

Nonequilibrium Phase Transitions in Lattice Models

This book provides an introduction to nonequilibrium statistical physics via lattice models.

Beginning with an introduction to the basic driven lattice gas, the early chapters discuss the relevance of this lattice model to certain natural phenomena and examine simulation results in detail. Several possible theoretical approaches to the driven lattice gas are presented. In the next two chapters, absorbing-state transitions are discussed in detail. The later chapters examine a variety of systems subject to dynamic disorder before returning to look at the more surprising effects of multi-particle rules, nonunique absorbing-states and conservation laws. Examples are given throughout the book, the emphasis being on using simple representations of nature to describe ordering in real systems. The use of methods such as mean-field theory, Monte Carlo simulation, and the concept of universality to study and interpret these models is described. Detailed references are included.

The book will be of interest to graduate students and researchers in statistical physics and also to researchers in areas including mathematics, chemistry, mathematical biology, and geology interested in collective phenomena in complex systems.

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Cambridge University Press
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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521480628

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First published 1999
This digitally printed first paperback version 2005

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Marro, Joaquín, 1945–
Nonequilibrium phase transitions in lattice models / Joaquín Marro, Ronald Dickman.
p. cm. – (Collection Aléa-Saclay. Monographs and texts in statistical physics)

Includes bibliographical references and index.

ISBN 0 521 48062 0 (hardbound)

1. Phase transformations (Statistical physics) 2. Lattice gas. 3. Lattice dynamics.

I. Dickman, Ronald. II. Title. III. Series.

QC175.16.P5M37 1999

530.13–dc21 98-29461 CIP

ISBN-13 978-0-521-48062-8 hardback

ISBN-10 0-521-48062-0 hardback

ISBN-13 978-0-521-01946-0 paperback

ISBN-10 0-521-01946-X paperback

Cambridge University Press
052101946X - Nonequilibrium Phase Transitions in Lattice Models
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Art is the lie that helps us see the truth
Pablo Picasso

To Julia and Adriana

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Preface

Nature provides countless examples of many-particle systems maintained out of thermodynamic equilibrium. Perhaps the simplest condition we can expect to find such systems in is that of a nonequilibrium steady state; these already present a much more varied and complex picture than equilibrium states. Their instabilities, variously described as nonequilibrium phase transitions, bifurcations, and synergetics, are associated with pattern formation, morphogenesis, and self-organization, which connect the microscopic level of simple interacting units with the coherent structures observed, for example, in organisms and communities.

Nonequilibrium phenomena have naturally attracted considerable interest, but until recently were largely studied at a macroscopic level. Detailed investigation of phase transitions in lattice models out of equilibrium has blossomed over the last decade, to the point where it seems worthwhile collecting some of the better understood examples in a book accessible to graduate students and researchers outside the field. The models we study are oversimplified representations or caricatures of nature, but may capture some of the essential features responsible for nonequilibrium ordering in real systems.

Lattice models have played a central role in equilibrium statistical mechanics, particularly in understanding phase transitions and critical phenomena. We expect them to be equally important in nonequilibrium phase transitions, and for similar reasons: they are the most amenable to precise analysis, and allow one to isolate specific features of a system and to connect them with macroscopic properties. Equilibrium lattice models are typically specified by their energy function or ‘Hamiltonian’ on configuration space; here, the models (we restrict our attention to lattice Markov processes or particle systems) are defined by a set of transition probabilities. Unlike in equilibrium, the stationary probability distribution is not known *a priori*.

In fact, lattice models of nonequilibrium processes have lately begun to multiply at a dizzying pace. Sandpiles, driven lattice gases, traffic models, contact processes, surface catalytic reactions, branching annihilating random walks, and sequential adsorption are just a few classes of nonequilibrium lattice models that have become the staple of the statistical physics literature. Despite the absence of general unifying principles for this varied set of models, it turns out that many of them fall naturally into one of a small number of classes. That one can now recognize ‘family resemblances’ amongst models encouraged us to attempt the present work. Some sort of schema, however incomplete and provisional, to the bewildering array of models under active investigation, should help to establish connections between seemingly disparate fields, and avert unnecessary duplication of effort. Since a general formalism, analogous to equilibrium statistical mechanics, is lacking for nonequilibrium steady states, the field presents a particular challenge to theoretical physics.

This book is based largely upon research by the authors, and their students and colleagues, during the past few years. We have by no means attempted a comprehensive survey of nonequilibrium phase transitions, not even of lattice models of such. We have little to say, for example, about self-organized criticality or surface growth problems. Clearly these subjects demand books in themselves. On the other hand, we have tried to do more than provide a compendium of recent results. We hope to present a set of examples with sufficient vividness and clarity that the reader will be convinced of their intrinsic interest, and be drawn to think about them, or to devise his or her own.

At certain points in the book we express our attitude regarding various issues, some of them controversial. But we have tried to point out the weakness or controversial nature of the arguments, within the limitations posed by our own lack of familiarity with certain methods and results. In other words, we don’t that claim ours is the definitive account of all problems considered here, and therefore encourage others to address the gaps or misconceptions they find in the present work.

It is a pleasure for us to thank many colleagues whose comments, suggestions, and corrections were valuable to us in producing this presentation. One or both of us have enjoyed discussions with Abdelfattah Achahbar, Juanjo Alonso, Dani ben-Avraham, Martin Burschka, John Cardy, Michel Droz, José Duarte, Richard Durrett, Jim Evans, Julio Fernández, Hans Fogedby, Pedro Garrido, Jesús González-Miranda, Peter Grassberger, Geoffrey Grinstein, Malte Henkel, Iwan Jensen, Makoto Katori, Peter Kleban, Norio Konno, Eduardo Lage, Joel Lebowitz, Roberto Livi, Antonio López-Lacomba, Maria do Céu Marques, José Fernando Mendes, Adriana Gomes Moreira, Miguel Angel Muñoz, Mário de Oliveira, Maya Paczuský, Vladimir Privman, Sid Redner, Maria Augusta dos Santos,

Cambridge University Press
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Beate Schmittmann, Tânia Tomé, Raúl Toral, Alex Tretyakov, Lorenzo Vallés, Royce Zia, and Robert Ziff. Our work during the last years has been partially supported by grants from the DGICYT (PB91-0709) and the CICYT (TXT96-1809), from the *Junta de Andalucía*, and from the European Commission.

Granada and New York

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