
Introduction

A biographical sketch

Walras's life in France

Léon Walras was the founder of the modern theory of general economic equilibrium.¹ He noted in his autobiography (1965, *I*, pp. 1–15),² from which most of the information in this sketch is drawn, that he was born on December 16, 1834, in Evreux in the Department of Eure in France, and christened Marie Esprit Léon. His father was Antoine Auguste Walras, a secondary school administrator and an amateur economist and literary critic; his mother was Louise Aline de Sainte Beuve, the daughter of an Evreux notary. After studying at the College of Caen from 1844 to 1850, he entered the *lycée* of Douai, where he received a *bachelier-ès-lettres* in 1851 and a *bachelier-ès-sciences* in 1853. He entered the School of Mines of Paris in 1854, but finding the course of preparation of an engineer not to his liking, he gradually abandoned his academic studies in order to cultivate literature, philosophy, and social science. Although those efforts resulted in a short story and a novel, *Francis Sauveur* (1858), it rapidly became apparent to him that social science was his true interest. Accordingly, in 1858 he agreed to his father's request that he give up literature and devote himself to economics, and promised to continue his father's investigations.³

During his youth in Paris, Walras became a journalist for *Journal des Economistes* and *La Presse* from 1859 to 1862, the author of a refutation (1860) on philosophical grounds of the normative economic doctrines of P.-J. Proudhon, an employee of the directors of the Northern Railway in 1862, and managing director of a cooperative association bank in 1865. He gave public lectures on cooperative associations in 1865, from 1866 to 1868 was co-

¹ For an account of the school of economic theory that he founded and references to the works of its members, see Walker 1996a.

² See the last section of this introduction for conventions on citations of Walras's writings.

³ Auguste chose the opposite of the plan that he recommended to Léon. Auguste decided to give up economics and to concentrate on literature for much of his career, returning briefly to economics only at the end of the 1850s, without adding anything of significance to his earlier work (see the intellectual biography of Auguste by Pierre-Henri Goutte and Jean-Michel Servet in Auguste Walras 1990, CXXI–CLXXII).

2 Introduction

editor and publisher with Léon Say of the journal *Le Travail*, a review devoted largely to the cooperative movement, and during those years gave public lectures on social topics (1868). After the failure of the association bank in 1868, he found employment with a private bank until 1870. During the 1860s he tried intermittently to obtain an academic appointment in France, but he lacked the necessary educational credentials, and the eleven economics positions in higher education in France were monopolized by orthodox economists who, he complained, passed their chairs on to their relatives. His fortunes ultimately changed as a result of his participation in 1860 in an international congress on taxation in Lausanne, for that brought him to the attention of Louis Ruchonnet, a Swiss politician who secured his appointment in 1870 to an untenured professorship of economics at the Academy (subsequently University) of Lausanne in Switzerland. He was made a tenured professor there in 1871, and held that position throughout his teaching career.

Walras's personal life was initially unconventional. He and Célestine Aline Ferbach (1834–79) formed a common law union in the late 1850s. She had a son, Georges, by a previous liaison, and she and Walras had twin daughters in 1863, one of whom died in infancy. In 1869 he married Célestine, thereby legitimizing their daughter, Marie Aline, and adopted Célestine's son. A long illness of Célestine's and the meagerness of Walras's salary made life very difficult for him for several years. His time and energy were sorely taxed not only by the need to care for his wife but by the need to supplement his salary by teaching extra classes, contributing to the *Gazette de Lausanne* and the *Bibliothèque Universelle*, and working as a consultant for La Suisse insurance company. Five years after Célestine's death in 1879, Walras married Léonide Désirée Mailly (1826–1900). The marriage was a happy one. Her annuity relieved his financial distress, and his situation was further improved in 1892 by an inheritance of 100,000 francs from his mother, which enabled him to pay debts incurred in publishing and disseminating his works and to buy an annuity of 800 francs.

Influences on his work. Walras's professional life was devoted to research and teaching. He frequently asserted that his research was a development of his father's, and that was true in some respects. It was under the influence of his father's classification of economic studies that Léon, as early as 1862, planned the division of his life's work into the study of pure theory, economic policies, and normative goals (Walras to Jules du Mesnil Marigny, December 23, 1862, in 1965, *I*, letter 81, pp. 117–24). He ultimately set forth those areas of study respectively in his book *Eléments d'économie politique pure*⁴ and his two collections of essays, *Etudes d'économie social* (1896c) and

⁴ The title of all editions of this work is shortened to *Eléments* throughout this book.

Introduction

3

Etudes d'économie politique appliquée (1898b). Léon adopted his father's classification of the factors of production into the services of labor, land, and capital goods, regarding the source of each service as a type of capital. He adopted his father's definition of capital as wealth that can be used more than once and of income as wealth that can be used only once, and revised his father's vague use of the term "extensive utility," using it to mean the quantity-axis intercept of a market demand curve. The topic of utility had been treated in French thought by writers such as the Abbe Fernandino Galiani (a Neapolitan diplomat at Versailles) and Etienne Bonnot de Condillac (a priest and philosopher), and it was given further development by Auguste and Léon Walras. Auguste used the word *numéraire* to mean an abstract unit of account, and Léon adapted the meaning of the word to his purposes. Auguste's philosophy of social justice and his belief in the desirability of nationalizing land were advocated by Léon throughout his adult life. Léon's major economic theories, however, were derived from his own original inspiration and from sources other than his father. Auguste's greatest contribution to Léon's development as an economist were to encourage him to study economics, to suggest that it should be a mathematical science (A.A. Walras 1831, ch. 18; Walras 1965, *I*, p. 493), and to give him access to a library of books on economics.

In that library was Antoine Augustin Cournot's *Recherches sur les principes mathématiques de la théorie des richesses* (1838), which Léon Walras credited with having demonstrated that economics could and should be expressed in mathematical form (Walras to A.A. Cournot, March 20, 1874, in 1965, *I*, letter 253, pp. 363–67; Walras to Henry L. Moore, January 2, 1906, in 1965, *3*, letter 1614, pp. 292–93; Walras 1905a). Cournot's work introduced Walras to the mathematical formulation of exchange between two locations, the theory of monopoly and the associated conditions for profit maximization, and the analysis of how prices are repeatedly changed in a search for equilibrium in a purely competitive market, all topics that Walras developed in his own work. The first demand curve that Walras beheld was Cournot's, and he found it immensely suggestive. He was critical of it, however, because he perceived that Cournot's postulate that the quantity demanded of a commodity is a function only of its own price is inaccurate if more than two commodities are exchanged, and that Cournot did not provide a theoretical rationale for the demand function. Those perceptions, Walras observed, were the starting point for his own inquiries (1965, *I*, p. 5; 1905a).

Other ingredients that went into the composition of Walras's theories were provided by Adam Smith, John Stuart Mill, François Quesnay, A.R.J. Turgot and Jean-Baptiste Say. Smith had revealed many of the consequences of unfettered competition and had formulated the concept of normal value. Mill had provided a supplement to and reinforcement of Cournot's and Smith's

4 Introduction

analyses of competitive pricing (Walras to Ladislaus von Bortkiewicz, February 27, 1891, in 1965, 2, letter 999, pp. 434–35), and also an extension and grand synthesis of classical doctrines that served Walras as a catalyst for critical studies. Quesnay, in his *Tableau économique*, had expressed the concept of a circular flow of income and the interdependence of the various parts of the economy. Turgot had clearly delineated the idea of the simultaneous and mutually determined general equilibrium of those parts. Say (1836) had suggested the distinction between the incomes of the factors of production and the demand for commodities. Walras sharpened those ideas and made them a fundamental part of his general-equilibrium models.

Achylle-Nicolas Isnard's *Traité des richesses* (1781), a book that Walras owned and that may have been in his father's library, was probably an important source of some of Walras's constructions (Jaffé 1969). Like Walras, Isnard was interested in determining equilibrium price ratios, set up a system of simultaneous equations of exchange showing the dependence of the value of each commodity on the values of the others, stressed the necessity of having as many independent equations as unknowns, and perceived that the use of a numeraire rendered his system determinate. Anticipating Walras's treatment of production, Isnard assumed given ratios of the inputs in a mathematical model and expressed the costs of production in equation form. Also like Walras, Isnard studied the allocation of capital among different uses, coming to the conclusion, as did Walras, that in equilibrium the net rate of income of different capital goods is the same.

Finally, Louis Poinso's *Eléments de statique* (1803) exerted a powerful influence on Walras. He first read that book when he was nineteen, and kept it at his bedside for decades (Walras to Melle Dick May, May 23, 1901, in 1965, 3, letter 1483, pp. 148–49). Poinso painted a picture of the mutual interdependence of a vast number of variables, of how the dynamic forces in physical systems eventuate in an equilibrium in which each object is sustained in its path and relative position. Inspired by the implications of Poinso's work, Walras conceived a magnificent project. He would emulate Poinso's vision and analysis in reference to the general equilibrium of the economic universe! That he tried to carry out the plan can be inferred from the striking similarity of the form of his work to Poinso's, with its careful delineation of functional dependences and parameters, its sets of simultaneous equations, and its equilibrium conditions. Equipped, therefore, with ideas that he could take as building blocks and points of departure, with enough geometry and algebra to put together mathematical statements of economic relationships and conditions – his use of calculus in the *Eléments* came after the first edition – and with the explicit objective of developing a mathematical theory of general equilibrium, Walras began his scholarly activity in Lausanne in 1870.

Introduction

5

Walras was an extremely conscientious teacher, but he was an uninspiring lecturer (Jaffé in 1965, 2, p. 560), and the students at Lausanne were interested in careers in law, not in economics, so he failed to develop disciples among them. Moreover, he was with increasing frequency afflicted by bouts of mental exhaustion and irritability that made it difficult for him to lecture and to read and write. In 1892 he took a leave to regenerate his strength in order to be able to continue teaching, but soon realized he would find the strain of returning to his tasks insupportable and retired in that year, being at that time fifty-eight years old. In 1899 and 1900 he revised the *Eléments* once more. After 1900 he made no theoretical contributions but wrote some articles, and in late 1901 and 1902 made some inconsequential changes to the *Eléments* that were ultimately incorporated into the text of the fourth edition (1900) to produce the edition of 1926. He died on January 5, 1910, in Clarens, Switzerland, and was buried there.

Many models

Léon Walras's theoretical work is complex in several ways. Economists imagine that he presented a general economic model composed of submodels that are smoothly integrated into a seamless whole. This book will show that his work on markets is quite otherwise. He constructed many models of markets. They differ in a variety of respects in addition to dealing with different types of commodities. In some of the models, oral pledges are made; in some, written pledges are made; in others, pledges are not made. Some of the models deal with stocks, some with flows; some involve a numeraire, some do not; some are barter models, some are monetary; some are quite realistic, some are very unrealistic; some are complete, some are incomplete, and some are incompatible.

The models are united, however, by having a single fundamental concern – namely, the behavior of participants in markets. Walras grouped the models into categories that he called the theory of exchange, the theory of production, the theory of capital formation, and the theory of money, but those names do not bring out the fact that the models all deal with markets. What he called the theory of exchange comprises the models of market behavior that he constructed on the assumption that there are given total amounts of the commodities available to be traded. It is traditional to reserve the term “theory of exchange” for that situation, but that should not obscure the circumstance that the other models also deal with exchange, as is shown by the following considerations. First, Walras incorporated the monetary model of exchange of fixed available total stocks, epitomized by his model of the securities market, into his mature comprehensive model, which includes the production and sale of consumer commodities and capital goods. Second,

6 Introduction

Walras indicated that all the models are concerned with pricing. He wrote that “by the *theory of exchange*” he had been able to achieve “the determination of the prices of consumer goods and consumable services” (1889, p. XIII). That was not a good description of his theory of exchange, inasmuch as it deals with situations in his models in which there are all types of commodities, including wholesale agricultural commodities and securities markets. Nevertheless, the statement makes clear that Walras regarded the determination of prices in a model of consumer commodities as an important part of the pricing process. “By the *theory of production*,” he explained, he had been able to achieve “the determination of the prices of primary materials and productive services” (ibid.), so the theory of production is also concerned with price determination. The production to which Walras referred takes place in factories and workshops, but the determination of the prices and rates of production and use of services and consumer goods is effectuated by the participants in the markets of the model.

Thus, Walras's theory of production is largely a model of market behavior, not of activities in workshops. The name “the theory of production” is therefore not really descriptive of the model on which he bestowed it, and it is badly named for the additional reason that production also occurs in the model in which there is capital formation. “By the *theory of capital formation*” in particular, Walras explained, he had been able to achieve “the determination of the prices of capital goods” (ibid.). That occurs in a comprehensive model that is likewise principally concerned with markets – namely, those in which are determined the prices, rates of production and sale of services and of consumer goods as well as of new capital goods. Similarly, Walras's theory of money is concerned with models of markets – namely, markets for loans (ibid, p. XVI). Walras accordingly described his mature comprehensive model, which includes all commodities and money, as concerned with the behavior of “traders” – that is, with the behavior of participants in markets (1889, p. XVII; 1896b, p. XVI).

It has been written many times that Walras built a theoretical edifice architectonically by adding a model to a first model and another to the resulting model, and so on, until the grand edifice was finished. That is not true. The model of monetary exchange indicates how the pricing process is conducted on any market day in the market for any commodity, and that the pricing process takes place in every one of the models. The model of the production and sale of consumer commodities mentioned in the previous paragraph is one in which only nondurable commodities, including services, are produced, and there are no inventories. It cannot be said that Walras added capital goods markets to that model to produce the next stage of his construction because in every edition of the *Eléments* he discarded the nondurable consumer commodities model when he took up new capital goods. He replaced most of its leading features with a new set of assumptions

Introduction

7

about consumer behavior, about the kind of consumer commodities that are produced, and about inventories, and put that different assemblage of phenomena into a model with savings and capital goods markets. In his mature phase – at the same time as when he put consumer commodities and capital goods into a model – he also added the holding of money balances and a loan and money market. Similarly, it is recognized in this book that in the mature phase of his theoretical activity Walras did not develop a series of increasingly complex barter models and then add money in a final grand model. On the contrary, the mature models of nondurable consumer commodities, of new capital goods and all types of consumer commodities, and, of course, the model of the securities markets are all monetary. The book will show what transpired during the course of his efforts to revise his model in the fourth edition of the *Eléments*. Suffice it to say at this point that in that edition he did try to develop a barter model with consumer commodities and capital goods, and only subsequently tried to add money to it.

Phases of theoretical activity

The history of Walras's theoretical work is also complex because he went through four phases of theoretical activity, and changed most of his models to some degree and in some cases extensively during each phase. Accordingly, in order to identify the subject matter of Parts I and II of this book and to make clear their places in the scheme of his writings, some preliminary classifications and explanations must be provided. The phases merge into each other, so the dates given for their beginnings and endings are approximations except as noted later regarding the end of the fourth phase. The appropriateness of categorizing a particular publication of Walras's as being in a particular phase may therefore be debatable if the date of publication comes at or near the date given for the beginning or the end of a phase. Nevertheless, the phases are clearly there; indeed, most of Walras's writings are strongly marked by the characteristics of the phase in which they were written.

The experimental phase: First, Walras underwent an experimental phase, one of apprenticeship, mistakes, and exploration during the 1860s until 1872. In this period he made a number of attempts to model markets, but his efforts were confused and unproductive. He lacked the basic constructions that are necessary for that task. In particular, he did not have a theory of demand, and consequently lacked a theory of the determination of market prices.

The creative phase: Second, Walras experienced a creative phase, one of great originality and intense and substantial theorizing, which spanned the years 1872 through 1877. During that phase he wrote four brilliantly original memoirs (1877c) that he developed into the core of the reasoning and into

8 Introduction

much of the specific language of the first edition of the *Eléments* and completed that work. Walras insisted to his publisher that the first part appear in 1874, before the second part (1877b) was completed, because he learned in May of that year that William Stanley Jevons had published a mathematical theory of utility and exchange that was similar to his own (J. d'Aulnis de Bourouill to Walras, May 4, 1874, in 1965, *I*, letter 267, p. 388), and he was anxious to establish the independence of his discoveries and his priority in regard to most of them. For those same reasons he paid for the publication of his books, and sent copies of them and of his articles to his many correspondents. This book will not examine Walras's second phase in itself, but many of its constructions will be studied because he carried most of them, with modifications, forward into his next phase of work. The material in the first edition of the *Eléments* that is incorporated into the second edition is legitimately considered as contributing to the models that Walras presented in his mature phase.

Although the contents of the first edition suffer by comparison with the mature phase of Walras's work, it should be noted that the division, placement, and naming of sections in the first edition are clearer and better conceived than in any of the subsequent editions. In the first edition, Walras began by examining exchange with fixed stocks in barter models and in monetary models. Then he introduced a section on money to make clear that he was going to present monetary models in the remainder of the volume. Then he developed a monetary model of the production and sale of services and nondurable consumer goods. Finally, he presented a comprehensive monetary model of economic growth in which capital goods and durable and nondurable consumer commodities are produced and used and the national product grows. By titling that model "Conditions and consequences of economic progress" (1877b, p. 274), Walras indicated that it was a model of economic growth.

The mature phase: Third, Walras underwent a mature phase of theoretical activity during the years 1878 through the mid-1890s. He presented the theoretical aspects of his work during that phase in the second edition (1889) of the *Eléments* and in essays written during the first half of the 1890s. He asserted in the second edition that he had not changed the basic character of his models of market structures and behavior, noting that his theoretical system was "the same as [his] doctrine was fifteen years ago"⁵ (1889, p. XII) and that was true in many fundamental respects. By the time he prepared the second edition, however, he had achieved a clearer view of certain aspects of the type of economy that he wanted to portray. Walras emphasized this in a

⁵ He presented that statement again in the introduction to the fourth edition, but his insertion of the written pledges model into that edition rendered the statement invalid.

Introduction

9

letter in which he described some of the major “modifications” he had introduced into that edition, including the models of tatonnement in the production of nondurables, the numeraire commodity, and new capital goods (see ch. 12), and additions or alterations to his work on exchange, the net income of capital and its rate of net income, the special equations of capital formation, and the theorem of the marginal utility of new capital goods (Walras to P.H. Wicksteed, April 9, 1889, in Walras 1965, 2, letter 877, pp. 307–9). He also rewrote his model of the money market and behavior in regard to money. Accordingly, his mature phase was one of clarification and extension of important facets of his models. He refined many of their aspects, adding to the brilliant efforts of his second phase in the light of his increased experience and knowledge, and bringing his theoretical system to the best state it was to attain.

Of course, this does not mean that he brought it to perfection, because he did not complete all his conceptions or work out all their ramifications or reconcile all their differences or eliminates all their incompatible aspects. It is not to be expected that a pioneer would present a completely finished work. As Walras said, he had confined himself to solving the problem of general economic equilibrium in its broad outlines and left to those who came after him the tasks of correcting the details and filling in the gaps (Walras to Irving Fisher, July 28, 1892, in 1965, 2, letter 1064, pp. 498–99). The examination of Walras's mature phase of theoretical work is interesting, however, because it reveals his contributions as they were before he contradicted and obscured them by forcing the written pledges model and other material of inferior quality into the fourth edition of the *Eléments*. The examination reveals a Walrasian system, unrecognized in this century and poorly understood in the last, which could have given rise to a stream of economic modeling that would have been a fruitful alternative to the one that actually transpired. By contrast with the reception given to the fourth phase of Walras's theorizing, his mature phase of modeling has been neglected since 1920 to such an extent that there does not exist, as far as I am aware, any modern exposition of it in the secondary literature,⁶ a situation this book is intended to remedy. From it will emerge a fully documented picture of Walras's mature market models that differs from the received view based on his written pledges models. It is thought by some that his work was normative; it will be shown that his mature models were positive in intent. It is thought that he was concerned only with statics; it will be shown that the mature models are much concerned with dynamic processes and disequilibrium. It is thought that his work had no institutional content; it will be shown that he posited an institutional framework for the markets in his models. It is thought that his

⁶ My essays on the subject are excluded from this remark.

10 Introduction

work is purely abstract theory; it will be shown that his mature models are rich in detail drawn from real markets.

The question arises as to where the third edition of the *Eléments*, published in 1896, fits into the present classification of Walras's theoretical endeavors. His capacity for creative intellectual work had begun to diminish during the years that immediately preceded its publication, a situation he was well aware of. He wrote at the beginning of that edition that he feared he was no longer capable of finishing the plan of work that he had earlier projected, and represented his incapacity as the reason for not making any changes in the lessons that he carried over from the second to the third edition (1896b, p. V, n. 1).⁷ Indeed, the only changes he made were some small alterations in the introduction and in the lessons, the elimination of four lessons on the applied theory of money, and the addition of three brief appendixes consisting of material that he had written during the period 1890 through 1895. The appendixes do not alter the character of the models of markets that Walras presented in the second edition, but they strengthen his mature theoretical system. The lessons in the third edition are therefore exactly the same as the corresponding lessons in the second edition, and the appendixes in the third edition are part of his mature work, so the edition is part of Walras's work during his mature phase of theoretical activity.

During that phase Walras presented the following models:

1. Barter models of the exchange of fixed total (market) stocks of commodities. These are all oral pledges models, as will be explained.
2. A monetary model of an organized purely competitive market with fixed total stocks of the assets. This is an oral pledges market.
3. A monetary model of the production and sale of productive services, primary materials, and nondurable consumer commodities, called "the nondurable consumer commodities model" in this book. It will be seen that its behavior is radically different from that of the written pledges model of nondurable consumer commodities that Walras introduced in the fourth edition of the *Eléments*.
4. A comprehensive monetary model of equilibration and equilibrium, called "the mature comprehensive model" in this book. This was Walras's crowning achievement.
 - a. The model includes integrated submodels of the production and sale of productive services, primary materials, consumer

⁷ At the beginning of his preface to the fourth edition and in a letter (Walras to Alfred Danbitch, January 22, 1900, in 1965, 3, letter 1441, p. 106), Walras noted that it was "really the third" (1900, p. V) precisely because the body of what is called the third edition is, with a few exceptions, identical to that of the second.