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0521023955 - Social Goals and Social Organization: Essays in Memory of Elisha Pazner

Edited by Leonid Hurwicz, David Schmeidler and Hugo Sonnenschein

Excerpt

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CHAPTER 1

A perspective*Leonid Hurwicz*

Because this volume is dedicated to the memory of Elisha Pazner, let me start with two reminiscences. I first met Elisha sometime in 1969 or 1970 when I was visiting at Harvard. Elisha consulted with me on problems related to his dissertation. Initially, we did not find it easy to communicate, but I was impressed by his original insights and depth. Then, after 1971, I was back at Minnesota and he was visiting at Northwestern. By that time we shared the interest in problems of implementation. We had conversations on several occasions, including at least one visit by Elisha to Minneapolis. He was, I remember, particularly concerned about the relationship of the positive “free rider” results due to Groves and Ledyard to my earlier negative results concerning incentive compatibility (extended to public goods by Ledyard and Roberts [1974]). I feel indebted to Elisha both for the stimulation to pursue this issue and clarification of many essential points.

Subsequently, Elisha’s interests moved toward the implementation of rules satisfying criteria of fairness as well as efficiency in environments involving production, especially in the presence of a nontransferable endowment such as labor. His pathbreaking contributions in this area are discussed and references given in two essays of the present book (Varian and Thomson; Postlewaite).

All the essays in this book qualify under the rubric of *normative economics*: they go beyond the *positive economics* designed to explain the observed economic phenomena and aim at the development of criteria to be used in judging economic policies and systems. Normative economics, in turn, has focused on two major issues: one examining the logic and merits of the various welfare criteria in terms of which the consequences

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Excerpt

[More information](#)2 **Leonid Hurwicz**

of economic actions may be judged, and the other examining – in the light of such criteria – the workings of various policies and mechanisms.

Among the essays in this volume, two – focused on welfare criteria (d'Aspremont; Varian and Thomson) – are of the first category; four – devoted to problems of implementation – are of the second category. Of the latter, one (Myerson) is devoted to Bayesian implementation, one (Muller and Satterthwaite) to dominance implementation, and two (Maskin; Postlewaite) to Nash implementation. Two essays (Kalai; Milgrom) are devoted to problems (bargaining, bidding) that are outside the present framework.

The first category of studies has led to the development of the concepts of orderings and other attributes of social welfare, social welfare functions, and performance (or: social choice) functions or correspondences (rules). Among the orderings of social states, the Pareto criterion is the best known and most frequently used. More recently, different notions of fairness, so important in Pazner's work, have come to play an important role. Among social welfare functions, those associated with the names of Bergson, Samuelson, and Arrow are of prime importance. Performance functions were perhaps first formalized and their importance stressed by Reiter and Mount in the context of informationally oriented models, and by Maskin in a game-theoretic (incentive-respecting) framework.¹

In the earlier literature (Bergson [1938], Lange [1942], Lerner [1944], Arrow [1951], Koopmans [1957], Debreu [1959], Arrow and Hahn [1971]), the central question asked was whether certain mechanisms (especially the competitive or monopoly mechanism) generated Pareto-optimal allocations, and – if so – for what categories of economic environments. Subsequently, the question was reversed: instead of regarding the mechanism as given and seeking the class of environments for which it works well, one seeks mechanisms that will work well for a given class of environments. Initially, in part because of the nature of Hayek, Mises, Lange, Lerner debates concerning the feasibility of socialism, the emphasis was on the informational (as distinct from incentival) aspects of economic mechanisms. One question posed was whether, for “classical”² environments, there exist mechanisms with the same optimality proper-

¹ Among related precursor concepts one should mention that of a social choice function $C(S)$ in Arrow [1951, 1963] and the notion of the choice function $C(S, R)$ in Sen [1970]. It is important to note that, in its contemporary version, the social performance correspondence need not be derived from any maximization process, and that its domain is a class of environments (n -tuples of individual characteristics) rather than merely feasible sets and preference profiles. In particular, the status quo (e.g., initial endowment) may be part of the specification of the environment. The idea of a more general concept of a social choice function is suggested in Arrow ([1963], footnote 34, p. 104).

² Satisfying the assumptions of convexity, divisibility, and so forth as, for instance, in Koopmans's [1957] Propositions 4 and 5.

ties as perfect competition but, in some sense, informationally more efficient than (or as efficient as) perfect competition. It was shown (Mount and Reiter [1974], Hurwicz [1977], Osana [1978]) that no mechanisms with a smaller message space would guarantee optimality. Analogous results for the Lindahl mechanism were obtained by Sato [1981]. A uniqueness result for the Walrasian mechanism was obtained by Jordan [1982] under the additional postulate of individual rationality.

But not everyone was satisfied with the analysis of performance of various mechanisms restricted to classical environments and to their purely informational properties. In particular, incentive properties in two types of nonclassical environments had engaged the economists' attention for a long time: technologies with increasing returns, and public goods. Indeed, in each of these areas alternative mechanisms had been suggested to supplement, or substitute for, the competitive market: for increasing returns, marginal cost pricing; for public goods, the Lindahl solution. (See Lindahl [1919], Hotelling [1938], Lange [1938], Lerner [1944].)

Each of these remedies was shown to have certain defects, and the analysis of these defects led to important theoretical developments. For our purposes, the problem of public goods serves as an excellent example.

Public goods

To begin with, standard theorems concerning the optimality properties of competitive equilibria assume the absence of public goods. Indeed, it is not obvious how the competitive equilibrium in an economy with public goods is to be defined. It seems clear, however, that any reasonable definition of a competitive market would not yield Pareto optimality in the presence of public goods. On the other hand, Lindahl equilibrium (this term is formally defined below) in public goods economies is well defined and, under the customary assumptions of convexity and so on, Pareto optimal (Foley [1970], Milleron [1972]). But, as pointed out by Samuelson [1954, 1955], it may be unrealistic to expect the behavior required of individuals in order that a Lindahl equilibrium prevail.

To understand the problem, let us illustrate the Lindahl equilibrium in Lindahl's own simple setting. In one scenario representing the Lindahl mechanism, there is an "auctioneer" proposing shares p_i ($i = 1, \dots, n$), $\sum_{i=1}^n p_i = 1$, of the aggregate cost of a public service to be borne by the n participants; in turn, the i th participant responds by specifying the level y_i of the public service that would maximize his or her utility given p_i ; agent i would then contribute $p_i y_i$ to cover the costs of public service. Equilibrium obtains when the shares p_i are so chosen that all agents desire the same level of public service, that is, $y_1 = \dots = y_n$.

4 **Leonid Hurwicz**

Let the i th agent's utility function in an economy, with two goods be $\bar{u}^i(x^i, y)$ where x^i is the amount of private good X held by i (after taxes) and y is the level of public service Y provided to every agent. Assume furthermore, as is often done, that Y can be produced by using X as the input; that the production function is one of constant returns; and that the units of measurement are chosen so that one unit of X produces one unit of Y . If the contribution of X (tax paid) by agent i is denoted by t^i , we have $x^i = \bar{x}^i - t^i$ where \bar{x}^i is the initial X -endowment of agent i . Assuming zero initial endowment of Y and production efficiency feasibility, we obtain the balance requirement

$$y = \sum_{i=1}^n t^i.$$

A *Lindahl equilibrium* for the economy $\bar{e} = (\bar{e}^1, \dots, \bar{e}^n)$, $\bar{e}^i = (\bar{u}^i, \bar{x}^i)$ may be defined as a vector

$$(x^{*1}, \dots, x^{*n}, y^*; p^{*1}, \dots, p^{*n})$$

such that

$$\sum_{i=1}^n p_i^* = 1$$

$$y^* = \sum_{i=1}^n (\bar{x}^i - x^{*i});$$

and for each $i = 1, \dots, n$,

$$p^{*i} \geq 0, \quad (x^{*i}, y^*) \text{ is individually feasible,}^3$$

$$p_i^* y^* + x^{*i} = \bar{x}^i$$

and

$$\bar{u}^i(x^{*i}, y^*) \geq \bar{u}^i(x^i, y)$$

for each individually feasible (x^i, y) satisfying the budget equality $p_i^* y + x_i = \bar{x}^i$.

The preceding defines the *Lindahl allocation correspondence* L on the class E of economies by

$$L(e) = \{(x^1, \dots, x^n, y): \text{for some } (p^1, \dots, p^n), \text{ the vector } (x^1, \dots, x^n, y; p^1, \dots, p^n) \text{ is a Lindahl equilibrium for } e\},$$

for every e in E . [Here, again, $e = (e^1, \dots, e^n)$, $e^i = (u^i, \bar{x}^i)$.]

³ That is, (x^{*i}, y^*) is in the i th agent's consumption set C^i , often chosen as the non-negative quadrant.

A perspective

5

The Lindahl scenario assumes that each agent treats the proposed shares p_i parametrically (i.e., as given) and that the agent's response y_i is based on the maximization of the agent's *true* utility function. But, as stressed by Samuelson, agents might find it to their advantage to respond with a value of y_i that does not maximize their true utility functions. That is, they might benefit by misrepresenting their preferences. Consequently, the Lindahl mechanism may fail to produce Pareto-optimal outcomes. If it does fail, is there another mechanism that would succeed where the Lindahl mechanism fails? In searching for such a mechanism, how is the problem to be formalized?

One answer is to be found in the theory of noncooperative games.⁴ Specifically, one may consider rules prescribing resource allocation behavior (such as Lindahl, or competitive profit maximization, or marginal cost pricing) as equivalent to direct-revelation mechanisms, giving rise to a special class of games (Hurwicz [1972]). But the terms "mechanism" and "direct revelation" must be defined.

A *mechanism* is defined by endowing each of the n participants with a *strategy domain*, with S^i denoting the domain of the i th agent, and the *outcome function* (also called *strategy outcome function* or *game form*) denoted by h ; this outcome function h specifies the resource allocation (x^1, \dots, x^n, y) resulting from any n -tuple of strategy choices. Thus the outcome function specifies the rules of the game; for the economy described previously, it may be written as follows:

$$(x^1, \dots, x^n, y) = h(s^1, \dots, s^n), \quad s^i \in S^i, \quad i = 1, \dots, n.$$

Formally, a mechanism is the ordered pair (S, h) , $S = S^1 \times \dots \times S^n$. The space of outcomes containing the range of h will be noted by Z .

Now consider an economy $e = (e^1, \dots, e^n)$ where e^i has the utility function u^i , and a mechanism $m = (S, h)$, $S = S^1 \times \dots \times S^n$. Let Γ denote the noncooperative game $(S, \varphi_{m,e})$ where the i th agent's payoff function is given by $\varphi_{m,e}^i(s) = u^i(h(s))$ for all s in S . Denote by $v_m(e)$ the set of Nash equilibria of the game Γ for the economy e .⁵ We say that the mechanism $m = (S, h)$ *Nash implements* a performance correspondence $F: E \rightarrow Z$ on E if it is the case that for every e in E , (1) the set $v_m(e)$ is

⁴ An important role in the development of public goods theory was played by the pioneering contributions of Drèze and de la Vallée Poussin [1969], Malinvaud [1969], and the subsequent dynamics-oriented literature. Unfortunately, I am unable to cover this work in the present essay.

⁵ $(s^1, \dots, s^n) = s^*$ in S is a *Nash equilibrium* of the game $\Gamma = (S, \varphi)$, $S = S^1 \times \dots \times S^n$, if, for every $i = 1, \dots, n$, it is the case that $\varphi^i(s^*) \geq \varphi^i(i, s^i/s^*)$ for all s^i in S^i , where $i, s^i/s^*$ denotes the n -tuple s^* with its i th entry s^{*i} replaced by s^i .

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[More information](#)

6 Leonid Hurwicz

not empty, and (2) $h(v_m(e)) \subseteq F(e)$. (This is sometimes called “weak” implementation.)

The general problem of designing a mechanism that yields Pareto-optimal outcomes in a given range E of environments can now be formally stated as that of finding a mechanism implementing the Pareto correspondence $P : E \rightarrow Z$ on E where $P(e)$ is the set of allocations that are Pareto optimal for e .

A *direct-revelation mechanism* in $E = E^1 \times \dots \times E^n$ may be defined by the following property: $S^i = E^i$, where E^i is the class of a priori admissible characteristics of agent i ($i = 1, \dots, n$).

A *natural*⁶ direct-revelation mechanism in $E = E^1 \times \dots \times E^n$ for the performance correspondence $F : E \rightarrow Z$ may then be defined as a direct-revelation mechanism in E such that, for $s = e$ [i.e., for $(s^1, \dots, s^n) = (e^1, \dots, e^n)$], the outcome $h(s)$ is an element of $F(e)$; that is, $h(s)|_{s=e} \in F(e)$. In particular, when F is a (single-valued) function and the mechanism m is natural for F , it is the case that $h(s)|_{s=e} = F(e)$. That is to say, in a natural direct-revelation mechanism, the outcome is one that would be desirable according to the performance function F if the agents were truthful. The corresponding game $\Gamma = (E, \varphi_{m,e})$ is called a *natural direct-revelation game* for F .

In such a natural direct-revelation game for F one would wish that the (unique) n -tuple of Nash equilibrium strategies be *truthful*; the mechanism is then called *straightforward*. This requirement can be written as

$$v_\Gamma(e) = \{e\}.$$

For clearly, in this case, Γ Nash implements F . (It does not follow that a mechanism must be straightforward to implement F .) A natural direct-revelation mechanism whose Nash equilibria are truthful is called *incentive compatible*.

Now it turns out (d’Aspremont and Gérard-Varet [1979b], Thm. 1; Dasgupta, Hammond, and Maskin [1979], Thm. 7.1.1) that if a mechanism is incentive compatible, its Nash equilibria are also dominance equilibria. But it is also known that, generally speaking, there do not exist mechanisms guaranteeing dominance equilibria for sufficiently broad classes of environments when F is Pareto optimal (i.e., a subcorrespondence of the Pareto correspondence).⁷ Hence the aim of constructing

⁶ The term *natural* is introduced to focus attention on the postulated property of the outcome function.

⁷ For economies with transferable utilities (of the form $u^i(x^i, y) = x^i + v_i(y)$) this non-existence follows (roughly) from the following results. (1) If for a given mechanism dominance equilibria exist, then there is an “equivalent” natural direct mechanism with

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Excerpt

[More information](#)**A perspective**

7

incentive-compatible equilibria in the sense of the preceding definition is too ambitious. (This is true not only for public goods economies, but also for private goods pure exchange economies. See the subsequent discussion.)

The recognition of this fact can lead to several alternative compromises. One might sacrifice Pareto optimality or postulate probabilistic (Bayesian) beliefs. This latter approach was pioneered by Harsanyi [1967, 1968]; its economic applications include contributions by d'Aspremont and Gérard-Varet [1979a,b] and Arrow [1977]. (See the essay by Myerson, this volume.) But we shall confine ourselves here to the study of another compromise, sacrificing the dominance equilibrium property. We shall still ask for Pareto optimality and Nash implementability but no longer in a natural direct-revelation game. Thus there is no longer any necessary relationship between S^i and E^i and no requirement corresponding to $h(s)|_{s=e} \in F(e)$. (In fact, if the game is not of direct-revelation type, $s = e$ no longer makes sense.)

In this broader class of games, the Nash implementation of a Pareto-optimal performance correspondence is no longer impossible. This was shown by Groves and Ledyard who were the first to construct a mechanism (not of direct-revelation type) that yielded Pareto-optimal Nash allocations in public goods environments. However, the performance correspondence of the Groves–Ledyard mechanism was not individually rational. It became, therefore, natural to ask whether some Pareto optimal individually rational correspondence could be Nash implemented. Since the Lindahl performance is both Pareto optimal and individually rational, the question could be answered affirmatively by showing that the Lindahl correspondence is Nash implementable. As it turned out, it is

truth as everyone's dominant strategy (Dasgupta, Hammond, and Maskin [1979], Thm. 4.1.1). (2) On "smoothly connected" domains V' of valuation functions v_i , any mechanism with truthful dominance equilibria is a "Groves mechanism" (Green and Laffont [1977, 1979]; Holmström [1979], Thm. 1). (3) A Groves mechanism is generically unbalanced (Green and Laffont [1979], Walker [1980], Thm. 1). (4) An unbalanced mechanism is not Pareto optimal.

Note that the case of transferable utilities is somewhat special. In this case there is a unique interior Pareto-optimal level \hat{y} of the public service, defined for differentiable utility functions by the Samuelson condition $\sum_{i=1}^n v'_i(\hat{y}) = 1$. (Uniqueness of \hat{y} follows from the fact that v'_i is assumed strictly decreasing.) For such economies it is possible to design incentive-compatible mechanisms that will yield a Pareto-optimal level \hat{y} of the public service (Clarke [1971], Groves [1970], Groves and Loeb [1975], Green and Laffont [1979]). However, the balance condition cannot in general be satisfied, hence the resulting allocation, say $(x^1, \dots, x^n, \hat{y})$ is not, in general, Pareto optimal.

8 **Leonid Hurwicz**

not difficult to construct mechanisms that Nash implement the Lindahl correspondence (Hurwicz [1979], Walker [1981]).⁸

For balanced implementation of the Lindahl correspondence when there are three or more agents, *smooth* mechanisms ignoring the individual feasibility condition were constructed by Hurwicz [1979s]⁹ and Walker [1981].¹⁰ The former has price-quantity proposals as strategic messages; that is, $m_i = (p_i, y_i)$, and uses a circular arrangement of participants so that each agent i is an auctioneer for his neighbor, say agent $i + 1$. Walker's arrangement is analogous but using a message space of smaller dimensions. Specifically, in a two-good, n -agent world with constant returns, the Walker strategy space is of dimension n while the Hurwicz strategy space is of dimension $2n$. Clearly, the Walker mechanism minimizes the dimensionality of the message space.

A (discontinuous) balanced mechanism with profile type strategy spaces where individual feasibility conditions *are* satisfied is constructed in Hurwicz, Maskin, and Postlewaite [1980]. Subject to additional conditions on the preferences, a Constrained Lindahl correspondence is feasibly implemented.

The mechanisms just mentioned are balanced, in the sense that, for every s in S , the balance requirement

$$y = \sum_{i=1}^n t^i$$

is satisfied.¹¹ But the requirement of individual feasibility [i.e., that $(x^i, y) \geq 0$] might have been violated. (The same problem arose in Schmeidler's [1976] and Hurwicz's [1979] mechanisms Nash implementing the Walrasian correspondence.) However, as shown in Hurwicz, Maskin, and Postlewaite [1980], mechanisms analogous to those in Maskin [1977] can be constructed to Nash implement Lindahl, Walras, and various other correspondences without violating either the balance or the individual feasibility requirements.

These findings raise the more general question: Which correspondences are Nash implementable without violating the feasibility (individual and balance) requirements? To a considerable extent the question

⁸ The trailblazing contribution to the analogous problem in private goods economies is due to Schmeidler [1976], who constructed a mechanism Nash implementing the Walrasian correspondence. However, see the subsequent text concerning the individual feasibility requirement.

⁹ A two-good economy (one public, one private) and constant returns technology.

¹⁰ Extended to arbitrarily many public and private goods and more general technologies.

¹¹ More explicitly, write $(t^1, \dots, t^n, y) = h(s) = (T^1(s), \dots, T^n(s), Y(s))$; then the balance requirement is that $Y(s) = \sum_{i=1}^n T^i(s)$ for all $s \in S$.

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Excerpt

[More information](#)**A perspective**

9

was answered in Maskin [1977] for cases where the feasible outcome set is a priori known to the designer of the mechanism. Maskin showed that (1) a Nash-implementable correspondence must be monotone;¹² and (2) if there are at least three agents, any correspondence that is monotone and has the “no-veto power”¹³ property is Nash implementable.

Analogous Nash-implementability conditions for cases where the feasible set is not a priori known to the designer are given in Hurwicz, Maskin, and Postlewaite [1980].¹⁴

It is to be noted that the implementability results in Maskin [1977] and Hurwicz, Maskin, and Postlewaite [1980] are proved by constructive methods. Thus, for example, when the feasible set is a priori known to the designer and the correspondence to be implemented is monotone and has the “no-veto power” property, Maskin’s proof of Nash implementability shows how to construct the outcome functions using preference profiles as the players’ strategic variables. In Hurwicz, Maskin and Postlewaite, the profiles (e^1, \dots, e^n) , where e^i involves endowments and/or production sets as well as preferences of the i agent, are used as the strategic variables.

Economists are particularly interested in implementing correspondences that are Pareto optimal¹⁵ and either individually rational or envy free (i.e., fair). Let F be Pareto optimal and individually rational. Somewhat surprisingly, it turns out (Hurwicz [1979b]) that if (i) F is Pareto optimal, individually rational, and continuous, (ii) E is sufficiently broad, and (iii) F is Nash implementable over E , then (a) in public goods economies $F \supseteq L$, while (b) in private goods pure exchange economies, $F \supseteq W$ (where W is the Walrasian correspondence).

An analogous result for private goods pure exchange economies concerning fairness is due to Thomson [1979], who showed that if (i) F is Pareto optimal, envy free, and continuous, (ii) E is sufficiently broad, and (iii) F is Nash implementable over E , then $F \supseteq W_{I(w)}$. [Here $W_{I(w)}$ is

¹² F is monotone when the following holds: if an outcome z is F -desirable for a preference profile \mathbf{R} , and another profile \mathbf{R}' is no less favorable to z than \mathbf{R} was, then z is also F -desirable for \mathbf{R}' (Maskin [1977]). (‘ z is F -desirable for the environment e ’ means that z is an element of $F(e)$; ‘ \mathbf{R}' is no less favorable to z than \mathbf{R} was’ means that $zR_i z'$ implies $zR'_i z'$ for all i and all z' .)

¹³ F has the “no-veto power” property when the following holds: if an outcome z is the most preferred one for at least $n - 1$ agents then z is F -desirable (Maskin [1977]).

¹⁴ In particular, an example due to Postlewaite (Hurwicz, Maskin, and Postlewaite [1980]) shows that in environments where boundary equilibria can occur, the Walras correspondence is not monotone and must be replaced by its “constrained” counterpart.

¹⁵ That is, correspondences F such that $F(e) \subseteq P(e)$ for all e in E where $P(e)$ is the set of Pareto-optimal outcomes in the environment e , and E is the class of environments over which implementation is sought.

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10 Leonid Hurwicz

the Walrasian correspondence from the equal initial endowment point $I(w)$ obtained by the redistribution of $w = (w^1, \dots, w^n)$; w^i is i 's initial endowment vector.¹⁶

Two-agent economies

It is worth noting that special difficulties arise with regard to balanced Nash implementation when there are only two agents ($n = 2$). Most mechanisms referred to in the preceding sections work only for three or more persons. It is possible,¹⁷ however, to Nash implement Walrasian and Lindahl correspondences when $n = 2$ by balanced (but discontinuous) outcome functions (Hurwicz [1979c], Miura [1982]¹⁸). That this cannot be done by *smooth* functions is shown in Reichelstein [1984]. (An analogous result for the Pareto-optimal correspondence has recently been obtained by Hurwicz and Hans Weinberger.)

Implementation through profiles as strategies (as in Maskin [1977] and Hurwicz, Maskin, and Postlewaite [1980]) has the disadvantage of using huge (indeed, infinite-dimensional) strategy domains, and discontinuous outcome functions. But it should be understood that the profile approach provides not merely the implementation of a particular performance correspondence but, rather, an *algorithm* for constructing outcome functions implementing a large class of performance correspondences.

When the specific correspondence to be implemented is known, it may be possible to do much better. In particular, for a pure exchange private goods economy, with three or more agents, and with endowments a priori known to the designer, Postlewaite and Wettstein [1983] have constructed a mechanism with a *continuous* outcome function where each agent's strategy domain is of finite dimension (equal to $2l + 1$ where l is the number of goods), a mechanism that Nash implements the Constrained Walrasian correspondence.

For *smooth* balanced mechanisms¹⁷ Reichelstein [1982] has found minimal dimensions of strategy spaces for Nash implementing the Walrasian correspondence in two-good economies with three or more

¹⁶ That is, for $e = (e^1, \dots, e^n)$, $e^i = (u^i, w^i)$, $i = 1, \dots, n$, we have $z \in W_{H(w)}(e)$ if and only if z is the Walrasian allocation in an economy with the same preferences (u^1, \dots, u^n) but with each agent's initial endowment equalized to $I(w) = \sum_{i=1}^n w^i/n$.

¹⁷ Ignoring the individual feasibility condition.

¹⁸ Miura points out an error in Hurwicz's [1979c] Lindahl-implementing outcome function; his modified outcome function does implement the Lindahl correspondence for two or more agents.