DYNAMICS OF MARKETS

Econophysics and Finance

Standard texts and research in economics and finance ignore the fact that there is no evidence from the analysis of real, unmassaged market data to support the notion of Adam Smith's stabilizing Invisible Hand. The neo-classical equilibrium model forms the theoretical basis for the positions of the US Treasury, the World Bank, the IMF, and the European Union, all of which accept and apply it as their credo. As is taught and practised today, that equilibrium model provides the theoretical underpinning for globalization with the expectation to achieve the best of all possible worlds via the deregulation of all markets.

In stark contrast, this text introduces a new empirically based model of financial market dynamics that explains volatility and prices options correctly and makes clear the instability of financial markets. The emphasis is on understanding how real markets behave, not how they hypothetically "should" behave.

This text is written for physics and engineering graduate students and finance specialists, but will also serve as a valuable resource for those with less of a mathematics background. Although much of the text is mathematical, the logical structure guides the reader through the main line of thought. The reader is not only led to the frontiers, to the main unsolved challenges in economic theory, but will also receive a general understanding of the main ideas of econophysics.

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The author is very grateful to the Austrian National Bank for permission to use the 1000 Schilling banknote as cover piece, and also to Schrödinger's daughter, Ruth Braunizer, and the Physics Library at the University of Vienna for permission to use Erwin Schrödinger's photo, which appears on the banknote.

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Mainly for my stimulating partner
Cornelia,
who worked very hard and effectively helping me to improve this text,
but also for our youngest son,
Finn.

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Preface

This book emphasizes what standard texts and research in economics and finance ignore: that there is as yet no evidence from the analysis of real, unmassaged market data to support the notion of Adam Smith's stabilizing Invisible Hand. There is no empirical evidence for stable equilibrium, for a stabilizing hand to provide self-regulation of unregulated markets. This is in stark contrast with the standard model taught in typical economics texts (Mankiw, 2000; Barro, 1997), which forms the basis for the positions of the US Treasury, the European Union, the World Bank, and the IMF, who take the standard theory as their credo (Stiglitz, 2002). Our central thrust is to introduce a new empirically based model of financial market dynamics that prices options correctly and also makes clear the instability of financial markets. Our emphasis is on understanding how markets really behave, not how they hypothetically "should" behave as predicted by completely unrealistic models.

By analyzing financial market data we will develop a new model of the dynamics of market returns with nontrivial volatility. The model allows us to value options in agreement with traders' prices. The concentration is on financial markets because that is where one finds the very best data for a careful empirical analysis. We will also suggest how to analyze other economic price data to find evidence for or against Adam Smith's Invisible Hand. That is, we will explain that the idea of the Invisible Hand is falsifiable. That method is described at the end of Sections 4.9 and 7.5.

Standard economic theory and standard finance theory have entirely different origins and show very little, if any, theoretical overlap. The former, with no empirical basis for its postulates, is based on the idea of equilibrium, whereas finance theory is motivated by, and deals from the start with, empirical data and modeling via nonequilibrium stochastic dynamics.

However, mathematicians teach standard finance theory as if it would be merely a subset of the abstract theory of stochastic processes (Neftci, 2000). There, lognormal pricing of assets combined with "implied volatility" is taken as the standard model.

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The "no-arbitrage" condition is regarded as the foundation of modern finance theory and is sometimes even confused with the idea of Adam Smith's Invisible Hand (Nakamura, 2000). Instead of following the finance theorists and beginning with mathematical theorems about "no-arbitrage," we will use the empirically observed market distribution to deduce a new dynamical model. We do not need the idea of "implied volatility" that is required when using the lognormal distribution, because we will deduce the empirical volatility from the observed market distribution. And, if a market perfectly satisfies a no-arbitrage condition, so is it, and if not, then so is it as well. We ask what markets are doing empirically, not what they would do were they to follow our wishes expressed as mathematically convenient model assumptions. In other words, we present a physicist's approach to economics and finance, one that is completely uncolored by any belief in the ideology of neo-classical economic theory or by pretty mathematical theorems about Martingales. One strength of our empirically based approach is that it exposes neo-classical expectations of stability as falsified, and therefore as a false basis for advising the world in financial matters.

But before we enter the realm of economics and finance, we first need to emphasize the difference of socio-economic phenomena with natural phenomena (physics, chemistry, cell biology) by bringing to light the underlying basis for the discovery of mathematical laws of nature. The reader finds this presented in Chapter 1 where we follow Wigner and discuss invariance principles as the fundamental building blocks necessary for the discovery of physical law.

Taking the next step, we review the globally dominant economic theory critically. This constitutes Chapter 2. We show that the neo-classical microeconomic theory is falsified by agents' choices. We then scrutinize briefly the advanced and very impressive mathematical work by Sonnenschein (1973a, b), Radner (1968), and Kirman (1989) in neo-classical economics. Our discussion emphasizes Sonnenschein's inadequately advertised result that shows that there is no macroeconomic theory of markets based on utility maximization (Keen, 2001). The calculations made by Radner and Kirman show that equilibrium cannot be located by agents, and that liquidity/money and therefore financial markets can not appear in the neo-classical theory.

Next, in Chapter 3, we introduce probability and stochastic processes from a physicist's standpoint, presenting Fokker–Planck equations and Green functions for diffusive processes parallel to Ito calculus. Green functions are later used to formulate market dynamics and option pricing.

With these tools in hand we proceed to Chapter 4 where we introduce and discuss the standard notions of finance theory, including the Nobel Prize winning Modigliani–Miller argument, which says that the amount of debt doesn't matter. The most important topic in this chapter is the analysis of the instability and lack

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of equilibrium of financial markets, based on the example provided by the standard lognormal pricing model. We bring to light the reigning confusion in economics over the notion of equilibrium, and then go on to present an entirely new interpretation of Black's idea of value. We also explain why an assumption of microscopic randomness cannot, in and of itself, lead to universality of macroscopic economic rules.

Chapter 5 presents standard portfolio selection theory, including a detailed analysis of the capital asset pricing model (CAPM) and an introduction to option pricing based on empirical averages. Synthetic options are also defined. We present and discuss the last part of the very beautiful Black–Scholes paper that explains how one can understand bondholders (debt owners) as the owners of a firm, while stockholders merely have options on the company's assets. Finally, for the first time in the literature, we show why Black and Scholes were wrong in claiming in their original path finding 1973 paper that the CAPM and the delta hedge yield the same option price partial differential equation. We show how to solve the Black–Scholes equation easily by using the Green function, and then end the chapter by discussing Enron, an example where the ratio of debt to equity did matter.

The main contribution of this book to finance theory is our (Gunaratne and McCauley) empirically based theory of market dynamics, volatility and option pricing. This forms the core of Chapter 6, where the exponential distribution plays the key role. The main idea is that an (x, t)-dependent diffusion coefficient is required to generate the empirical returns distribution. This automatically explains why volatility is a random variable but one that is perfectly correlated with returns x. This model is not merely an incremental improvement on any existing model, but is completely new and constitutes a major improvement on Black–Scholes theory. Nonuniqueness in extracting stochastic dynamics from empirical data is faced and discussed. We also show that the "risk neutral" option pricing partial differential equation is simply the backward Kolmogorov equation corresponding to the Fokker–Planck equation describing the data. That is, all information required for option pricing is included in the Green function of the market Fokker–Planck equation. Finally, we show how to price options using stretched exponential densities.

In Chapter 7 we discuss liquidity, reversible trading, and replicating, self-financing hedges. Then follows a thermodynamic analogy that leads us back to a topic introduced in Chapter 4, the instability of financial markets. We explain in this chapter why empirically valid thermodynamic analogies cannot be achieved in economic modeling, and suggest an empirical test to determine whether any market can be found that shows evidence for Adam Smith's stabilizing Invisible Hand.

In Chapter 8, after introducing affine scaling, we discuss the efficient market hypothesis (EMH) in light of fractional Brownian motion, using Ito calculus to formulate the latter. We use Kolmogorov's 1962 lognormal model of turbulence to

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show how one can analyze the question: do financial data show evidence for an information cascade? In concluding, we discuss Levy distributions and then discuss the results of financial data analyses by five different groups of econophysicists.

We end the book with a survey of various ideas of complexity in Chapter 9. The chapter is based on ideas from nonlinear dynamics and computability theory. We cover qualitatively and only very briefly the difficult unanswered question whether biology might eventually provide a working mathematical model for economic behavior.

For those readers who are not trained in advanced mathematics but want an overview of our econophysics viewpoint in financial market theory, here is a recommended "survival guide": the nonmathematical reader should try to follow the line of the argumentation in Chapters 1, 2, 4, 5, 7, and 9 by ignoring most of the equations. Selectively reading those chapters may provide a reasonable understanding of the main issues in this field. For a deeper, more critical understanding the reader can't avoid the introduction to stochastic calculus given in Chapter 3. For those with adequate mathematical background, interested only in the bare bones of finance theory, Chapters 3–6 are recommended. Those chapters, which form the core of finance theory, can be read independently of the rest of the book and can be supplemented with the discussions of scaling, correlations and fair games in Chapter 8 if the reader is interested in a deeper understanding of the basic ideas of econophysics. Chapters 6, 7 and 8 are based on the mathematics of stochastic processes developed in Chapter 3 and cannot be understood without that basis. Chapter 9 discusses complexity qualitatively from the perspective of Turing's idea of computability and von Neumann's consequent ideas of automata and, like Chapters 1 and 2, does not depend at all on Chapter 3. Although Chapter 9 contains no equations, it relies on very advanced ideas from computability theory and nonlinear dynamics.

I teach most of the content of Chapters 2–8 at a comfortable pace in a one-semester course for second year graduate students in physics at the University of Houston. As homework one can either assign the students to work through the derivations, assign a project, or both. A project might involve working through a theoretical paper like the one by Kirman, or analyzing economic data on agricultural commodities (Roehner, 2001). The goal in the latter case is to find nonfinancial economic data that are good enough to permit unambiguous conclusions to be drawn. The main idea is to plot histograms for different times to try to learn the time evolution of price statistics.

As useful background for a graduate course using this book, the students have preferably already had courses in statistical mechanics, classical mechanics or nonlinear dynamics (primarily for Chapter 2), and mathematical methods. Prior background in economic theory was neither required nor seen as useful, but the students

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are advised to read Bodie and Merton's introductory level finance text to learn the main terminology in that field.

I'm very grateful to my friend and colleague Gemunu Gunaratne, without whom there would be no Chapter 6 and no new model of market dynamics and option pricing. That work was done together during 2001 and 2002, partly while I was teaching econophysics during two fall semesters and also via email while I was in Austria. Gemunu's original unpublished work on the discovery of the empirical distribution and consequent option pricing are presented with slight variation in Section 6.1.2. My contribution to that section is the discovery that γ and ν must blow up at expiration in order to reproduce the correct forward-time initial condition at expiration of the option. Gemunu's pioneering empirical work was done around 1990 while working for a year at Tradelink Corp. Next, I am enormously indebted to my life-partner, hiking companion and wife, former newspaper editor Cornelia Küffner, for critically reading this Preface and all chapters, and suggesting vast improvements in the presentation. Cornelia followed the logic of my arguments, made comments and asked me penetrating and crucial questions, and my answers to her questions are by and large written into the text, making the presentation much more complete. To the extent that the text succeeds in getting the ideas across to the reader, then you have her to thank. My editor, Simon Capelin, has always been supportive and encouraging since we first made contact with each other around 1990. Simon, in the best tradition of English respect and tolerance for nonmainstream ideas, encouraged the development of this book, last but not least over a lively and very pleasant dinner together in Messina in December, 2001, where we celebrated Gene Stanley's 60th birthday. Larry Pinsky, Physics Department Chairman at the University of Houston, has been totally supportive of my work in econophysics, has financed my travel to many conferences and also has created, with the aid of the local econophysics/complexity group, a new econophysics option in the graduate program at our university. I have benefited greatly from discussions, support, and also criticism from many colleagues, especially my good friend and colleague Yi-Cheng Zhang, who drew me into this new field by asking me first to write book reviews and then articles for the econophysics web site www.unifr.ch/econophysics. I'm also very much indebted to Gene Stanley, who has made *Physica A* the primary econophysics journal, and has thereby encouraged work in this new field. I've learned from Doyne Farmer, Harry Thomas (who made me realize that I had to learn Ito calculus), Cris Moore, Johannes Skjeltorp, Joseph Hrgovcic, Kevin Bassler, George Reiter, Michel Dacorogna, Joachim Peinke, Paul Ormerod, Giovanni Dosi, Lei-Han Tang, Giulio Bottazzi, Angelo Secchi, and an anonymous former Enron employee (Chapter 5). Last but far from least, my old friend Arne Skjeltorp, the father of the theoretical economist Johannes Skjeltorp, has long been a strong source of support and encouragement for my work and life.

I end the Preface by explaining why Erwin Schrödinger's face decorates the cover of this book. Schrödinger was the first physicist to inspire others, with his Cambridge (1944) book *What is Life?*, to apply the methods of physics to a science beyond physics. He encouraged physicists to study the chromosome molecules/fibers that carry the "code-script." In fact, Schrödinger's phrase "code-script" is the origin of the phrase "genetic code." He attributed the discrete jumps called mutations to quantum jumps in chemical bonding. He also suggested that the stability of rules of heredity, in the absence of a large N limit that would be necessary for any macroscopic biological laws, must be due to the stability of the chromosome molecules (which he called linear "aperiodic crystals") formed via chemical bonding à la Heitler–London theory. He asserted that the code-script carries the complete set of instructions and mechanism required to generate any organism via cellular replication, and this is, as he had guessed without using the term, where the "complexity" lies. In fact, *What is Life?* was written parallel to (and independent of) Turing's and von Neumann's development of our first ideas of complexity. Now, the study of complexity includes economics and finance. As in Schrödinger's day, a new fertile research frontier has opened up.

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April 9, 2003