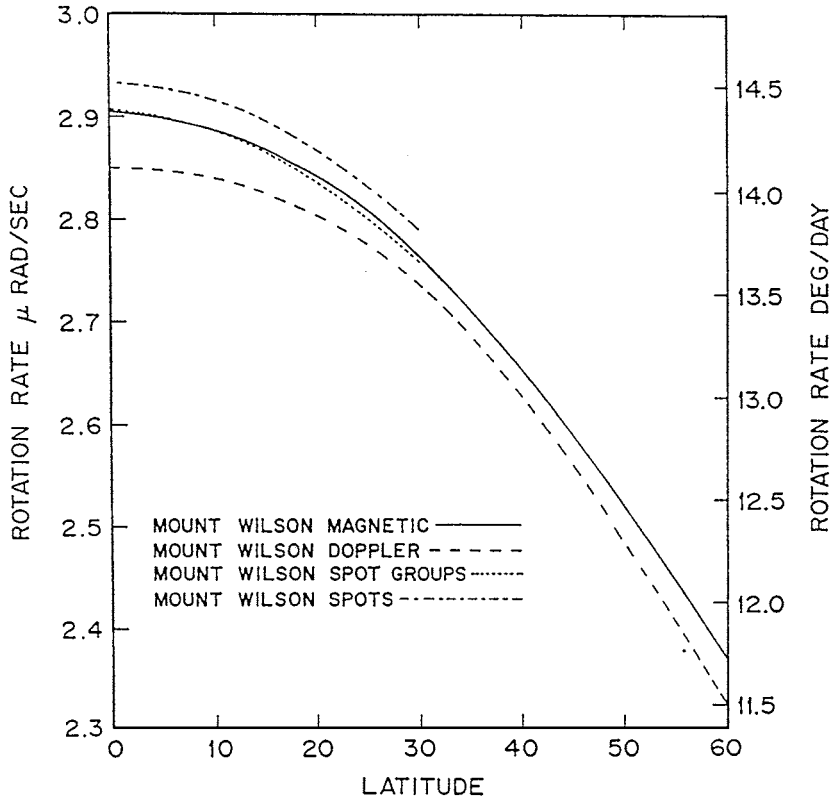


Fig. 1.1. Comparison of the solar differential rotation obtained by different methods. *Source:* Howard, R., *Annu. Rev. Astron. Astrophys.*, **22**, 131, 1984. (By permission. Copyright 1984 by Annual Reviews.)

smoothed curves obtained by fitting the data to expansions in the form

$$\Omega = A + B \sin^2 \phi + C \sin^4 \phi. \quad (1.3)$$

The decrease of angular velocity with increasing heliocentric latitude is clear. However, it is also apparent that different techniques for measuring the solar surface rotation rate yield significantly different results. In particular, the sunspot groups rotate more slowly in their latitudes than individual sunspots. Note also that the rotation rate for the magnetic tracers is intermediate between that for the individual spots and that for the photospheric plasma. It is not yet clear whether these different rotation rates represent real differences of rotation at various depths in the solar atmosphere or whether they reflect a characteristic behavior of the tracers themselves.

Chromospheric and coronal rotation measurements have also been reported in the literature. It seems clear from these results that the latitudinal gradient of angular velocity depends very much on the size and lifetime of the tracers located above the photosphere. To be specific, the long-lived structures exhibit smaller gradients than the short-lived ones, and the very long-lived coronal holes rotate almost uniformly. These noticeable differences remain poorly understood.

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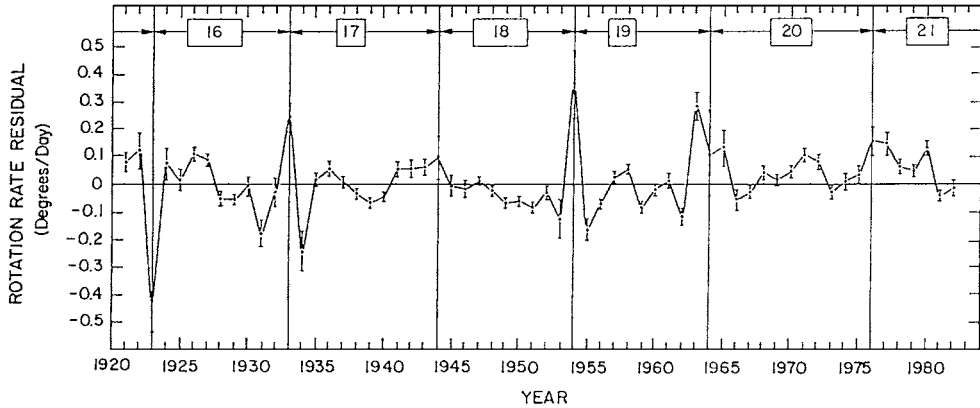


Fig. 1.2. Residuals of annual average sunspot rotation rates for the period 1921–1982. Solar cycle maxima timing and length are denoted by numbered boxes. Vertical lines denote year of sunspot minimum. *Source:* Gilman, P. A., and Howard, R., *Astrophys. J.*, **283**, 385, 1984.

Figure 1.1 merely illustrates the *mean* properties of the solar surface differential rotation. As was originally shown by Howard and LaBonte (1980), however, analysis of the residual motions in the daily Doppler measurements made at Mount Wilson suggests the presence of a *torsional oscillation* of very small amplitude in the photosphere. This oscillation is an apparently organized pattern of zonally averaged variations from a mean curve for the differential rotation, as defined in Eq. (1.3). The amplitude of the residuals constituting the torsional oscillation is of the order of 5 m s^{-1} . It is a traveling wave, with latitude zones of fast and slow rotation, that originates near the poles and moves equatorward over the course of a 22-year cycle. The latitude drift speed of the shear is of the order of 2 m s^{-1} . In the lower heliocentric latitudes, the torsional shear zone between the fast stream on the equator side and the slow stream on the pole side is the locus of solar activity. This coincidence strongly suggests that this torsional oscillation is somewhat related to the solar activity cycle.

Variations of the solar surface rotation rate over individual sunspot cycles have been reported by many investigators. Detailed analyses of the Mount Wilson sunspot data for the period 1921–1982 suggest that *on average* the Sun rotates more rapidly at sunspot minimum.* A similar frequency of rotation maxima is also seen in the Greenwich sunspot data for the years 1874–1976. The variability of the mean rotation rate is illustrated in Figure 1.2, which exhibits peaks of about $0.1 \text{ degree day}^{-1}$ in the residuals near minima of solar activity. The Mount Wilson data also show variations from cycle to cycle, with the most rapid rotation found during cycles with fewer sunspots and less sunspot area.

* A similar result was obtained by Eddy, Gilman, and Trotter (1977) from their careful analysis of drawings of the Sun made by Christopher Scheiner (during 1625–1626) and Johannes Hevelius (during 1642–1644). During the earlier period, which occurred 20 years before the start of the Maunder sunspot minimum (1645–1715), solar rotation was very much like that of today. By contrast, in the later period, the equatorial velocity of the Sun was faster by 3 to 5% and the differential rotation was enhanced by a factor of 3. These results strongly suggest that the change in rotation of the solar surface between 1625 and 1645 was associated, as cause or effect, with the Maunder minimum anomaly.

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Very recently, Yoshimura and Kamby (1993) have found evidence for a *long-term* periodic modulation of the solar differential rotation, with a time scale of the order 100 years. This modulation was observed in the sunspot data obtained by combining Greenwich data covering the period 1874–1976 and Mitaka data covering the period 1943–1992. Their analysis suggests that there exists a well-defined periodic variation in the overall rotation rate of the photospheric layers. To be specific, it is found that the surface rotation rate reaches a maximum at solar cycle 14, decreases to a minimum at cycle 17, and increases again to reach a maximum at cycle 21. Moreover, the time profile of the long-term modulation of the solar rotation is quite similar to the time profile of the solar-cycle amplitude modulation, but the two profiles are displaced by about 23 years in time. Further study is needed to ascertain whether this long-term modulation is strictly periodic or part of a long-term aperiodic undulation.

Several observational efforts have been made to detect a *mean* north–south motion on the Sun’s surface. Unfortunately, whereas the latitudinal and temporal variations of the solar rotation are reasonably well established, the general features of the meridional flow are still poorly understood. Three different techniques have been used to measure these very slow motions: (i) the Doppler shift of selected spectral lines, (ii) the displacement of magnetic features on the solar disk, and (iii) the tracing of sunspots or plages. A majority of Doppler observations suggests a poleward motion of the order of 10 m s^{-1} , whereas others differ in magnitude and even in direction. Doppler data obtained with the Global Oscillation Network Group (GONG) instruments in Tucson from 1992 to 1995 indicate a poleward motion of the order of 20 m s^{-1} , but the results also suggest that the Sun may undergo episodes in which the meridional speeds increase dramatically. The analysis of magnetic features shows the existence of a meridional flow that is poleward in each hemisphere and is of the order of 10 m s^{-1} , which agrees with most of the Doppler measurements. On the contrary, sunspots or plages do not show a simple poleward meridional flow but a motion either toward or away from the mean latitude of solar activity, with a speed of a few meters per second. Analysis of sunspot positions generally shows equatorward motions at low heliocentric latitudes and poleward motions at high latitudes. Several authors have suggested that these discrepancies might be ascribed to the fact that different features are anchored at different depths in the solar convection zone. Accordingly, the meridional flow deep into this zone might be reflected by the sunspot motions, whereas the meridional flow in the upper part of this zone might be reflected by the other measurements. As we shall see in Section 5.2, these speculations have a direct bearing on the theoretical models of solar differential rotation.

1.2.2 *Helioseismology: The internal rotation rate*

The Sun is a very small amplitude variable star. Its oscillations are arising from a huge number of discrete modes with periods ranging from a few minutes to several hours. The so-called five-minute oscillations, which have frequencies between about 2 mHz and 4 mHz, have been extensively studied. They correspond to standing *acoustic* waves that are trapped beneath the solar surface, with each mode traveling within a well-defined shell in the solar interior. Since the properties of these modes are determined by the stratification of the Sun, accurate measurements of their frequencies thus provide a new window in the hitherto invisible solar interior.

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To a first approximation, the Sun may be considered to be a spherically symmetric body. In that case, by making use of spherical polar coordinates (r, θ, φ) , we can write the components of the Lagrangian displacement for each acoustic mode in the separable form

$$\xi = \left(\xi_r P_l^m, \xi_h \frac{dP_l^m}{d\theta}, \xi_h \frac{P_l^m}{\cos\theta} \frac{\partial}{\partial\varphi} \right) \cos(m\varphi - \omega_{n,l}t), \quad (1.4)$$

where $P_l^m(\cos\theta)$ is the associated Legendre function of degree l and order m ($-l \leq m \leq +l$). The eigenfunctions $\xi_r(r; n, l)$ and $\xi_h(r; n, l)$ define the radial and horizontal displacements of the mode. Both functions depend on the integer n , which is related to the number of zeros of the function ξ_r along the radius, and the integer l , which is the number of nodal lines on the solar surface. Because a spherical configuration has no preferred axis of symmetry, these eigenfunctions are independent of the azimuthal order m , so that to each value of the eigenfrequency $\omega_{n,l}$ correspond $2l + 1$ displacements. Rotation splits this degeneracy with respect to the azimuthal order m of the eigenfrequencies. Hence, we have

$$\omega_{n,l,m} = \omega_{n,l} + \Delta\omega_{n,l,m}. \quad (1.5)$$

Since the magnitude of the angular velocity Ω is much less than the acoustic frequencies $\omega_{n,l}$, perturbation theory can be applied to calculate these frequency splittings. One can show that

$$\Delta\omega_{n,l,m} = m \int_0^R \int_0^\pi K_{n,l,m}(r, \theta) \Omega(r, \theta) r dr d\theta, \quad (1.6)$$

where the rotational kernels $K_{n,l,m}(r, \theta)$ are functions that may be derived from a *non-rotating* solar model for which one has calculated the eigenfrequencies $\omega_{n,l}$ and their corresponding eigenfunctions. Given measurements of the rotational splittings $\Delta\omega_{n,l,m}$, it is therefore possible, in principle, to solve this integral equation for the angular velocity.

Measurement of the rotational splitting $\Delta\omega_{n,l,m}$ provides a measure of rotation *in a certain region* of the Sun. In fact, the acoustic modes of progressively lower l penetrate deeper into the Sun, so that the information on the angular velocity in the deeper layers is confined to splittings of low- l modes. Similarly, because only when an acoustic mode is quasi-zonal can it reach the polar regions, the information on the angular velocity at high heliocentric latitudes is confined to splittings of low- m modes. Since the measured splittings for the low- l and low- m modes have comparatively larger relative errors, determination of the function $\Omega(r, \theta)$ thus becomes increasingly difficult with increasing depth and increasing latitude.

Several groups of workers have observed the splittings of acoustic frequencies that arise from the Sun's differential rotation. Figures 1.3 and 1.4 illustrate the inverted solution of Eq. (1.6) based on frequency splitting determinations from the latest GONG data (1996). Note that the equatorial rotation rate presents a steep increase with radius near $r = 0.7R_\odot$, thus suggesting the possibility of a discontinuity near the base of the convection zone. Note also that the equatorial rotation rate peaks near $r = 0.95R_\odot$, before decreasing with radius in the outermost surface layers. Figure 1.4 illustrates the latitudinal dependence of the inverted profile. In the outer convection zone, for latitude $\phi < 30^\circ$, the rotation rate is nearly constant on cylinders, owing to a rapidly rotating

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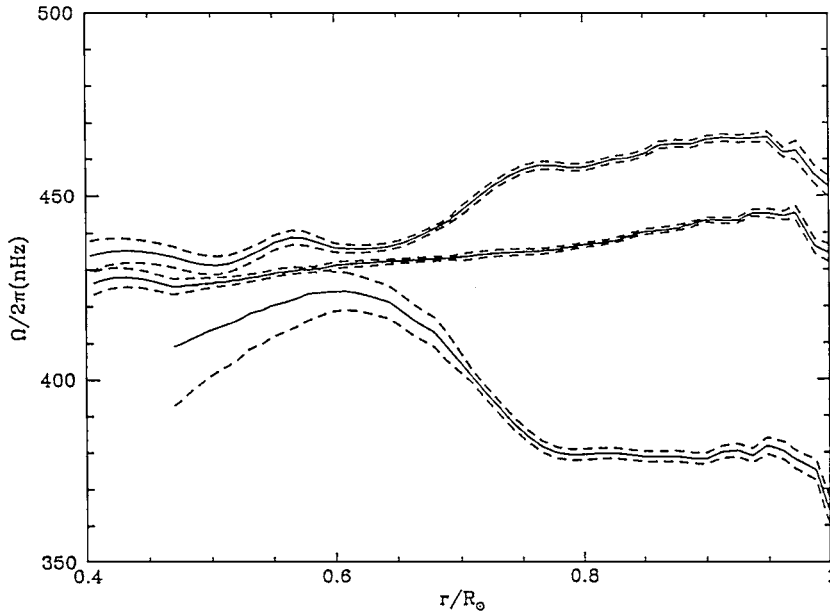


Fig. 1.3. Solar rotation rate inferred from the latest GONG data (1996). The curves are plotted as a function of radius at the latitudes of 0° (top), 30° (middle), and 60° (bottom). The dashed curves indicate error levels. *Source*: Sekii, T., in *Sounding Solar and Stellar Interiors* (Provost, J., and Schmider, F. X., eds.), I.A.U. Symposium No 181, p. 189, Dordrecht: Kluwer, 1997. (By permission. Copyright 1997 by Kluwer Academic Publishers.)

belt centered near $r = 0.95R_\odot$. At higher latitudes, however, the rotation rate becomes constant on cones. The differential character of the rotation disappears below a depth that corresponds to the base of the convection zone. This solution agrees qualitatively with the inverted profiles obtained by other groups. Perhaps the most interesting result of these inversions is that they show no sign of a tendency for rotation to occur at constant angular velocity on cylinders throughout the outer convection zone.

In summary, several inversion studies indicate that the rotation rate in the solar convection zone is similar to that at the surface, with the polar regions rotating more slowly than the equatorial belt. Near the base of the convection zone, one finds that there exists an abrupt unresolved transition to essentially uniform rotation at a rate corresponding to some average of the rate in the convection zone. This shear layer, which is known as the *solar tachocline*, is centered near $r = 0.7R_\odot$; recent studies indicate that it is quite thin, probably no more than $0.06R_\odot$. The actual rotation rate in the radiative core remains quite uncertain, however, because of a lack of accurately measured splittings for low- l acoustic modes. Several investigators have found that from the base of the convection zone down to $r \approx 0.1\text{--}0.2R_\odot$ their measurements are consistent with uniform rotation at a rate somewhat lower than the surface equatorial rate. Not unexpectedly, the rotation rate inside that radius is even more uncertain. Some studies suggest that the rotation rate of this inner core might be between 2 and 4 times larger than that at the surface. According to other investigators, however, it is more likely that this inner core rotates with approximately the same period as the outer parts of the radiative core. I shall not go into the disputes.