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THE MOTION OF BUBBLES AND DROPS IN REDUCED GRAVITY

Orbiting spacecraft provide a valuable laboratory for experiments on physical and biological systems in a reduced gravity environment. Materials processing experiments have commonly involved the growth of crystals from the melt or solution and the processing of alloys and composites. Biological experiments have been performed on a variety of subjects, including protein crystal growth, bioreactors, and the adaptation of humans to extended periods of weightlessness. In these studies, fluid masses containing bubbles and drops are encountered routinely. This book is the first to provide a clear, thorough review of the motion of bubbles and drops in reduced gravity, particularly motion caused by variations in interfacial tension arising from temperature gradients on their surfaces. The emphasis is on theoretical analysis from first principles; experimental results are discussed and compared with predictions where appropriate. Students and researchers interested in fluid mechanics in reduced gravity will welcome this state-of-the-art reference.

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Dedicated to our parents

Sita Lakshmi and R. K. Ramasubramaniam

and

Saraswathi and D. Ramaswamy

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Errata

The correct versions in each case are given below.

Page 8, line 20: proportional to the square of the distance

Page 26, line 2 below Equation (2.2.8): dimensions of force per unit volume

Page 41, Figure 2.6.3: upper case Φ should be replaced by lower case ϕ

Page 116, line 2 below Equation (4.7.5): Ferrers' functions

Page 172, line 3 from the bottom: for T_1', but … equation for T_1' of

Page 231, lines 4 and 5 below Equation (5.1.5): spindle-shaped surfaces when $\eta > \frac{\pi}{2}$, apple-shaped surfaces when $\eta < \frac{\pi}{2}$,

Page 430, Equation (9.6.24): The correct form is given below.

$$G_1 = Be^{-\chi Pr\eta}\left(1+\sum_{n=1}^{\infty} C_n e^{-n\chi\eta}\right) \qquad (9.6.24)$$

Prepared by R. S. Subramanian and R. Balasubramaniam

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Preface

This monograph is principally about the motion of bubbles and drops caused by variations in interfacial tension arising from temperature gradients on their surfaces. We have attempted to provide a reasonably comprehensive picture of the progress and the current status of research on this subject. It is our opinion that, in the long run, this driving force for the motion of bubbles and drops will prove to be as ubiquitous in a reduced gravity environment as gravity is on the surface of Earth.

The book is divided into four parts. In Part One, we introduce the reader to the role of gravity and interfacial tension in the motion of bubbles and drops in Chapter One and cover the governing equations in Chapter Two. Part Two is devoted to the motion of isolated bubbles or drops and contains two chapters. Some important aspects of the motion of bubbles and drops due to gravity, which is a familiar body force, are treated in Chapter Three. This is done for the purpose of completeness in coverage and to provide contrast where needed with features of the motion driven by the interface, which is discussed in Chapter Four. In Part Three, which is composed of three chapters, we discuss the interactions of bubbles and drops with each other and with neighboring boundaries. General solutions are given in Chapter Five and are then used in Chapters Six and Seven, which are devoted to body-force-driven motion and motion driven by the interface, respectively. In Part Four, two chapters cover topics that are closely related to the main theme. Chapter Eight deals with mass transport to bubbles and drops in reduced gravity conditions, and Chapter Nine is devoted to motion that occurs in a body of fluid due to interfacial tension gradients on its free surface. Although the emphasis in this work is on theoretical analysis, we have presented and discussed experimental results wherever appropriate and possible.

We hope that scholars who choose to work on bubbles and drops, on fluid mechanics in reduced gravity, and on interfacial phenomena will find this book useful. We have employed a level that is suitable for advanced students in engineering and science with the expectation that some of this material may be used in courses dealing with transport phenomena associated with motion driven by the interface. Also, the topics covered should be of interest to scientists studying the processing of materials in reduced gravity.

We have used the following system for numbering equations, figures, and tables. In each section, equations are numbered sequentially, beginning with 1. The identification number assigned to an equation also includes the chapter number and the section number, separated by periods. Thus, an equation number has the form $C.S.N$, where C designates the chapter number, S is the section number, and N stands for the sequential

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number within the section. Regarding symbols, we have used a uniform convention throughout the book, to the extent possible. In some instances, however, it has been necessary to use the same symbol with different connotations in different parts of the book. Definitions of the symbols used commonly are given when they are first introduced and repeated when needed.

We are grateful to our mentors, to our present and former students, and to our colleagues, too numerous to mention individually, who have offered valuable suggestions along the way. We alone are responsible for any errors and omissions. We would appreciate readers informing us about any errors that they may find. We wish to thank Florence Padgett of Cambridge University Press for her consistent encouragement and support, Nancy Mieczkowski of GRAFIXWORKS.COM, Lorain, Ohio, for preparing the drawings, and Erin Subramanian, Potsdam, New York, for preparing the bulk of the equations. We are indebted to our spouses and children for the patience and understanding they have displayed during the years when we devoted time toward the preparation of this work. Finally, we wish to express our appreciation to the National Aeronautics and Space Administration for steady support of our research program in numerous ways and to the European Space Agency for their kind hospitality during the years when we collaborated with them on the design and conduct of the flight experiments.

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