ELECTROMECHANICS OF PARTICLES
ELECTROMECHANICS OF PARTICLES

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University of Rochester
To my wife, Mary, and to my daughters, Laura and Audrey
Contents

Preface xiii
Nomenclature xvii

1 Introduction 1
1.1 Background and motivation 1
1.2 Objectives of this book 2
1.3 Limitations and caveats 3

2. Fundamentals 5
2.1 Introduction 5
   A. Electromechanics of particles 5
   B. Force on an infinitesimal dipole 6
   C. Torque on a dipole 8
2.2 Lossless dielectric particle in an electric field 9
   A. Induced multipolar moments 9
   B. Effective dipole moment of dielectric sphere in
dielectric medium 10
   C. Conducting sphere in uniform DC field 12
   D. Lossless spherical shell in lossless medium with
uniform field 12
   E. Lossless dielectric sphere in lossless dielectric medium
in field of point charge 14
2.3 Dielectric particle with loss in an electric field 16
   A. Homogeneous sphere in an AC electric field 17
   B. Shells with ohmic loss in an AC electric field 20
   C. Summary 21
2.4 Effective moment calculation of force and torque 24
   A. Hypothesis and definitions 24
   B. Lossless particles 25
   C. Particles with ohmic (or dielectric) loss 28
2.5 Theory of Sauer and Schlögl 30
3 Dielectrophoresis and magnetophoresis

3.1 Introduction

3.2 DEP phenomenology for lossless spherical particle

3.3 Frequency-dependent DEP phenomenology

3.4 DEP levitation

3.5 Magnetophoresis

3.6 Applications of dielectrophoresis and magnetophoresis

4 Particle rotation

4.1 Introduction

4.2 Theory for particle rotation

Contents

B. Electric torque on homogeneous dielectric particle
   with ohmic loss 86
C. Turen’s bifurcation theory 88
4.3 Rotational (relaxation) spectra 92
   A. General theory 92
   B. Sample spectra 94
   C. Argand diagrams 98
4.4 Quincke rotation in DC electric field
   A. Theory 98
   B. Experimental results 100
4.5 Rotation of magnetizable particles 101
   A. Induction 101
   B. Nonlinear effects 103
4.6 Applications of electrorotation 105
   A. Cell characterization studies 106
   B. Cell separation 108
   C. Practical implications of the Quincke effect 108
5 Orientation of nonspherical particles 110
   5.1 Introduction 110
   5.2 Orientation for lossless homogeneous ellipsoids
      A. Isotropic ellipsoid in a uniform electric field 111
      B. Alignment torque expressions 113
      C. Two special cases: prolate and oblate spheroids 115
      D. DEP force on ellipsoidal particle 118
   5.3 Orientation for lossy dielectric ellipsoids
      A. Theory for homogeneous particles 119
      B. Alignment behavior of homogeneous particles 120
      C. Orientation of layered particles 121
   5.4 Experimental orientational spectra 124
   5.5 Static torque on suspended particle 125
      A. Basic model for torque calculation 126
      B. Rotational torque on suspended particle 128
      C. Alignment torque on suspended particle 130
   5.6 Orientation of magnetizable particles 132
      A. Magnetically linear particles with anisotropy 132
      B. Prolate spheroidal crystals 134
      C. Thin disks and laminae 135
      D. Isotropic particle with remanent magnetization 135
   5.7 Applications of orientational phenomena 136
      A. Dielectric particles 136
<table>
<thead>
<tr>
<th></th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B. Magnetizable particle applications</td>
</tr>
<tr>
<td>6</td>
<td><strong>Theory of particle chains</strong></td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction to chaining and review of previous work</td>
</tr>
<tr>
<td>6.2</td>
<td>Linear chains of conducting spheres</td>
</tr>
<tr>
<td></td>
<td>A. Solution using the method of images</td>
</tr>
<tr>
<td></td>
<td>B. Solution using geometric inversion</td>
</tr>
<tr>
<td></td>
<td>C. Alignment torque for chain of two identical conducting spheres</td>
</tr>
<tr>
<td></td>
<td>D. Spheres of unequal radii</td>
</tr>
<tr>
<td></td>
<td>E. Intersecting spheres</td>
</tr>
<tr>
<td></td>
<td>F. Discussion of results for short chains of contacting particles</td>
</tr>
<tr>
<td>6.3</td>
<td>Chains of dielectric (and magnetic) spheres</td>
</tr>
<tr>
<td></td>
<td>A. Simple dipole approximation</td>
</tr>
<tr>
<td></td>
<td>B. Accuracy considerations</td>
</tr>
<tr>
<td></td>
<td>C. General expansion of linear multipoles</td>
</tr>
<tr>
<td></td>
<td>D. Experimental measurements on chains</td>
</tr>
<tr>
<td>6.4</td>
<td>Frequency-dependent orientation of chains</td>
</tr>
<tr>
<td>6.5</td>
<td>Heterogeneous mixtures containing particle chains</td>
</tr>
<tr>
<td></td>
<td>A. Mixture theory</td>
</tr>
<tr>
<td></td>
<td>B. Suspensions of chains</td>
</tr>
<tr>
<td>6.6</td>
<td>Conclusion</td>
</tr>
<tr>
<td>7</td>
<td><strong>Force interactions between particles</strong></td>
</tr>
<tr>
<td>7.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>7.2</td>
<td>Theory</td>
</tr>
<tr>
<td></td>
<td>A. Force between conducting spheres</td>
</tr>
<tr>
<td></td>
<td>B. Force between dielectric spheres</td>
</tr>
<tr>
<td></td>
<td>C. Current-controlled interparticle forces</td>
</tr>
<tr>
<td>7.3</td>
<td>Experiments</td>
</tr>
<tr>
<td></td>
<td>A. Interparticle force measurements</td>
</tr>
<tr>
<td></td>
<td>B. Measurements on longer chains and planar arrays</td>
</tr>
<tr>
<td></td>
<td>C. Nonlinear effects</td>
</tr>
<tr>
<td>7.4</td>
<td>Electrostatic contributions to adhesion</td>
</tr>
<tr>
<td></td>
<td>A. Phenomenological force expression</td>
</tr>
<tr>
<td></td>
<td>B. Image force contributions</td>
</tr>
<tr>
<td></td>
<td>C. Detachment force contribution</td>
</tr>
<tr>
<td></td>
<td>D. Induced moment contributions</td>
</tr>
<tr>
<td></td>
<td>E. Generalized model of Fowkes and Robinson</td>
</tr>
<tr>
<td>7.5</td>
<td>Mechanics of chains and layers</td>
</tr>
<tr>
<td></td>
<td>A. Chains of magnetizable particles</td>
</tr>
</tbody>
</table>
Contents

B. Mechanics of particle beds 209
7.6 Discussion of applications 211
A. Electrofusion of biological cells 211
B. Chaining in electrorheological fluids 212
C. Electrofluidized and electropacked beds 214
D. Magnetopacked and magnetofluidized beds 215
7.7 Closing prospect 216

Appendix A: Analogies between electrostatic, conduction, and magnetostatic problems 218
Appendix B: Review of linear multipoles 222
Appendix C: Models for layered spherical particles 227
Appendix D: Transient response of ohmic dielectric sphere to a suddenly applied DC electric field 236
Appendix E: Relationship of DEP and ROT spectra 238
Appendix F: General multipolar theory 248
Appendix G: Induced effective moment of dielectric ellipsoid 251
References 253
Index 263
Preface

As a consequence of their electrical and/or magnetic properties, all particles experience forces and torques when subjected to electric and/or magnetic fields. Furthermore, when they are electrically charged, polarized, or magnetized, closely spaced particles often exhibit strong mutual interactions. In this book, I focus on these particle–field interactions, referred to collectively as particle electromechanics, by delineating common phenomenology and by developing simple yet general models useful in predicting electrically and magnetically coupled mechanics. The objective is to bring together diverse examples of field–particle interactions from many technologies and to provide a common framework for understanding the relevant electromechanical phenomena. It may disappoint some readers to learn that, despite the rather general definition offered for particle electromechanics, I restrict attention to particles in the size range from approximately 1 micron (10^{-6} m) to 1 millimeter (10^{-3} m). Though many of the ideas developed here indeed carry over into the domain of ultrafine particles, the lower limit recognizes that other phenomena, such as van der Waals forces and thermal (Brownian) motion, become important below one micron. The upper limit is consistent with a reasonable definition for a classical particle.

Chapter 1 introduces the subject, provides a definition for particle electromechanics, and adds some caveats to inform the reader of the book’s limitations. Chapter 2 unveils the fundamental effective moment concept employed throughout the book in the calculation of electromechanical forces and torques. It also uses multipolar expansion methods to solve for the induced moments for a particle experiencing a strongly nonuniform field and exploits the analogy between electrostatic and magnetostatic problems to reveal how the results for a dielectric particle can be applied to a magnetizable particle. Chapter 3 harnesses the effective moment method to derive dielectrophoretic (DEP) and magnetophoretic (MAP) force expressions for small particles in nonuniform AC electric and magnetic fields, respectively. The frequency-dependent DEP force on dielectric particles with ohmic or dielectric loss is examined with reference to
Preface

dielectrophoretic levitation. Particle rotation is the subject of Chapter 4, where the effective moment method aids us in calculating the frequency-dependent torque on a particle due to a rotating electric field. Spontaneous Quincke rotation (in a static, DC electric field) is considered as a special case. The behavior of magnetizable and electrically conductive particles in a rotating magnetic field is examined by drawing on the close analogy to rotating machines. Chapter 5 concerns nonspherical (ellipsoidal) particles in uniform fields, principally the frequency-dependent orientation of lossy dielectric ellipsoids in AC electric fields and the alignment of magnetic crystals in a DC magnetic field.

The remainder of the book focuses on the mutual interactions between closely spaced particles. Chapter 6 describes methods for determining the effective multipolar moments of linear particle chains. These effective moments provide the means to calculate net DEP forces and electrical torques exerted upon chains by electric or magnetic fields. Several different approaches useful in modeling the particle interactions are covered, including the method of images and method of inversion, both applicable only to conductive particles, and a straightforward multipolar expansion technique, applicable to dielectric and magnetizable particles. This chapter also examines the effect of nonlinear magnetization upon the interactions of ferromagnetic particle chains and introduces a simple model for heterogeneous mixtures of particles agglomerated into short chains.

Chapter 7 exploits the particle interaction models of the previous chapter to determine the mutual forces of attraction between closely spaced particles. Experimental force measurements obtained with magnetizable spheres are compared to predictions of a linear multipolar expansion. One section of this chapter, devoted to the electrostatic contributions to surface adhesion, highlights practical methods for estimating these forces in realistic situations. An extensive review of some particulate technologies where chaining is important concludes this chapter.

A nomenclature section lists all important algebraic symbols used in the mathematical expressions. All literature references, cited by author and date of publication in the text, are listed alphabetically at the end of the book. A large set of appendixes at the end of the book covers the details of certain mathematical derivations and results. The choice of SI units for all variables and mathematical expressions throughout the text reflects the significant interdisciplinary character of particle electromechanics as well as the personal choice of the author.

This book evolved from lecture notes used by the author in a graduate course entitled “Particle Electromechanics” first offered at the University of Rochester in 1985. Among many who helped bring it into being, I am privileged to mention Dr. William Y. Fowlkes, Dr. Kelly S. Robinson, and Dr. Bijay Saha of Eastman Kodak Company; Dr. Ruth D. Miller of Kansas State University; John Kraybill
Preface

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This book is based in part upon research supported by a series of grants from the Particulate and Multiphase Processes Program of the National Science Foundation, the Copy Products Research and Development Division of Eastman Kodak Company, and the Hitachi Research Laboratories. I am deeply grateful for this support and also for two travel grants from the North Atlantic Treaty Organization. Finally, a sabbatical leave granted by the University of Rochester during the fall of 1990 provided the opportunity to initiate the task of writing.

Having benefited from those who have encouraged, taught, or studied with me, I can lay full claim only to the errors found in this book. Concerning these errors, readers are kindly requested to bring them to my attention.
Nomenclature

\(A\) area of charge patch, Equation (7.14)
\(\vec{A}\) magnetic vector potential, Equation (3.61)
\(A, B\) coefficients of dipole solution for potential used in Equations (2.8a,b)
\(A_r B_s\) coefficients of multipolar potential terms in Equations (2.17a,b)
\(A_r B_s C_n D_m\) coefficients of multipolar terms for potential of layered sphere in field of point charge (Appendix C)
\(B\) magnetic flux density
\(D\) spacing of source charges \(\pm Q\) shown in Figure 6.2
\(D\) electric displacement vector
\(D(\omega)\) complex quantity for conducting magnetizable sphere in time-varying field defined by Equation (3.64)
\(\vec{E}\) electric field vector
\(E_{\text{gap}}\) electric field in gap between closely spaced particles
\(E_{\text{max}}\) breakdown-limited electric field between closely spaced particles
\(E_0\) externally applied electric field
\(E_p, E_z\) parallel and perpendicular components of electric field
\(F\) force vector
\(F_{m,n}\) attractive force between two aligned linear multipoles of orders \(m\) and \(n\) in Equation (7.1)
\(G\) feedback gain coefficient in Equation (3.43)
\(H\) magnetic intensity vector
\(H_c\) coercive magnetic field value
\(H_{\text{gap}}\) magnetic field in gap between closely spaced particles
\(I\) moment of inertia of spherical particle used in Equation (4.23)
\(I_{\text{erm}}\) modified half-integer order Bessel functions of first kind
**Nomenclature**

- $J$: electric current density vector
- $K$: Clausius–Mossotti functions as defined by Equation (2.13)
- $K^{\infty}$: polarization coefficient of nth order linear multipole defined by Equation (2.23)
- $K_0$, $K_\infty$: low- and high-frequency limits of complex Clausius-Mossotti function
- $K_x$, $K_y$, $K_z$: polarization factors along three principal axes of ellipsoidal particle, Equation (5.27)
- $K$: surface electric current density (Appendix C)
- $L_d(\omega) = R^3 D/2$: magnetization coefficient for conductive magnetizable sphere, Equation (3.63)
- $L_e$: characteristic length of levitation electrodes
- $L_x$, $L_y$, $L_z$: depolarization factors along x, y, and z axes for ellipsoid defined by Equation (5.4)
- $L_{xy}$, $L_{yz}$, $L_{zx}$: depolarization factors parallel and perpendicular to long axis of prolate spheroidal particle, Equation (5.13)
- $\overrightarrow{M}$: magnetization vector
- $M_n$: normal component of magnetization vector
- $M_n$: remanent magnetization per unit volume
- $N$: integer, number of spheres in linear chain
- $N_p$: number of particles distributed in sphere of radius $R_0$
- $P$: levitation equilibrium point: $(x_p, y_p, z_p)$
- $P$: polarization vector
- $P_n(\cos \theta)$: Legendre polynomial of order $n$
- $Q$: source charge for method of images
- $R$: radius of spherical particle
- $R_a$, $R_b$: radii of two identical intersecting spheres shown in Figure 6.10
- $R_a$, $R_b$: radii of two interacting spherical particles shown in Figure 6.8b
- $R_m = R/\sqrt{\alpha_2 \sigma_2/2}$: magnetic Reynolds number of conductive spherical particle used in Equation (3.68)
- $R_0$: radius of equivalent sphere of homogeneous permittivity $\varepsilon_0$ defined in Figure 6.22
- $S_1$, $S_2$: functions defined by Equations (7.19a,b)
- $T$: temperature
- $T^c$: vector torque of electrical origin
- $T^\eta$: viscous drag torque on rotating particle defined by Equation (4.7)
- $T^m$: vector torque of magnetic origin
- $V$: voltage, or particle volume
<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0$</td>
<td>equilibrium voltage for levitated particle at point $P$ defined by Equation (3.34c)</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>coefficients of magnetostatic potential solution in Equations (3.46a,b)</td>
</tr>
<tr>
<td>$Z_1, Z_2$</td>
<td>coefficients defined by Equations (G.1) and (G.2)</td>
</tr>
<tr>
<td>$a = R_1/R_2$</td>
<td>ratio of radii of spherical shell in Figures 2.3 and 3.4a</td>
</tr>
<tr>
<td>$a, b, c$</td>
<td>semimajor axes of ellipsoidal particle shown in Figure 5.1</td>
</tr>
<tr>
<td>$b$</td>
<td>damping coefficient in Equations (3.33a,b)</td>
</tr>
<tr>
<td>$c$</td>
<td>constant used in Equation (6.59)</td>
</tr>
<tr>
<td>$c_m$</td>
<td>cell membrane capacitance per unit area in Figures 3.3a and 3.4a</td>
</tr>
<tr>
<td>$c_x, c_y, c_z$</td>
<td>direction cosines</td>
</tr>
<tr>
<td>$d$</td>
<td>vector distance between charges in finite dipole</td>
</tr>
<tr>
<td>$d_n$</td>
<td>distance of $n$th image charge from midplane of particle chain</td>
</tr>
<tr>
<td>$e$</td>
<td>eccentricity of spheroidal particle used in Equation (5.15)</td>
</tr>
<tr>
<td>$f = q_j/q$</td>
<td>patch fraction defined in Section 7.4B</td>
</tr>
<tr>
<td>$f(N), f(\Theta)$</td>
<td>distribution functions for chain mixtures used in Section 6.5B</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity (9.81 m/s²)</td>
</tr>
<tr>
<td>$g_m$</td>
<td>transmembrane conductance</td>
</tr>
<tr>
<td>$i, j$</td>
<td>integer indices</td>
</tr>
<tr>
<td>$j = \sqrt{-1}$</td>
<td>square root of minus one</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann’s constant (1.38 • 10⁻²³ J/deg K)</td>
</tr>
<tr>
<td>$m$</td>
<td>magnetic dipole moment vector</td>
</tr>
<tr>
<td>$m, n$</td>
<td>integer indices</td>
</tr>
<tr>
<td>$m_b$</td>
<td>buoyant mass of particle in fluid</td>
</tr>
<tr>
<td>$m_{	ext{eff}}$</td>
<td>effective magnetic dipole moment</td>
</tr>
<tr>
<td>$m_{	ext{perm}}$</td>
<td>permanent moment of magnetized particle in air, Section 5.6D</td>
</tr>
<tr>
<td>$m_{\text{rem}}$</td>
<td>remanent moment of magnetized particle in Equation 4.37</td>
</tr>
<tr>
<td>$h$</td>
<td>unit normal to surface</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>electric dipole moment vector</td>
</tr>
<tr>
<td>$p^{(0)}$</td>
<td>moment of $n$th order linear multipole</td>
</tr>
<tr>
<td>$p_{	ext{eff}}$</td>
<td>effective electric dipole moment</td>
</tr>
<tr>
<td>$p_{\text{eff}} = p_{\text{eff}}(p_0)$</td>
<td>normalized effective electric dipole moment</td>
</tr>
<tr>
<td>$p_N$</td>
<td>dipole moment of linear chain of $N$ perfectly conducting spheres</td>
</tr>
<tr>
<td>$p_0 = 4\pi\varepsilon_0 R^2 E_0$</td>
<td>dipole moment of perfectly conducting sphere</td>
</tr>
</tbody>
</table>
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0, P_\perp$</td>
<td>components of dipole moment parallel and perpendicular to long axis of prolate spheroid or chain</td>
</tr>
<tr>
<td>$P_0, P_\perp$</td>
<td>parallel and perpendicular effective electric dipole moments normalized to $P_0$</td>
</tr>
<tr>
<td>$q$</td>
<td>electric charge</td>
</tr>
<tr>
<td>$q_c$</td>
<td>charge at center of dielectric particle, Figure 7.15</td>
</tr>
<tr>
<td>$q_n$</td>
<td>magnitude of $n$th image charge within conducting particle chain</td>
</tr>
<tr>
<td>$q_p$</td>
<td>charge on dielectric particle, Figure 7.15</td>
</tr>
<tr>
<td>$r$</td>
<td>position vector</td>
</tr>
<tr>
<td>$s$</td>
<td>complex frequency variable used in Equation (E.1); variable of integration in Section 5.2</td>
</tr>
<tr>
<td>$s_1, s_2, s_3, \ldots$</td>
<td>poles of $K(s)$ defined in Appendix E</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>unit step function</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>feedback voltage defined by Equation (3.34c)</td>
</tr>
<tr>
<td>$x$</td>
<td>variable of integration in Kramers–Kronig relations</td>
</tr>
<tr>
<td>$\hat{\xi}, \hat{\eta}, \hat{\zeta}$</td>
<td>unit vectors in Cartesian coordinate system</td>
</tr>
<tr>
<td>$x_1, x_2, x_3$</td>
<td>Cartesian coordinates $(x, y, z)$ in indicial notation (Section 3.4A)</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>uniform thickness of layer on spherical particle, Appendix C</td>
</tr>
<tr>
<td>$\Delta_1, \Delta_2, \Delta_3$</td>
<td>thicknesses of confocal layers in ellipsoid shown in Figure 5.4a</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>center-to-center spacing of interacting spheres in Figure 7.2</td>
</tr>
<tr>
<td>$\Sigma_1, \Sigma_2, \Sigma_3$</td>
<td>summations defined by Equations (6.28a,b,c)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>closed surface of integration</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>electrostatic potential function</td>
</tr>
<tr>
<td>$\Psi^1, \Psi^2$</td>
<td>electromechanical potential function in Equation (3.24)</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>magnetostatic potential functions defined by Equations (3.46a,b)</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>angular velocity of particle</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>equilibrium rotational speed of particle defined by Equation (4.6)</td>
</tr>
<tr>
<td>$\Omega_t$</td>
<td>terminal angular velocity of Quincke rotation defined by Equation (4.27)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>lagging angle between rotating field and dipole moment shown in Figure 4.2; angle of intersection for spheres in Figure 6.10</td>
</tr>
<tr>
<td>$\alpha, \beta, \gamma$</td>
<td>indices representing right-hand, ordered sequence of Cartesian coordinates: $x, y, z, x, \ldots$</td>
</tr>
</tbody>
</table>
Nomenclature

- \( \alpha_1, \alpha_2, \ldots \): coefficients of field expansion along axis of axisymmetric field from Equation (3.32)
- \( \beta = R_b/R_a \): ratio of radii of chained spheres shown in Figure 6.8a
- \( \gamma = \alpha/b \): ratio of axes of spheroid shown in Figure 5.2a
- \( \delta = \Xi - 2R \): gap spacing between two interacting particles
- \( \varepsilon, \varepsilon' \): dielectric permittivity
- \( \varepsilon = \varepsilon' + \sigma j \omega \): tensor permittivity of anisotropic particle
- \( \varepsilon' \): complex permittivity
- \( \varepsilon'' \): real part of complex permittivity
- \( \varepsilon'_e \): negative imaginary part of permittivity
- \( \varepsilon'_s \): effective permittivity of spherical shell, Equation (C.4)
- \( \varepsilon'_c \): complex surface permittivity used in Equation (C.14)
- \( \varepsilon_0 \): permittivity of dielectric substrate in Figure 7.12
- \( \varepsilon_0 \): permittivity of free space (8.854 \times 10^{-12} \text{ F/m})
- \( \varepsilon_c, \sigma_c \): permittivity and conductivity of cell cytoplasm, Section 3.3E
- \( \varepsilon_{c_e}, \sigma_{c_e} \): permittivity and conductivity of cell wall, Section 3.3E
- \( \zeta \): distance of charge \( q \) from center of sphere in Figure 2.4
- \( \zeta(n) \): Riemann-zeta function of integer argument \( n \)
- \( \eta_1 \): dynamic viscosity of fluid medium
- \( \theta \): polar angle in spherical coordinates
- \( \theta_p, \theta_{\perp} \): polarization coefficients for chains of unequal conductive particles in Equations (6.27a,b)
- \( \kappa = \varepsilon \varepsilon_0 \): dielectric constant
- \( \kappa_0 \): dielectric constant of substrate in Figure 7.15
- \( \kappa = \kappa' - j\kappa'' \): complex dielectric constant
- \( \lambda \): exponent of electric field defined in Section 7.2C
- \( \lambda_n \): coefficients of neutralizing charges, Equation (6.12)
- \( \mu \): magnetic permeability
- \( \mu_0 \): permeability of free space (4\(\pi\) \times 10^{-7} \text{ H/m})
- \( \mu_0 = \mu_\parallel / \mu_\perp \): relative permeability
- \( \xi = (\kappa_\perp - \kappa_\parallel) / (\kappa_\parallel + \kappa_\perp) \): coefficient of images in dielectric plane used in Section 7.4B
- \( \xi_m \): distance of neutralizing charges from midplane in Figure 6.4c
- \( \rho \): radial coordinate in cylindrical coordinates
- \( \rho_1, \rho_2 \): mass densities of fluid and particle
- \( \rho_{pl} \): planar spacing parameter in Figure 7.16b
- \( \sigma \): electrical conductivity
- \( \sigma_f \): free electrical surface charge
- \( \sigma_\Sigma \): surface electrical conductivity
- \( \tau = \varepsilon / \sigma \): charge relaxation time
- \( \tau_{pl}, \tau_0, \ldots, \tau_N \): relaxation times of layered particle
- \( \tau_c = \varepsilon_c / \sigma_c \): time constant used in Equation (3.12)
Nomenclature

\( \tau_m = c_m R/\sigma_c \)  
time constant used in Equation (3.12)

\( \tau_{MW} \)  
Maxwell–Wagner relaxation time defined by Equation (2.32)

\( \tau_0 \)  
relaxation time constant defined by Equation (2.32)

\( \tau_r, \tau_r, \tau_r \)  
Maxwell–Wagner relaxation time constants along the three principal axes of ellipsoid, Equation (5.45)

\( \phi \)  
azimuthal angle in spherical coordinates

\( \chi \)  
magnetic susceptibility tensor

\( \chi_r, \chi_\perp \)  
parallel and perpendicular susceptibilities of prolate spheroids, Equation (5.53)

\( \chi^* \)  
critical value of magnetic susceptibility in Equation (5.54a,b)

\( \omega \)  
radian frequency of electric or magnetic field

\( \omega_c \)  
critical transition frequency between + and – DEP defined by Equation (3.41)