Non-Classical Problems in the Theory of Elastic Stability

When a structure is put under an increasing compressive load, it may become unstable and buckle. Buckling is a particularly significant concern in designing shell structures such as aircraft, automobiles, ships, or bridges. This book discusses stability analysis and buckling problems and offers practical tools for dealing with uncertainties that inherently exist in real systems. The techniques are based on two competing yet complementary theories, which are developed in the text. First, the authors present the probabilistic theory of stability, with particular emphasis on reliability. Both theoretical and computational issues are discussed. Second, they present the alternative to probability based on the notion of "anti-optimization," a theory that is valid when the necessary information for probabilistic analysis is absent, that is, when only scant data are available. Design engineers, researchers, and graduate students in aerospace, mechanical, marine, and civil engineering and theoretical and applied mechanics who are concerned with issues of structural reliability and integrity will find this book a particularly useful reference.

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Deterministic, Probabilistic and Anti-Optimization Approaches

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of Professor Dr. Ir. Warner Tjardus Koiter
– colleague, co-author, mentor, and friend

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Preface: Why Still Another Book on Stability?

The preface is the most important part of the book. Even reviewers read a preface.
Philip Guedalla

There are at present numerous books available on the theory of stability and its applications to structures. One author even remarked sarcastically that if they were put in a single bookcase, it would buckle under their weight. We do not complain that ". . . of the making of many books there is no end" (Ecclesiastes 12:12), rather we ask a natural question: Is there a legitimate place for a new book in this field?

The answer to this question is affirmative, if a book has its unique, distinct characteristics. We have chosen to deal with non-classical problems. To the best of our knowledge, none of the subjects, touched upon in this monograph, have been discussed exclusively in the existing books on buckling analysis. Thus we feel that this book will not be *just* another new book on buckling. Indeed, most existing books may be classified as belonging to one of the following two categories: textbooks – which often look very much alike, maybe not without reason, since the subject is the same – and monographs – which have an encyclopedic nature, trying to comprise an uncomprisable – to cover all or nearly all pertinent topics. This latter task of listing all the results (even only those of major importance) appears to be impossible indeed.

The purpose of this book is to present two competing theories, which incorporate ever-present uncertainty in the stability applications of the real world. These uncertainties are first and foremost due to unavoidable initial imperfections, deviations of the structure from its intended, nominal, ideal shapes. Other uncertainties are the material characteristics and/or realizations of the boundary conditions. These topics are almost never touched upon in the texts or monographs. Here we first present the probabilistic theory of stability. The bridge is made between a description of random imperfections as random fields, its description as random vectors, and the Monte Carlo methods. Special emphasis is devoted to evaluation of reliability, the probability that the structure will not fail prior to preselected load level; reliability concept is the powerful tool that needs to be introduced in the practical design of uncertain structures, if the probabilistic paradigm is adopted. The book presents a unified probabilistic theory of stability. It elucidates both the theoretical and computational aspects in a single package.

Special emphasis is placed on asymptotic evaluations and non–Monte Carlo analytical methods.

Along with the probabilistic theory of stability, we expose the alternative to probability, based on the notion of "anti-optimization"; this theory is valid when the necessary information for the probabilistic analysis is absent and only the scant data are available. Such a theory appears to be of prime importance for estimating the worst case scenario with limited data present. Thus, these two theories complement each other. Ideally, if sufficient data are available, one would resort to the probabilistic theory; however, realistically, in most circumstances its alternative, anti-optimization, should be preferred.

The main objective of this book is, therefore, to provide a strong impetus to both theorists and practitioners so that they will become acquainted with the probabilistic and anti-optimization theories as the most practical tools for dealing with the uncertainties that are present whether theorists or practitioners want to acknowledge them or not and whether they know the appropriate analyses or not. Uncertainty analysis is thus not a luxury but rather a mere necessity to rigorously reflect the working conditions of the real systems. This book, therefore, fills the gap that exists in the present-day literature and practice. In fact, engineers and researchers can no longer pretend that their deterministic approaches represent the truth, that directly, without invoking the uncertainty aspects, could be utilized by the practicing engineers. Recognition of this fact is extremely important not only to the practitioners but also for the theorists for they would try, it is hoped, in their future analyses, to devote additional effort to the rigor of their models, especially when the overlooked uncertainty may drastically change their results obtained for ideal, maybe even never existing, situations.

From the engineering point of view, the developments in stability of structures can be divided into two periods. The first period dates from 1744, when Leonhard Euler communicated to the world his famous formula for the buckling load of a column. The second dates from 1945, when Warner Koiter submitted his PhD thesis to the Delft University of Technology. He uncovered the disastrous effect of initial geometric imperfections on the load-carrying capacity of shell structures. During the two centuries of the post-Euler, pre-Koiter era, engineers and scientists made outstanding contributions in shedding light on the buckling of structures. New problems arose with the technological revolution that took place in the twentieth century. With the modern need to develop lightweight vehicles, the buckling concept has become even more eminent. Lorenz, Southwell, and Timoshenko generalized Euler's classical formula for columns to the case of thin cylindrical shells. Fortunately, buckling specialists were interested in experimental validation of their findings. This led to a surprising and rather disappointing observation: The experimental results did not vindicate the theoretical investigations, most of the data were much lower than what the linear analyses predicted. Koiter's teachings appeared then to explain that the unavoidable imperfections, deviations from the nominally ideal shape, play a dominant role in the drastic reduction of the load-carrying capacity of cylindrical shells and some other structures. Thus, the notion of *imperfection sensitivity* came into being. This is how the buckling topic emerged into healthy adolescence after two hundred years of childhood. The theoretical works of Budiansky and Hutchinson at Harvard University, of Thompson at

University College, London, and of G. W. Hunt of University of Bath, experimental studies by Singer at the Technion–Israel Institute of Technology, combined theoretical-numerical-experimental investigations by Arbocz at the Delft University of Technology, to name just a few, have been spectacularly instrumental in providing this old yet still mysterious field with new impetus and ideas.

Practical and theoretical people in the aeronautics and aerospace fields, who must design real shells and who have invested considerable funds in the research on shell structures and their stability problems, still have not fully adopted the concept of imperfection sensitivity but are using the so-called *knockdown factor*. This appears to be the case since basically engineering scientists did not present to them a means of incorporating theoretical findings into practice. Knockdown factor, which is defined so that its product with the classical buckling load yields a lower bound to all existing experimental data, was compiled primarily by Weingarten, Seide, and Peterson. This is both welcome and disappointing. It is welcome given that many of the shell structures we employ perform extremely responsible functions; it is better to overdesign than underdesign, if we must choose between these two alternatives and do not want to take excessive risks. On the other hand, it is disappointing that the results of several decades of research are mostly ignored and unreflected in engineering practice and that His Excellency the Knockdown Factor is reigning in the kingdom of the designers.

It has been recognized since 1958, when V. V. Bolotin of the Moscow Power Engineering Institute introduced the notion of *randomness* into the theory of elastic stability, that for the buckling theories to become practical, they must be combined with analysis of the uncertain initial imperfections. We are aware that there are no two identical structures, even if they are produced by the same manufacturing process. The apparent reluctance of the designers to take advantage of theoretical findings stems from the fact that most theoretical and/or numerical imperfection studies are conditional on detailed a priori knowledge of the geometric imperfections of the particular structure that is available. In an ideal case, the imperfections can be measured and incorporated into the analyses to predict the buckling loads. For example, Horton of Georgia Institute of Technology tested a large-scale shell with a diameter of 60 ft, and Arbocz and Williams measured the imperfection profile of 10-ft diameter stiffened shells at the NASA Langley Research Center. More recently, Ariane interstage shells (some of them 80 ft in diameter) were measured in the Fokker Aircraft Company. This approach, which is justified for single prototype-like structures, is impractical as a general method for behavior prediction. Information on the type and magnitude of imperfections of a particular structure would be too specific and hardly applicable to other realizations of the same structure, even if they were produced by the same manufacturing process.

With these considerations in view and also bearing in mind the large scatter of the experimental results, it becomes clear that practical applications of the imperfection-sensitivity theories are conditional on their being fused with a probabilistic analysis of the imperfections and buckling loads. This is because engineers do not want to overdesign or underdesign the structures. Apparently, the knockdown factor approach penalizes ingeniously designed shells, those with fewer imperfections.

However, appreciation of the probabilistic approach is not sufficient to solve the problem. Indeed, the early studies on random imperfections fell into two classes. The first class considered overly simplified models of single-degree-of-freedom finite structures with the initial imperfection amplitude as a random variable and with attendant straightforward analysis. The second class treated the initial imperfections as random fields (i.e., random functions of both the axial and circumferential coordinates). To facilitate the purely (and often restrictive) analytical treatment, researchers adopted far-reaching assumptions regarding the probabilistic nature of initial imperfection (statistical homogeneity and ergodicity); they also treated infinitely long structures rather than ones of finite length. Thus a new thinking was necessary to discard negative characteristics of existing probabilistic treatments, while retaining and reinforcing their positive characteristics. We take the liberty of quoting Johann Arbocz's extensive review (1981) on the past, present, and future of shell stability analysis: ". . . it was not until 1979, when Elishakoff published his reliability study on the buckling of a stochastically imperfect finite column on a nonlinear elastic foundation, that a method has been proposed, which made it possible to introduce the results of the initial imperfection surveys routinely into the analysis." Basically, the idea was to utilize the Monte Carlo method for solution of the stability problem of axially compressed cylindrical shells with random initial imperfections. The latter are expanded in terms of the buckling modes of the perfect structure, and then the Fourier coefficients are treated as random variables. Next, using a special numerical procedure, the Fourier coefficients of the desired large sample of random initial imperfection shapes are simulated after a sufficiently large sample of initial imperfection measurements become available. This is followed by a deterministic buckling load of each of the simulated shells, with subsequent statistical analysis.

Meaningful probabilistic analysis is conditional on probabilistic characterization of the input variables and functions. This can be performed only when appropriate measurement data are available. Fortunately, Babcock and Arbocz at the California Institute of Technology, as well as some other investigators realized the need for large-scale experiments, as a result of which several *initial-imperfection data banks* have been developed (those at the Delft University of Technology and the Technion–Israel Institute of Technology being the most notable). Instead of making unnecessary and often restrictive assumptions regarding the properties of the initial imperfections, these data banks are amenable to direct statistical analysis, since they provide indispensable tools for further probabilistic analysis.

One must recognize that, at first glance, the Monte Carlo method may not appear to be attractive analytically; it may turn out to be numerically time-consuming especially for some highly complex structures. The answer to such a possible reservation is that the Monte Carlo method is the only *universal* technique for probabilistic analysis of structures and that its use does not entail some of the heavily simplified solution procedures that are often required for idealized structural models.

Complex multi-mode non-linear deterministic analysis should be balanced by maximum resemblance to real-life situations. As Timothy Ferris instructively put it, "science is said to proceed on two legs, on the theory (or, loosely, of deduction) and the other of observation and experiment (or induction). Its progress, however, is less often a commanding stride than kind of halting stagger – more like the path of a wandering minstrel than the straight-ruled trajectory of a military marching band." We trust that the direct introduction of the results of experiments into the probabilistic analysis puts us on the right track, combining both the deductive and the inductive facets of engineering. This became feasible by employing a numerical tool closely resembling the experiments themselves, namely by the Monte Carlo method. At the same time, we do not advocate an abandonment of analytical techniques that, although applicable, may complement the Monte Carlo method especially where small imperfections are involved. The Monte Carlo method needs very accurate numerical techniques to evaluate buckling loads of each shell in the ensemble of shells; careful numerical techniques like STAGS, BOSOR, PANDA, and those based on multi-mode imperfection methods are consistent with the reliability analysis.

Can one use probabilistic modeling if the data are extremely limited? This is an intriguing question. The answer is affirmative for those who make a fetish of the probabilistic models, but we feel that it should be negative. Indeed, the central premise of this book is the need for sound theoretical, experimental, and numerical analyses. Rigorous deterministic analysis is a cornerstone of every meaningful probabilistic treatment of a problem at hand. Theoretical analysis elucidates the physical phenomenon but may not be feasible without some idealizations. Experimental analysis then provides the data used by the numerical analyst to create statistical "brothers and sisters" of the measured shells. According to the *John Wiley Dictionary of Scientific Terms*, *reliability* is "the probability that a component part, equipment, or system will satisfactorily perform its intended function under given circumstances, such as environmental conditions, limitations as the operating time, and frequency and thoroughness of maintenance, for a specified period of time." In the structural-stability context, reliability is the probability that the structure will sustain loads in excess of the specified level. Here either extremely high reliabilities or equivalently extremely low probabilities of failure are needed. To perform such a refined analysis, refined data and authentic analytical and numerical tools are called for. In the absence of experimental data (as may well be the case with variation of the elastic moduli), probabilistic methods cannot be recommended for design purposes. Fortunately, there exists a new discipline, called *convex modeling of uncertainty* (or, in a more general context, *set-theoretical modeling of uncertainty*), developed by Ben-Haim and Elishakoff for applied-mechanics problems. This discipline does not seek to make something out of nothing (i.e., it does not create the probabilistic model out of extremely limited or absent data), but it does represent an alternative technique for such special yet often encountered situations. Set-theoretical uncertainty in effect represents *anti-optimization* under uncertainty: It operates with the least favorable scenarios based on the limited experimental data describing uncertain variables or functions.

As we saw earlier, accurate determination of the reliability or the least favorable buckling loads calls for a threefold effort in: (a) deepening theoretical insight into the phenomenon, (b) collecting experimental information, and (c) conducting proper numerical analysis. Choice of a suitable model of uncertainty is also a prominent decision to be made, based on the amount and character of the information.

This monograph reflects this philosophy. Neither a textbook nor an encyclopedia, it covers the deterministic, probabilistic, and set-theoretical approaches for buckling of structures. Chapters 1, 2 (except Sections 2.1 and 2.2), and 5 are written by three of us; the first two sections of Chapter 2 are based on our joint papers with Professor Koiter of the Delft University of Technology on the effect of thickness variation in perfect or imperfect isotropic shells. The last two sections of Chapter 2 represent a generalization for the case of composite shells. Chapters 3 (except Section 3.2), 4, 6, and 7 are written by the first author; Section 3.2 is based on our joint paper with Professor Masanobu Shinozuka of the University of Southern California. Chapters 3 and 4 are based on the work of the first author (Section 3.1), and his joint studies with Professor Johann Arbocz of the Delft University of Technology (Sections 3.3, 3.4, and 4.2), Professor Koyohiro Ikeda of Tohoku University and Professor Kazuo Murota of the Kyoto University (Section 4.1) and other investigators. Contents of works of Professor Wei-Chau Xie of the University of Waterloo and of Dr. Jun Zhang and Professor Bruce Ellingwood of Johns Hopkins University constitute the central part of the last two sections of Chapter 4. Section 3.5 is based on our joint investigation with Dr. David Bushnell of Lockheed Company. Section 5.3 is based on the joint paper by the first and third authors with Professor Guoqiang Cai of the Florida Atlantic University.

The organization of the present book is as follows: The first two chapters are devoted to some new deterministic problems. Chapter 1 discusses the mode localization in the deterministic setting, an extremely recent topic in the buckling context. In particular, we consider multi-span columns and plates, with unavoidable misplacements of the stiffeners location. Generally, a misplacement can be regarded as one type of imperfection, although it does not lead to as drastic a change in the buckling load as geometric imperfections do in shell buckling. As a matter of fact, in many experimental studies (e.g., of Singer or of Tenerelli and Horton) of the shell buckling, only localized buckling modes were observed. In strong ring-stiffened shells, two or three buckles emerged when the critical load was reached and were confined to a single span. In weak ring-stiffened shells, buckles appeared around the whole circumference but extended only through part of the spans. Chapter 1 advocates that, if each span is treated as an element of a periodic structure, the localization phenomena can be explained by using the analytical finite difference calculus. Chapter 2 is devoted to the influence of thickness variation on the buckling of perfect or imperfect, isotropic or composite, circular cylindrical shells.

Chapters 3 and 4 deal with stochastic buckling of structures with random imperfections. Chapter 3 focuses on the Monte Carlo method, whereas Chapter 4 discusses approximate analytical and numerical techniques, including the asymptotic analysis, the first-order second-moment method, the mode localization due to random misplacements, and the finite-element method for structures with random material properties.

Convex modeling of uncertainty in buckling problems is the focal point of Chapter 5. Here the realistic situation of data scarcity is considered, and the minimum buckling loads in an ensemble of plates and shells with uncertain material properties are derived. Chapter 6 discusses the Godunov-Conte shooting method (representing an extended version of the paper by Elishakoff and Charmats), whereas Chapter 7 deals with the application of computerized symbolic algebra – an able and obedient "servant" of the present-day researchers. It constitutes an extended version of the paper by Elishakoff and Tang.

We hope that the monograph will prove useful for researchers, engineers, and senior graduate students specializing in aeronautical, aerospace, mechanical, civil, nuclear, and marine engineering. We hope that our "pluralistic" philosophy will have a definite impact on the entire engineering profession. As Abraham Maslow remarks, "If the only tool you have is a hammer, you tend to treat everything as if it were a nail." We strongly trust that the modern analyst cannot afford to confine himself/herself to only one of the trio, namely, (a) deterministic, (b) probabilistic, and (c) set-theoretical *Weltanschauung*. Instead, we, as engineers must be pragmatic and flexible in choosing the most appropriate methods, consistent with available experimental information.

Some pertinent questions remain still to be tackled. Two of the central questions were posed by Professor Bernard Budiansky in his correspondence with Professor Johann Arbocz (Arbocz and Singer, 2000): "Are we necessarily doomed to accept forever the unhappy coexistence of efficient shell design and imperfection-sensitivity? Or are there some structural design secrets to be discovered that will retain minimum weight and rid us of the curse of imperfection-sensitivity?"

This book deals with how to live with the above curse most efficiently, combining deterministic probabilistic and anti-optimization "cures," in the hope that some "genes" will be uncovered that will make the multiplicity of studies on the "curse" unnecessary even if still instructive.

We thank (a) Elsevier Science Publishers for allowing us to reproduce Figures 3.13–3.29 from the article by I. Elishakoff and J. Arbocz, "Reliability of Axially Compressed Cylindrical Shells with Random Axisymmetric Imperfections," *International Journal of Solids and Structures*, Vol. 18, 563–85, 1982; Figures 4.1–4.4 from the article by K. Ikeda, K. Murota, and I. Elishakoff, "Reliability of Structures Subject to Normally Distributed Initial Imperfections," *Computers and Structures*, Vol. 59, 463–9, 1995; Figures 5.10–5.14 from the study by I. Elishakoff, G. Q. Cai, and J. H. Starnes, Jr., "Nonlinear Buckling of a Column with Initial Imperfection via Stochastic and Non-Stochastic Convex Models," *International Journal of Nonlinear Mechanics*, Vol. 29, 71–82, 1994; Figure 3.34 from the book *Buckling of Structures – Theory and Experiment* (Elsevier, 1988), edited by I. Elishakoff, J. Arbocz, C. D. Babcock, Jr., and A. Libai; (b) AIAA Press for permission to reproduce Figures 4.5–4.6 from the article by I. Elishakoff, S. Van Manen, P. G. Vermeulen, and J. Arbocz, "First-Order Second-Moment Analysis of the Buckling of Shells with Random Initial Imperfections," *AIAA Journal*, Vol. 25, 1113–17, 1987; Figures 4.7–4.11 from the paper by W.-C. Xie, "Buckling Mode Localization in Randomly Disordered Multispan Continuous Beam," *AIAA Journal*, Vol. 33, 1142–9, 1995; (c) American Society of Civil Engineering for permission to reproduce Figures 4.12–4.14 from the paper by J. Zhang and B. Ellingwood, "Effects of Uncertain Material Properties on Structural Stability," *Journal of Structural Engineering*, Vol. 121, 705–16, 1995; and (d) American Society of Mechanical Engineering for the permission to reproduce Figures 3.30–3.33 and 3.35–3.37 from the paper by I. Elishakoff and J. Arbocz, "Reliability of Axially Compressed Cylindrical Shells with General Nonsymmetric Imperfections," *Journal of Applied Mechanics*, Vol. 52, 122–8, 1985.

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The responsibility for any imperfections in this monograph, be they random or otherwise, rests solely upon the authors. We would appreciate hearing your comments via electronic mail (*ielishak@me.fau.edu*), fax (561-297-2825), or regular correspondence.