

## Index.

### A

**abbreviations** (of formulas in MSL), 233

**absolute formula**, 104

**abstract data types**, 225

**abstraction**, 187, 215

**abstractor operator**, 88, 187, 205

**accessibility relation**, 305, 307-308, 336

**addition**, 129-131, 134

  in induction models, 140, 142

**algebraic**

  closure, 165(def), 171, 201

  procedure, 171

**algebraically**

  defined relations, 173, 199(def)

**algebras for logic**, 225

**algorithm**, 70

**alphabet**

  of FOML, 309(def)

  of FTT language  $\lambda$ -F $_{\omega}$ , 206(def)

  of FTT language  $\lambda c$ -F $_{\omega}$ , 206(def)

  of MSL, 231(def)

  of MSL $\Box$ , 282(def)

  of MSL $\square$ , 312(def)

  of MSL $\Diamond$ , 328(def)

  of MSL $\nabla$ , 329(def)

  of SOL language  $L_2$ , 11(def)

  of  $\lambda$ -SOL language  $\lambda$ - $L_2$ , 10(def)

  of PML, 295(def)

  of PDL, 336(def)

  of RTT language  $T_{\omega}$ , 188(def)/

**alternative presentation of standard semantics**, 39

**arbitrary**

  permutations, 202(def)

  cartesian product, 202(def)

**antecedent**, 75(def), 241

**artificial intelligence**, 263

**assignment**

  of FOML(S5), 310(def)

  of MSL $\Diamond$ , 329(def)

  of RTT, 191(def)

  of SOL, 30(def)

  of MSL, 235(def)

**assumptions**

  (when choosing classical logic), 97

**axiom(s)**

  of arithmetical induction, 2(def)

  of comprehension, 3(def)

  of constructibility, 103(def), 106

  of choice, 6, 53, 106, 150

  of Dedekind's cut, 57

  of dense ordering, 52

  of description, 218, 219

  for equality, 187, 218, 219

  for equals Substitution, 218, 219

  of extensionality, 6, 55-57, 73, 75, 78,

  87-88, 150, 181, 184, 186, 190, 218(def),

  219, 278, 281, 285, 287, 289, 317, 325-326,

  347

  of finiteness, 2, 57

  of induction (second order), 116

  of infinity, 2, 52

  of lambda conversion, 218-219

  of MODO, 317

  of MODO(S4), 317

  Only Two Truth Values, 218-219

  of Peano, 116(def)

  of PDL, 340

  of replacement, 60

  of separation, 60

  of well ordering, 3

[modal  $\sim$ ], 301-302, 305, 308, 311, 317, 340

**axiomatic**

  calculi, 298

  set theory, 179

**axiomatizable**, 51, 52, 53, 57, 175, 269, 299, 300, 302  
 class, 51(def), 269  
 logic, 300(def)  
 (only up to isomorphism), 269  
 property, 51(def)  
 [categorically ~], 52(def)  
 [finitely ~ (logic)], 300(def)  
 [finitely ~ (class)], 51(def)  
 [finitely ~ (property), (SOL)], 50(def)

**axiomatization**, 293  
 [completeness of an ~], 293  
 [recursive ~], 287

**B**

**background set theory**, 1-6, 49-51, 97, 99, 110-112, 148, 150

**behave**, 281

**Beth's definability theorem**, 217

**binary relation**  
 on states, 338  
 variables, 10

**Birkhoff theorem**, 224

**boolean**  
 model, 194  
 operation(s), 45(def), 318

**bound variable**  
 (SOL), 17  
 (MSL), 234

**bounded formula**, 104(def)

**C**

**calculus**  
 for ETT (type theory), 218(def)  
 for FOML(S5), 311(def)  
 for K, S4, S5 (of PML), 302(def), 323  
 of MSL (many-sorted logic), 241(def)  
 of PDL, 340(def)  
 of PML (propositional modal logic), 298, 299(def), 301(def)

for RTT (type theory with lambda,  $\lambda\text{-C}_\omega$ ), 192(def)  
 for SOL (sequent calculi  $C_2$ ,  $C_2^-$ ,  $\lambda\text{-C}_2$ ), 75-80 (def)  
 [complete ~], 71, 90, 168, 245, 290, 307, 326, 327, 333, 350  
 [Gentzen's ~], 223  
 [incomplete ~], (see *incompleteness*)  
 [sound ~], 71, 78, 157, 168, 195, 244, 273, 290, 307-308, 326-327, 333, 350  
 [strongly complete ~], (see *strong*)  
 [weak complete ~], (see *weak*)

**canonical**  
 model (PML), 275-276, 295, 306(def), 307-309  
 general structure (PML), 324(def)

**Cantor's paradox**, 184

**cardinality**  
 of a language (SOL), 13(def), 245  
 of a structure (MSL), 231(def)

**categorical/ly**, 4, 51-53, 60, 119, 121  
 axiomatizable, 52(def)

**categoricity**  
 of Peano theory, 116, 122, 128-129

**characteristic function**, 39

**choice**  
 [axiom of ~], 6, 53, 106, 150

**class**  
 -model (of set theory), 99, 103(def)  
 [axiomatizable ~], 51(def), 269

**closed**  
 expression, 17(def)  
 term (of MSL), 234(def)  
 term (of SOL), 17(def)  
 under definability, 278, 281, 283

**closure**  
 [algebraic ~], 165(def), 171, 201  
 [existential ~ (of a formula)], 18(def)  
 [reflexive ~], 339  
 [transitive ~], 339/

**Cohen's model**, 99, 150

- coincidence lemma**
  - (FOML), 311
  - (MSL), 238
  - (SOL), 62
- commutative operation**, 145, 147
- compact logic**, 5-6, 175, 221, 335
- compactness**, 5, 53, 60, 61, 112, 118, 120, 159, 176, 222, 224, 225, 246, 256, 262, 269, 271, 272, 289, 321, 322, 327, 333, 341
  - theorem, 60, 112, 159, 222, 269
    - of FOML(S5), 333
    - of MSL, 246, 256, 262
    - of PML (K, S4), 321, 322
    - of SOL, 5, 60
    - of SOL with general semantics, 289
- complete**
  - axiom system, 6
  - (deductive) calculus, 5, 6, 71, 90, 168, 245, 290, 307, 326, 327, 333, 350
  - ordered field, 57
  - second order logic, 153
  - set of sentences, 128
  - [semantically  $\sim$ ], 127
  - [strongly complete  $\sim$ ], 5, 245, 307
- completeness**, 5-6, 59-62, 120, 128-129, 148, 151, 153, 157-159, 168-170, 173-176, 193-197, 223, 240, 245, 273-275, 289, 291, 293, 309, 326-327, 333-334, 350
  - of an axiomatization, 293
  - of CAL(XL), 275(def)
  - of FOML(S5), 333, 334
  - of K (of PML), 326
  - of MSL, 240, 245-256
  - of PML, 309
  - of SOL with general structures, 289, 290
  - of S4, 327
  - of type theory (RTT), 193
- comprehension**, 3, 6, 13-14, 56-57, 73-74, 79, 84-88, 94, 102, 148, 151, 153, 157-158, 161, 164-170, 173, 175-176, 181, 183-184, 190, 193, 195, 263, 278, 281, 285, 289, 309, 314-315, 317-320, 324-326, 342, 350
  - axioms, 3(def)
    - in  $\lambda$ -notation, 88(def)
  - principle for classes (in GBC), 99
  - rule, 79(def)
  - schema, 79(def)
  - theorem, 87
- [weakening the  $\sim$  axiom]**, 75, 153, 173, 314, 317, 347
- computer**
  - aided problem solving, 226
  - programs, 335
  - science, 226, 263
- congruence**, 136-145, 247
  - (relation), 27-29
  - in Peano models, 136
  - on natural numbers, 136
- connectives**
  - of SOL, 10
  - of MSL, 229-233
- [extensional  $\sim$ ]**, 30
- [intensional  $\sim$ ]**, 30
- consequence (relation)**
  - on frames (SOL), 149, 156(def)
  - in functional frames (type theory), 210(def)
  - global (PML), 306(def)
  - global (PDL), 339(def)
  - in Kripke models (PML), 306(def)
  - local (PML), 306(def)
  - local (PDL), 339(def)
  - for MSL, 236(def)
  - notion (XL), 268, 269
  - for PDL, 339(def)
  - in relational frames (type theory), 195(def)
  - for SOL, 33-35(def)
  - for type theory, 192(def)
- consequent**, 75(def)
- consistency**, 242-251, 254-255, 300-303
  - (MSL), 242-245, 248, 254-255
  - (PML), 300, 301
  - of normal logics contained in S5, 303
  - [finiteness of  $\sim$  lemma], of MSL, 243, 248
  - [maximal  $\sim$  theorems]
    - of MSL, 243
    - of PML, 300-301
- consistent**, 98, 106, 109-110, 113, 223, 242-257, 275, 300, 301, 303-307, 324-326
  - modal logic, 300(def)
  - normal logic, 303
  - maximal, 300(def)
- constructibility axiom**, 103(def), 106

**constructible**, 102-106, 109-111, 152  
 universe, 102(def)

**contingency**  
 (of propositions), 292

**continuum hypothesis**, 3(def), 58  
 [generalized  $\sim$ ], 3(def), 58, 98-99, 106, 110, 113

**contradictory**  
 set of formulas, 34, 242(def), 240, 243-244, 247-249  
 modal logic, 291, 299, 300(def)

**conversion**  
 of variables into parametres, 203  
 of structures  
   direct, 265-266, 272, 277, 288  
   reverse, 269, 272  
   SOL into MSL $\Delta$ , 280, 283(def), 285(def), 289  
   FOML(S5) into MSL $\Diamond$ , 329  
   MSL into FOL, 223, 257-258  
   FOL into MSL, 259  
   PML into MSL $\Box$ , 315(def), 320(def)  
   PDL into MSL $\bowtie$ , 342

**correct reasoning**, 34

**correspondence theory**, 291

**countable**, 3, 245, 246, 255

**Craig's interpolation lemma for MSL**, 222

**cummulative hierarchy**, (see *hierarchy*)

**cut rule**, 217

**D**

**degree** (of a predicate), 12(def)

**Dedekind's cut axiom**, 57

**deduction**, 61  
 (ETT), 219(def)  
 exercises, 88-90, 129  
 (MSL), 241-242  
 (SOL), 75(def), 80  
 theorem, 299, 323, 326  
 [finiteness of  $\sim$ ], 91, 96

**definability**  
 theorem (of Beth), 217  
 in dynamic logic, 339(def)

**definable** (sets/relations), 3, 40-46, 94, 99, 101-103, 118-122, 130, 151, 169 (see also *relation(s)*)  
 closure, 165-166(def), 172, 200  
 in dynamic logic, 339(def)  
 in a frame, 161, 163-165  
 in a relational frame (type theory), 195-196(def)  
 in RTT, 181  
 [algebraically  $\sim$ ], 173, 199  
 [parametrically  $\sim$ ], (see *relation(s)*)

**definition**  
 of program languages, 225  
 by recursion on EXPR, 15(def), 234(def)

**denotation**, 32, 55, 155, 159-161, 167, 191-192, 195, 197, 210, 235, 255  
 of expressions (RTT), 191(def)  
   (FTT), 210(def)  
   (MSL), 235(def)  
   (RTT with lambda), 192(def)  
 of terms and predicates  
 (SOL), 32(def)  
   in frames, 155(def)  
   in general structures, 167(def)

**dense ordering**, 52

**derivable**  
 rules (SOL), 80  
 sequent, 80

**descriptions**  
 of states, 294  
 [axiom of  $\sim$ ], 218, 219

**deterministic** (program), 340

**difference**, 171, 198

**disjoint universes property**, 278, 283, 285

**downward Löwenheim-Skolem**  
 for weak-SOL, 176

**dynamic**  
 logic, 75, 225-227, 264, 295, 335-351  
 situations, 291

[definability in  $\sim$  logic], 339(def)  
 [extended  $\sim$  frames], 345(def)

**E**

**efficient** (proof theory), 226

**elementary equivalence**, 66

**eliminating (superfluous variables)**, 203

**embedding**, 25(def)

**enumerability**, 262, 267, 272, 289, 321, 323, 327, 332-333  
 of FOML(S5), 332  
 of  $K$ , 321  
 of MSL, 262  
 of S4, 323  
 of SOL (with general structures), 289  
 of XL, 267, 272

**environment**, 47, 50

**Epimenides paradox (liar's paradox)**, 50, 97, 185

**equality**, 2, 7-8, 11, 14-15, 32, 37, 53-57, 73, 75, 77, 84-87, 100, 112, 155, 159, 175, 187-190, 195-197, 200-201, 209, 213, 214, 215-219, 228-230, 323, 241, 247, 249-250, 274, 282, 310, 328, 329  
 in ETT, 215-219  
 -free SOL, 8(def), 14, 37, 53-55, 78, 159  
 in MSL, 228-230, 323, 241, 247, 249-250  
 in RTT, 187-189(def), 190, 195  
 in SOL, 11, 32  
 [axioms for  $\sim$ ], 187, 218, 219  
 [reflexivity of  $\sim$ ], 77, 241

**equals** (substitution lemma), 242

**equational (theory of types (ETT))**, 214-219

**equipollent**, 3

**equivalen(ce/t)**

between programs, 335  
 in frames, 156-157  
 relation, 27-28, 55-56, 137, 229, 247, 249, 250, 252  
 [elementary  $\sim$ ], 66  
 [logical  $\sim$ ], 36-37, 192, 210, 217, 236

[secondary  $\sim$ ], 66  
 [semantic  $\sim$ ], 212, 265-267, 269  
 SOL-MSL $\square$ , 288-290  
 FOML-MSL $\Diamond$ , 329-332  
 XL-MSL#, 265

**erasure function**, 303(def)

**euclidean relation**, 308

**existential closure** (of a formula), 18(def)

**exponentiation**, 129-135, 140-145  
 in Peano models, 131-132(def)  
 in induction models, 140-145(def)

**expressing** (properties of programs in PDL), 339

**expressions**

of SOL, 12(def)  
 of  $T\omega$ , 188(def)  
 of  $\lambda$ - $F\omega$ , 206(def)  
 of MSL, 231(def)  
 of MSL $\square$ , 282(def)  
 of MSL $\square$ , 312(def)  
 of MSL $\Diamond$ , 328(def)  
 of MSL $\#$ , 343(def)  
 [closed  $\sim$ ], 17(def)

**expressive power**, 59-60, 97, 99, 112, 118, 121, 127, 151, 174-175, 181, 186, 193, 309

**extended**

SOL, 7(def) SOL  
 dynamic frames, 342, 345(def), 346-347

**extensional**

connective, 30  
 model, 278, 283

**extensionality**, 6, 55-57, 73, 75, 78, 87-88, 150, 181, 184, 186, 190, 218-219, 278, 281, 285, 287, 289, 317, 325-326, 347

**rule**

(in SOL), 78(def)  
 (in ETT), 218(def)

**F**

**falsity**, 10, 221

**finitely (axiomatizable)**

logic, 300(def)  
 class (of structures in SOL), 51(def)  
 property (SOL) , 50, 52(def)

**finiteness**, 2, 3, 5, 52, 56, 57, 149, 170, 237  
 axiom, 2, 57  
 property, 170  
 of consistency, 235  
 (MSL), 243, 248

**first order**

logic (FOL), 6, 215  
 modal logic (FOML) , 297  
 and many-sorted logic, 297  
 as many-sorted logic, 315  
 relation (see *relation(s)*)  
 [proper extension of  $\sim$  logic], 218

**formula (s)**

of FOML , 310(def)  
 of MSL, 231(def)  
 of PDL, 337(def)  
 of PML, 296(def)  
 of SOL, 11(def)  
 [abbreviation of a  $\sim$  of MSL], 233  
 [absolute  $\sim$ ], 104  
 [absolute  $\sim$  relative to a theory], 102(def)  
 [bounded  $\sim$ ], 104(def)  
 [independent  $\sim$ ], 35  
 [logically equivalent  $\sim$ ], 36  
 [persistent  $\sim$ ], 99-100, 105(def), 108, 111, 114  
 [persistent  $\sim$  relative to a theory], 102(def)  
 [relativization of a  $\sim$ ], 104(def)  
 [unsatisfiable  $\sim$ ], 33(def)  
 [valid  $\sim$ ], (see *valid*)

**formal language**, (see *language* )

**frame**, 74, 78, 148, 152-164, 167-170, 172-174, 182, 193, 194-196, 207, 210-215, 264, 278, 309, 314-315, 342-347  
 built on PD-structures, 345(def)  
 built on PM-structures, 315(def)  
 -consequence, 156(def)  
 -equivalent, 156(def)  
 -model,  
   (of SOL), 155(def)  
   (of type theory), 195  
 -satisfiable, 156(def)  
 -valid, 156(def)  
 [extended dynamic  $\sim$ ], 345  
 [functional  $\sim$ (type theory)], 207(def)

**[general  $\sim$ ], 154(def)**

[relations in a  $\sim$ ] (see *relation(s)* )  
 [relational  $\sim$ (type theory)], 194(def)  
 [second order  $\sim$ ], 154(def)

**free (variable)**

(SOL), 17(def)  
 (MSL), 234(def)

**function**

variables (SOL),7, 9  
 constants (SOL), 8, 10-12, 15  
 [characteristic  $\sim$ ], 39

**functional**

(theory of types, FTT), (see *type* )  
 general structures, 209(def)  
 hierarchy of sets, 207(def), 211-212

**fundamental (relations)**, 171-172, 198-199**G****general**

frames, 154(def)  
 model  
   (of SOL) 148, 150, 153, 289  
   (of type theory), 194  
 structures, 6, 74, 79, 121, 148, 151-153, 161, 164-171, 173, 186, 194, 196-203, 209, 217, 277-278, 281, 284, 288, 315, 316  
 (SOL), 165(def), 172(def)  
 algebraically defined, 171-174(def)  
 [canonical  $\sim$  structure], 324(def)  
 [functional  $\sim$  structure], 209(def)  
 [relation definable in a  $\sim$  structure of type theory] (see *relation(s)* )  
 [relational  $\sim$  structure], 196(def), 193-197

**generalized continuum hypothesis**, 3, 58  
 98-99, 106, 110, 113**Gentzen's calculus**, 223**Gödel's model**, 99**H****heterogeneous structures**, (see *many-sorted* )**hierarchy**

of sets (Zermelo-Fraenkel), 47-49, 102, 152

of sets (Gödel-Bernays-Neumann), 102  
 of types  
 [functional  $\sim$ ], 207(def), 211-212  
 [relational  $\sim$ ], 180, 190, 191(def), 195, 200,  
 278

**higher order** (logic), 1-219

**Hoare-Floyd logic**, 227

**homogeneous structures**, 149

**homomorphic image**, 24(def), 25-29, 64,  
 122, 126-127, 130-133, 140, 142-144, 220

**homomorphism**, 24(def), 25, 27-29, 64, 122,  
 125-127, 130-133, 140, 142, 144, 220  
 onto, 25(def)  
 strong, 25(def)

## I

**IF THEN** (programs), 339

**identity**, 2, 32, 34, 37, 39, 53-56, 67, 73, 88,  
 127, 133-134, 137, 139, 157, 159-160, 175,  
 187, 190, 195-197, 200-202, 209, 211-215,  
 228, 230, 247, 249-250, 258, 285-287, 329,  
 345  
 in FOL, 54  
 of individuals, 2(def)  
 in equality-free SOL, 54(def)  
 of relations  
 in equality-free SOL, 55(def)  
 in frames, 159(def)

**incompleteness**, 5-6, 59-60, 62, 94-114,  
 128-129, 151, 169, 175, 193  
 of type theory, 6  
 of standard SOL (strong sense), 94-114  
 of standard SOL (weak sense), 94-114,  
 128-129  
 of C<sub>2</sub>- in general structures, 169  
 of standard type theory (RTT), 193  
 of C<sub>2</sub>- in standard structures, 95

**index set** (SORT), 229

**induction**, 2, 14, 37-38, 62, 116-119, 122,  
 125-126, 130, 132, 135-140, 140-144,  
 233-234  
 axiom, 116-121, 124, 129-130  
 model, 116(def), 116-119, 122, 125-126,

130, 132, 135-144  
 [arithmetical  $\sim$ ], 2(def)  
 [proofs by  $\sim$ ], 14(def)

**infinitary** (logic), 227

**infinity**, 2, 52

**intensional** (connectives), 30

**interpolation lemma**, 222

**interpretation**

(FOML), 311(def)  
 (FTT), 207(def)  
 in Kripke models, 305(def)  
 (MSL), 235(def)  
 (PDL), 337(def)  
 (PML), 310(def)  
 (RTT), 190(def)  
 (SOL), 31(def)

**isomorphism**, 25(def), 25-26, 29, 57, 66-68,  
 126-128, 136, 176-178, 239-240, 246, 269,  
 285-288

theorem, 66-68  
 of Peano models, 126-128

## K

**K** logic, 302(def)

**knowledge representation of design**, 226

**Kripke**

models, 305(def), 294, 305-309, 336, 338  
 semantics, 294-295  
 structures (PDL), 338(def)

## L

**lambda**, 79, 93-94, 160-161, 167-168, 170,  
 186-187, 192-193, 197, 206, 216, 218-219  
 -conversion, 218(def)

**language**

ETT, 214(def)  
 FTT, 206(def)  
 many-sorted (MSL), 231(def)  
 MSL $\Box$ , 281(def)  
 MSL $\Box$ , 312(def)  
 MSL $\Diamond$ , 328(def)

- MSL $\Box$ , 342(def)
  - PDL, 336(def)
  - RTT, 188(def)
  - RTT with lambda, 192(def)
  - SOL, 11(def)
  - [object  $\sim$ ], 36, 50, 97-98, 183, 186
  - Leibniz's principle** (indiscernibility) 8, 54, 55, 85, 159, 175, 189, 209
  - liar's paradox**, 50, 97, 185
  - Lindenbaum's lemma**, 245-247, 301
  - logically equivalent formulas**, 36(def), 36-37, 192, 210, 217, 236
  - logic**
    - programming, 226
    - of programs, 227
    - [axiomatizable  $\sim$ ], 300(def)
    - [compact  $\sim$ ], (see *compact*)
    - [complete  $\sim$ ], (see *completeness*)
    - [dynamic  $\sim$ ], 335-351
    - [higher order  $\sim$ ], 1-219
    - [Hoare-Floyd  $\sim$ ], 227
    - [infinitary  $\sim$ ], 227
    - [K  $\sim$ ], 302(def)
    - [KT  $\sim$ ], 302(def)
    - [KT4  $\sim$ ] (Lewis system S4), 302(def)
    - [KT5  $\sim$ ] (Lewis system S5), 302(def)
    - [many-sorted  $\sim$ ], 220-262
    - [modal  $\sim$ ], 291-334
    - [multivalued  $\sim$ ], 227
    - [nonabsolute  $\sim$ ], 2
    - [non-classical  $\sim$ ], 227
    - [partial  $\sim$  (many-sorted)], 227
    - [second order  $\sim$ ], 1-179
    - [S5  $\sim$ ], 302(def)
    - [temporal  $\sim$ ], 227
    - [unsorted  $\sim$  (single sorted, one sorted)], 158, 221, 223, 257
  - logical**
    - consequence, (see *consequence*)
    - equivalence
      - (in SOL), 36(def)
      - (in RTT), 192(def)
      - (in MSL), 236(def)
  - Łos theorem**, 224
  - Löwenheim-Skolem theorem**
    - (in K), 321
  - (in MSL)**, 221-222, 224-225, 246, 256, 262
  - (in SOL)**, 5, 6, 60-61, 112, 148, 153, 159
  - (in S4)**, 323
  - (in FOML-S5)**, 327, 333
  - weak SOL** (downward), 175-176
  - (in XL)**, 269, 271-271, 289
- M**
- Main Theorem**, 269-272, 275
    - (for FOML-S5), 332
    - (for K), 321
    - (for SOL), 283, 288, 290
    - (for S4), 322
  - many-sorted**, 9, 56, 73-74, 149, 151, 153, 158-159, 173, 220-262, 277-291, 312-327, 328-334, 342-351
    - logic, 220-258
    - point of view, 149, 151
    - structures, 229-231
      - (MSL and FOML(S5)), 328-334
      - (MSL and PDL), 342-351
      - (MSL and PML), 312-327
      - (MSL and SOL), 277-291
      - MSL $\Box$ , 281(def)
      - MSL $\square$ , 312
      - MSL $\Diamond$ , 328(def)
      - MSL $\wedge$ , 342(def)
    - calculus, 72
  - material implication paradoxes**, 293
  - mathematical universe**, 47, 49-53, 58-59, 69, 97-99, 103, 106-108
  - maximal**
    - consistency, 242-251, 254-255, 300-303
    - theorem
      - of MSL, 243
      - of PML, 300-301
  - meaning of non-standard**, 151
  - membership**, 46-47, 49, 98-100, 165, 171, 198, 200, 265, 278-287, 303, 312, 314, 343
    - (in SOL), 171(def)
    - (in type theory), 198(def), 200(def)
  - metalanguage**, 4, 24, 49, 52, 58
  - metaproperties** (of PDL), 326

- modal**, 301-302, 305, 308, 311, 317, 340
  - axioms
    - K, 301(def), 305, 308, 323, 340
    - distribution of  $\Box$  with respect to  $\rightarrow$ , 301(def)
    - Df $\Diamond$ , 302(def)
    - D, T, B, 4, 5, 302(def)
  - logic, 30, 37, 75, 227, 291-334, 298(def), 336-338, 340, 342
  - operator symbol, 296(def), 294, 296-298, 303, 310, 337
  - system, 299(def)
  - theorem, 298(def)
- [normal ~ logic], 302(def)
  - K, 302(def), 305, 308, 323, 326, 340
  - S4, 302(def), 304, 322-324, 326-327
  - S5, 302(def)
- [pure ~ logic], 309(def)
- model**
  - of FOML(S5), 309, 310(def), 311(def)
  - of MSL, 236(def)
  - of set theory (GBN), 103
  - of set theory (ZFC), 59-60, 102(def)
- [boolean ~], 194
- [canonical ~], 275-276, 295, 306(def), 307-309
- [class- ~], (set theory), 99, 103(def)
- [Cohen's ~], 99, 150
- [extensional ~], 278, 283
- [frame- ~]
  - (of SOL), 155(def)
  - (of type theory), 195
- [general ~]
  - (of SOL) 148, 150, 153, 289
  - (of type theory), 194
- [Gödel's ~], 99
- [induction ~], 116(def), 116-117, 125-126, 136-140
- [Kripke ~], 305(def)
- [non-standard ~], (of SOL), 30, 152
  - of arithmetic, 120-122
- [normal ~], (in MSL), 228-230, 236
- [Peano's ~], 116(def), 116-117, 125-129, 136
- [standard ~], (of SOL), 33(def), 64(def)
- [weak ~], 176(def)
- MODO**, 317-320
- MODO(S4)**, 317-320
- monadic** (second order language), 7
- monotony**
  - on hypothesis, 299(def)
  - on logics, 299(def)
- multi-modal logic**, 336
- multivalued logic**, 227
- multiplication**
  - [operation of ~]
    - in induction models, 140
    - in Peano models, 131

**N****naive set theory**, 176**natural**

- deduction calculi, 298
- language, 8, 34
- logic, 264
- numbers, 46, 115-116, 118, 120, 127, 129, 176, 178
- semantics, 73, 136, 150, 153
- [segments of ~ numbers], 123(def)

**naturalness**, 181, 222, 224**necessary**, 291-293**necessitation**, 301-304, 311, 314, 329, 333, 337, 340**necessity**, 292, 294, 298, 309

- [logical ~], 292
- [physical ~], 292
- [moral ~], 292
- [temporal ~], 292
- [Leibnizian sense of ~], 298, 309

**non-standard**

- models, 30, 118, 120-122, 151-152, 176
- numbers, 119
- semantics, 7, 8, 30, 35, 78, 151, 159, 207
- structures, 6, 30, 44, 56, 148, 151, 154, 194, 226, 278
- view, 149-150
- [meaning of ~], 151

**normal**

- modal logic, 302(def)
  - determined by a class of structures, 308(def)

structure, 56, 229-230, 236(def), 259  
 [sound ~ modal logic], 307(def)  
 [strongly complete ~ modal logic], 307(def)  
 [weakly complete ~ modal logic], 307(def)

**O**

**object language**, 36, 50, 97-98, 183, 186  
**one-sorted logic**, 221, 224, 257-262

**operation**  
 constants (SOL), 6, 8-10, 100, 186, 188, 211, 278  
 of addition, 129, 130, 131, 134  
     in induction models, 140, 142  
 of arbitrary permutations, 202(def)  
 of arbitrary cartesian product, 202(def)  
 of complement, 202(def)  
 of complementary of the projection of the complement, 202(def)  
 of complement of the symmetrical difference, 202(def)  
 of identification, 202(def)  
 of multiplication, 131, 140  
     in induction models, 140  
     in Peano models, 131  
 of exponentiation, 129-135, 140-145  
     in induction models, 140-145(def)  
     in Peano models, 131-132(def)  
 of parameterization, 202(def)  
 [boolean ~], 45(def), 318  
 [commutative ~], 145, 147

**operator**  
 [abstractor ~], 88, 187, 205  
 [modal ~], 296(def), 294, 296-298, 303, 310, 337  
 [program ~], 336-337  
 [selector ~], 206, 208, 215, 218  
 [test ~], 337  
 [ $\vdash_B$  ~], 299

**ordered field**, 57

**ordering**  
 [dense ~], 52  
 [well ~], 3

**P**

**paradox(es)**, 182-186  
 [Cantor's ~], 184  
 [Epimenides (liar's ~)], 50, 97, 185  
 [logic or mathematical ~], 183, 189  
 [material implication ~], 293  
 [Russell's ~], 183  
 [semantic or epistemological ~], 183, 189  
 [Skolem's ~], 153

**parametrically definable**  
 relation, 40, 44, 46, 101, 121, 161, 164, 197-198, 200-203, 209  
 set, 101(def)

**parametrization**  
 (of a relation), 202(def)

**partial**  
 function, 122-125, 123(def)  
 many-sorted logic, 227

**Peano**  
 arithmetic, 115-148  
 axioms, 4, 89, 115, 116(def), 136, 139  
 model, 113(def), 119, 121, 122, 125, 126, 131, 132  
 theory  
     (first order), 112-118  
     (second order), 112-118  
 [axiomatizability of ~ structures], 116, 122, 128-129  
 [isomorphism of ~ structures], 126-128  
 [congruence relation in ~ structures], 136  
 [recursive operations in ~ models], 132

**permutation** (relation)  
 [arbitrary ~]  
     (in SOL), 172(def)  
     (in type theory), 198(def), 200, 202

**persistent formulas**, 99-100, 105(def), 108, 111, 114  
 relative to a theory, 102(def)

**possible/possibility**, 291-295, 309-310

**power**  
 set, 1, 3, 23, 48(def), 58-59, 112, 148, 150-151, 184-185, 278, 314  
 [expressive ~], 59-60, 97, 99, 112, 118, 121, 127, 151, 174-175, 181, 186, 193, 309

**predicate** (SOL), 11(def)

**pre-structures**, 150(def)

**primitive recursion**, 129-136, 140-147

**principal interpretations**, 224

**products**

direct and reduced, 224

**program(s)**

logic, 335

algorithm, 335

operator, 336-337

verification, 225

of PDL, 337(def)

[computer ~], 335

**programing languages**, 335

**projection relation**

type theory, 199, 201(def)

SOL, 172(def)

**proof** (in the calculus of PML), 288(def)

**propert(y/ies)**

of programs (in PDL), 339-340(def)

of the domain, 51

[axiomatizable ~], 51(def)

[finitely axiomatizable ~], 50, 52(def)

**propositional**

calculus, 273, 303

dynamic logic (PDL), 75, 173, 226-227, 335-351, 340(def)

logic, 30, 70, 180

modal logic, 75, 297-303, 312-328

symbols, 8

theorem (of PML), 298

type, 186, 206, 215

type theory, 194

variables, 215

rules, 273

**PD-structure**, 338

**PM-structure**, 305(def)

**pure**

modal logic, 309(def)

SOL, 8, 51, 53

## Q

**quantifiers**, 6-7, 11, 17-18, 23, 34, 37, 69, 73, 75, 104, 161, 173, 187, 188, 190, 209, 215, 231, 234, 273, 279, 282, 312, 319, 320, 328, 343  
 (in ETT), 215-218(def)  
 (in FOML), 221  
 (in SOL), 10  
 [relativization of ~], (MSL), 221, 257-258

## R

**real numbers**, 57, 58

**reasoning**, 34, 49, 69-71, 90, 112, 220-221, 226, 241, 335  
 about computations, 220  
 [correct ~], 34  
 [process of ~], 34

**recursion**

theorem, 125

[definition by ~], 15(def), 16, 234(def)  
 [primitive ~], 129-136, 140-147

**recursive**

axiomatization, 287

operations in Peano models, 132

**reduction**

of MSL to FOL, 257-263

**reflexive/reflexivity**, 18, 27, 37, 46, 52, 56, 77, 85, 241, 249, 308, 322-323, 326-327, 339, 344, 348  
 closure, 339  
 of equality, 77, 241

**relation(s)**

constants

(in FOML), 310(def)

(in SOL), 11(def)

variables (in SOL), 10(def)

[accessibiluty ~ ], 305, 307-308, 336

[binary ~ on states], 338

[fundamental ~]

membership, 171, 198(def)

difference, 171, 198(def)

cartesian product, 171, 198(def)

permutation 1, 172, 198(def)

permutation 2, 172, 198(def)

projection, 172, 199(def)  
 singletons, 172, 199(def)  
 [serial ~], 308

**relation(s) (in a structure)**

**in a frame**

[ $\mathcal{A}$ -definable first order  $\sim$  using  $\lambda\text{-}L_2$ ], 163(def)  
 [ $\mathcal{A}$ -definable second order  $\sim$  using  $\lambda\text{-}L_2$  (or  $L_2$ )], 163(def)  
 [defines  $\sim$  in the structure  $\mathcal{A}$  using  $L$ ], 163(def)  
 [first order  $\sim$  on  $\mathcal{A}$ ], 162(def)  
 [first order  $\sim$  into  $\mathcal{A}$ ], 162(def)  
 [parametrically  $\mathcal{A}$ -definable first order  $\sim$  using  $\lambda\text{-}L_2$ ], 163(def)  
 [parametrically  $\mathcal{A}$ -definable second order  $\sim$  using  $\lambda\text{-}L_2$ ], 163(def)

**in a relational frame of type theory**

of type  $\alpha$  of  $\mathcal{D}$ , 195(def)  
 into  $\mathcal{D}$ , 195(def)  
 [ $\mathcal{D}$ -definable  $\sim$  using a language  $T_\omega$ ], 195(def)  
 [parametrically  $\mathcal{D}$ -definable  $\sim$  using  $T_\omega$ ], 196(def)  
 [algebraically defined  $\sim$ ], 199

**in relational general structures of type theory**, 201(def)

intersection, 201(def)  
 universes, 201(def)  
 complement, 202(def)  
 complement of the difference, 202(def)  
 complements of the symmetrical difference, 202(def)  
 arbitrary permutations, 202(def)  
 arbitrary cartesian product, 202(def)  
 complementary of the projection of the complement, 202(def)  
 identity relation, 202(def)  
 parametrization, 202(def)  
 identification, 202(def)

**in a SOL-structure**

[ $\mathcal{A}$ -definable first order  $\sim$  using  $\lambda\text{-}L_2$ ], 42(def)  
 [ $\mathcal{A}$ -definable second order  $\sim$  using  $\lambda\text{-}L_2$  (or  $L_2$ )], 43(def)  
 [defines  $\sim$  in the structure  $\mathcal{A}$  using  $L$ ], 42(def)  
 [first order  $\sim$  on  $\mathcal{A}$ ], 40(def)

[first order  $\sim$  into  $\mathcal{A}$ ], 40(def)  
 [first order and globally definable  $\sim$ ], 51(def)  
 [second order and globally definable  $\sim$ ], 51(def)

[parametrically  $\mathcal{A}$ -definable first order  $\sim$  using  $\lambda\text{-}L_2$ ], 44(def)  
 [parametrically  $\mathcal{A}$ -definable second order  $\sim$  using  $\lambda\text{-}L_2$ ], 44(def)  
 [proper second order  $\sim$ ], 41(def)  
 [second order  $\sim$  on  $\mathcal{A}$ ], 41(def)  
 [second order  $\sim$  into  $\mathcal{A}$ ], 41(def)

**relational**

hierarchy of sets, 180, 190, 191(def), 195, 200, 278  
 standard structure (of RTT), 190(def)  
 standard hierarchy of types, 190(def)  
 frame for type theory, 193(def)  
 general structure (for type theory), 193-197  
 (algebraic definition), 197-205

theory of finite types (RTT), 187-198

**relative** (interpretation of set theory into itself), 101(def)

**relativization**

of a formula, 104(def)  
 of quantifiers, 221, 257-258

**replacement**

axiom, 60  
 rule, 218

**representation**

theorem, 264-269, 319, 322  
 (for K), 319  
 (for S4), 322  
 [knowledge  $\sim$ ], 226

**restricted SOL**, 7

**reverse conversion** (of structures), (see *conversion*)

**rule(s) (of the deductive calculi)**

Comprehension schema (CS), 79  
 Deduction rule (D), 82  
 Doble negation rule (DN), 82  
 Eliminating biconditional in the consequent (EBC), 84  
 Eliminating conjunction in the consequent, (ECC), 84

- Eliminating individual generalization in the consequent (EGC), 83
- Equals substitution for individuals (ES), 77, 85, 242
- Extensionality (Ext), 78
- Hypothesis introduction (HI), 76, 241
- Introducing abstraction in the antecedent (IAA), 79, 193
- Introducing abstraction in the consequent, 79, 193
- Introducing biconditional in the consequent (IBC), 84
- Introducing conjunction in the consequent (ICC), 84
- Introducing conditional in the consequent (I $\rightarrow$ C), 84
- Introducing disjunction in the antecedent (IDA), 77, 241
- Introducing disjunction in the consequent (IDC), 77, 241
- Introducing individual generalization in the consequent (IGC), 83
- Introducing individual particularization in the antecedent (IPA), 77, 241
- Introducing relation quantification in the antecedent (IPA)<sup>n</sup>, 77
- Introducing individual particularization in the consequent (IPC), 77, 241
- Introducing relation quantification in the consequent (IPC)<sup>n</sup>, 78
- Monotony (M), 76, 241
- Rule of "modus ponens" (MP) 81, 299, 340
- Rule of "modus tolens" (MT), 81
- Non contradiction (NC), 76, 241
- Proof by cases (PC), 76, 241
- Replacement (R), 218
- Reflexivity of equality for individuals (RE) 77, 241
- Second non contradiction rule (SNC), 81
- Rule of transitivity (T), 81
  
- Russell's paradox**, 183
  
- S**
  
- satisfaction**  
 (SOL), 32(def), 33(def)  
 [concept of ~], 30
  
- satisfiable**, 33-36, 156  
 formula (SOL), 33(def)  
 set of formulas (SOL), 33(def)  
 (in a frame), 156(def)
  
- satisfiability**, 33, 156, 192, 195, 210, 236, 292, 306, 339  
 (FTT, functional frames), 210(def)  
 (MSL), 236(def)  
 (PML, in Kripke models), 306(def)  
 (PDL), 339(def)  
 (RTT, standard), 192(def)  
 (RTT, relational frames), 195(def)  
 (SOL, standard), 33(def)  
 (SOL, on frames), 156(def)
  
- schema**  
 (PML), 296(def)
  
- second order**  
 calculus, 6, 75-80  
 equivalent, 66  
 expressible, 47  
 language, 7-16  
 logic, 1-179
  - (extended SOL), 7
  - ( $\lambda$ -SOL), 8
  - (non-standard), 155
  - (pure SOL), 8
  - (restricted SOL), 7
  - (standard SOL), 47
  - (SOL), 7(def)
  - (weak SOL), 174
 Peano axioms, 116-123  
 relations, 171-172
  
- [monadic ~ language], 7  
 [binary ~ language], 7  
 [ $n$ -ary ~ language], 7
  
- segments**  
 (of natural numbers), 123(def)
  
- selector**  
 operator, 206, 208, 215, 218  
 (in ETT), 218(def)
  
- semantic(s)**  
 conversion of MSL into FOL, 258-263  
 equivalence, 212, 265-267, 269
  - RTT to FTT, 210
  - FOML-MSL◊, 310(def), 329-332
  - SOL-MSL□, 288-290
  - XL-MSL#, 265
 on frames, 155  
 of modal logic, 310  
 of PDL, 337(def)  
 (standard SOL), 30-62
  
- [alternative presentation of standard ~], 39  
 [Kripke's ~], 294-295

- [non-standard  $\sim$ ], (see *non-standard*)
- semantically complete**, 124
- sentence**
  - (of SOL), 17
  - (of MSL), 234
- [closed expression], 17, 234
- separation**
  - [axiom of  $\sim$ ], 60
- sequent**, 74-94, 232
  - calculi (SOL), 75-94
  - rules
    - (of SOL), 75-80(def)
    - (derivable in SOL), 80-84
    - (of MSL), 241-242(def)
  - [derivable  $\sim$  (SOL)], 78(def)
  - [derivable  $\sim$  (MSL)], 232(def)
- serial**
  - (relation), 308
- set theory**, 1-6, 47, 49-51, 59-60, 70-71, 97-114, 148-152, 181-185
  - [axiomatic  $\sim$ ], 179
  - [background  $\sim$ ], 1-6, 49-51, 97, 99, 110-112, 148, 150
  - [class-model of  $\sim$ ], 99, 103(def)
  - [naive  $\sim$ ], 176
  - [relative interpretation of  $\sim$  into itself], 99
  - [Zermelo-Fraenkel  $\sim$ ],
    - (see *Zermelo-Fraenkel*)
- Skolem's paradox**, 53, 149
- signature**,
  - FTT, 205(def)
  - MSL, 229(def)
  - MSL $\Box$ , 312(def)
  - MSL $\Diamond$ , 328(def)
  - MSL $\lambda$ , 342(def)
  - MSL $\sigma$ , 281
  - RTT, 188(def)
  - SOL, 10(def)
- simple type theory**, 181
- simultaneous substitution**, 18(def)
  - lemma, 64
- singletons**, 54, 172, 199
  - SOL, 172(def)
- type theory**, 199(def)
- SOLO<sup>2</sup>**, 347(def)
- sound/soundness**, 71-72, 90-94, 109-114, 157-159, 165, 168-171, 174, 195, 197, 219, 244-245, 273, 275, 289, 290
  - of CAL(XL), 273(def)
  - of comprehension, 94
  - condition, 165
  - in frames (SOL), 157
  - in general structures (SOL), 168
  - of K, 326
  - of lambda rules (SOL), 93
  - of MSL, 244-245
  - of normal modal logics, 307(def), 308, 326-327
  - in relational frames (type theory), 195
  - of S<sub>4</sub>, 327
  - of S<sub>5</sub>, 333
  - of SOL, 91, 290
- standard**
  - consequence, 96, 158, 167
  - numbers, 118-122
  - structures, (see *structures*)
  - [relational  $\sim$  structure (of RTT)], 190(def)
  - [relational  $\sim$  hierarchy of types], 190(def)
- state**, 220, 294, 305, 310, 336, 337, 340, 342
  - (PML), 305(def), 310(def)
  - (PDL), 337(def)
- strong(ly)**
  - homomorphism, 24(def)
  - (sense of completeness), 5-6, 60-61, 71, 73, 94, 96, 112, 120, 148, 153, 157, 175-176, 193, 224, 241, 245, 256, 289, 307, 342
  - (sense of soundness), PDL, 340
  - version of lemma 2, 270, 288, 320, 332
  - complete normal modal logic, 307(def)
- structures**
  - built on PM- $\sim$ , 314-315
  - built on PD- $\sim$ , 344(def)
  - (of FOML-S<sub>5</sub>), 310
  - (of FTT), 207-209
  - (of MSL), 229(def), 227-231
    - MSL $\Box$ , 314-315(def)
    - MSL $\sigma$ , 283-288
    - MSL $\Diamond$ , 329(def)
    - MSL $\lambda$ , 344-345
  - (of PDL), 338(def)

- (of PML or Kripke's models or PM- $\sim$ ), 305(def)
- (of SOL), 1-2, 6, 22-30, 31, 33, 35, 37, 40-47, 50-57, 64, 66, 69
  - (general), 165(def), 172(def), 6, 74, 79, 164-173
  - (standard), 22(def), 2, 6, 22-30, 35, 37, 69, 90
  - (of weak-SOL), 174
- RTT, 190-192, 193-210
  - (general), 196(def), 197-205
  - (standard), 190(def)
- [algebraic definition of relational general  $\sim$ ]
  - (in SOL), 172-173
  - (in RTT), 197-205
- [axiomatizability of a class of  $\sim$ , (SOL)], 51(def)
- [axiomatizability of a property of the domain of a  $\sim$ , (SOL)], 51(def)
- [categorically axiomatizable  $\sim$ , (SOL)], 52, 57(def)
- [convention for simplifying standard  $\sim$ ], 23
- [conversion of  $\sim$ ], (see "conversion")
- [definable relations on/into/of  $\sim$ ], 40-47, (see also *relations*)
- [identity in a  $\sim$ , (SOL)], 54
- [induction  $\sim$ ], 135-140
- [infinite  $\sim$ , (SOL)], 52
- [isomorphic  $\sim$ ], 25(def), 66
- [finite axiomatizability of a class of structures], 52(def)
- [homogeneous  $\sim$ ], 149
- [homomorphic  $\sim$ ], 24(def)
- [embedding], 25(def)
- [Kripke  $\sim$ ], 294, 305-309, 336, 338
- [Peano  $\sim$ ], 116-118
- substructure**, 24
- subset**
  - [concept of  $\sim$ ], 150-152
- substitution**
  - (in MSL), 236-238
    - of a term (MSL), 236-238(def)
  - (in SOL), 18-22
    - of a term (SOL), 18-19(def)
    - of a predicate, 20-21(def)
- lemma
  - SOL, 64-66
  - MSL, 238
- [equals  $\sim$  lemma], 239
- [simultaneous  $\sim$ ], 22(def)
- [simultaneous  $\sim$  lemma], 18, 65
- symmetric**
  - (relation), 308(def)
- syntactical**
  - translation, (see "translation")
  - incompleteness, 128
- system** (of modal logic), 299
- T**
- Tarski's theorem of indefinability of truth, 50, 97
- temporal**
  - interpretation (of modal operators), 292
- term**
  - (of MSL), 232(def)
  - (of SOL), 11(def)
  - [closed  $\sim$ ], 17(def), 234(def)
- test operator**, 337
- theorem**
  - (modal), 298(def)
- theory**
  - (in modal logic), 299
  - of vector spaces, 220
  - of groups, 220
  - of rings, 220
  - of types, (see *types*)
- traditions of modal logic**, 293-294
  - syntactical, 293
  - algebraic, 293
  - model-theoretic, 294
- transitiv(e/ity)**, 100, 104-105, 111, 308, 322-323, 326-327, 339, 344-348, 54, 81, 250
  - class of sets, 100, 104(def)
  - closure, 339
  - rule, 81
  - subclass, 105, 111
  - substructure, 105
- translating (function)**
  - FOML(S5) into MSL $\Diamond$ , 328(def)
  - MSL into one-sorted FOL, 257(def)
  - PDL into MSL $\Box$ , 343(def)

- PML into MSL $\Box$ , 313(def)  
 RTT to FTT, 212(def)
- translation**, 263-276  
 of modal formulas, 313  
 SOL into MSL $\Box$ , 283(def)  
 of the axioms for normal modal logics, 313-314
- true**  
 formula, 103(def), 236(def), 311(def)  
 (at a state), 311(def)
- truth**, 2, 7, 30, 34, 50, 69, 95, 98-99, 103, 107-108, 160, 183, 190, 209, 216, 218-219, 246, 261, 266, 270, 272, 275, 277, 286, 291-295, 305, 307, 309, 311, 329, 335-337, 339(def)  
 in a class structure, 103(def)  
 of formulas of the object language, 50  
 function, 7, 30, 39, 293  
 in a model  
 (FOML(S5)), 311(def)  
 (PDL), 339(def)  
 of a persistent formula, 105  
 value, 39, 95, 100, 111, 160, 190, 209, 305, 329  
 standard, 30  
 universe, 227-228, 329
- [binary relation of  $\sim$ ], 107  
 [many-sorted  $\sim$ ], 261  
 [relativized  $\sim$  value], 291  
 [third  $\sim$  value], 161, 227  
 [undefinability of  $\sim$ ], 50, 97  
 [unspecified  $\sim$  value], 161, 227
- type**, 9-11, 22, 31, 39, 43-44, 51, 63  
 of individuals, 10  
 of operation, 10  
 of n-ary relations, 10  
 theory, 6, 112, 120, 148, 163, 166, 171, 180-189  
 ETT, 186, 187-190, 190-192, 214-219  
 FTT, 187, 205-207, 207, 210  
 RTT, 186, 187-190, 190-192  
 RTT with lambda, 192-193
- [finite  $\sim$ ], 180  
 [equational  $\sim$  theory], see "ETT"  
 [functional  $\sim$  theory], see "FTT"  
 [relational  $\sim$  theory], see "RTT"  
 [stratified  $\sim$ ], 269
- typed ( $\lambda$ -calculus)**, 8, 193, 218
- U**
- uncountable, 5, 57-58, 61
- undefinability of identity in frames, 159
- unification of domains, 222, 258(def)
- unifier logic, 226
- unifying**  
 framework, 263  
 approach, 264
- union**  
 (relation), 202(def)  
 (operation), 202(def)
- unique readability theorem**, 15
- universal**  
 closure (of a formula), 18(def)  
 class, 102, 184, 186  
 operations, 130, 140, 144, 147
- universe**  
 of individuals (SOL), 22(def)  
 n-ary relations, 23(def)  
 of sort i, 230  
 [mathematical  $\sim$ ], 47, 49-53, 58-59, 69, 97-99, 103, 106-108
- unsatisfiable** (formula), 33(def)
- unsorted logic** (single sorted, one sorted)], 158, 221, 223, 257
- unsoundness in frames**  
 SOL, (C<sub>2</sub>), 157-158  
 SOL with  $\lambda$ , ( $\lambda$ -C<sub>2</sub>), 160-161
- unspecified truth value**, 161, 227
- V**
- valid** (formula)  
 standard SOL, 35(def)  
 frame/general SOL, 156(def)  
 standard RTT, 192(def)  
 frame/general RTT, 195(def)  
 FTT, 210(def)  
 MSL, 236(def)

**Index.**

385

---

PML, 306(def)  
FOML-S5, 311(def)  
PDL, 339(def)

**validity**  
(see *valid*)

**variable**  
[bound  $\sim$ ]  
(SOL), 17(def)  
(MSL), 234(def)  
[free  $\sim$ ]  
(SOL), 17(def)  
(MSL), 234(def)

## W

**weak(ly)**  
(sense of completeness), 5, 60, 62, 71-73,  
94, 96-115, 148, 150, 153, 157, 176, 193  
second order logic, 174-179  
complete normal modal logic, 307(def)

**well ordering**, 3

**while programs**, 335

## Z

**Zermelo-Fraenkel**  
axioms, 49, 58  
hierarchy of sets, 47, 49, 102, 152  
set theory, 49, 59-60, 98, 106, 150

## Index of authors.

### A

Ackermann, 1  
 Andréka, 225, 264  
 Andrews, 165, 180, 215  
 Aristotle, 291

### B

Barwise, 5, 103  
 Bergstra, 225  
 Bernays, 98, 102, 103  
 Beth, 182  
 Birkhoff, 224  
 Boethius, 292  
 Boolos, 295  
 Boudreaux, 165  
 Bowen, 295, 310  
 Brower, 182  
 Bull, 291, 293, 295  
 Burmeister, 222

### C

Cantor, 58, 184  
 Carnap, 181, 291, 294, 295, 309/283, 294, 297  
 Chellas, 295  
 Chrysippus, 192  
 Church, 1, 2, 4, 8, 73, 181, 182, 183, 187, 205  
 Cirilis, 225  
 Cohen, 98, 99, 106, 110, 114, 148  
 Cohn, 224, 226  
 Copi, 182  
 Craig, 222  
 Cresswell, 310

### D

Dedekind, 5, 115  
 Denyer, 8  
 Diodorus Cronus, 291, 292  
 Doets, 4, 60  
 Dziergowski, 224

### E

Ebbinghaus, 4, 5, 49, 75, 223, 241  
 Ehrlig, 225  
 Enderton, 62, 221, 222, 262  
 Epimenides, 184, 186  
 Euclides, 291

### F

Feferman, 5, 222  
 Fermat, 115  
 Fine, 263, 295  
 Floyd, 227  
 Flum, 4, 5, 49, 75, 97  
 Fraenkel, 47, 49, 59, 98, 102  
 Frege, 1, 115, 294

### G

Gabbay, 264  
 Gallin, 225  
 Gandy, 181  
 Gentzen, 223  
 Gergely, 226  
 Givant, 336  
 Gödel, 62, 72, 97, 98, 99, 102, 103, 106, 110, 113, 128, 152, 181  
 Goguen, 225, 226  
 Goldblatt, 264, 291, 310, 341  
 Gurevich, 7

### H

Harel, 335, 341  
 Heijenoort, 5  
 Heraclitean philosophers, 5  
 Henkin, 4, 5, 73, 75, 78, 112, 115, 120, 121, 130, 135, 136, 148, 151, 165, 173, 187, 194, 215, 225, 226, 227, 246, 263, 264, 294, 336  
 Hilbert, 1  
 Hintikka, 291  
 Hoare, 227  
 Hook, 223  
 Huertas, 227

**Index of authors.**

387

Hughes, 310

**O**

Orłowska, 336

**J**Janssen, 226  
Jónsson, 294**P**

Pascal, 115  
 Pasztor, 5, 225  
 Peano, 115, 121, 129  
 Peirce, 116  
 Plato, 291  
 Pratt, 225, 295  
 Prior, 291

**K**

Kamin, 225  
 Kanger, 291  
 Kneale, 291, 292  
 Knuth, 225  
 Kreisel, 4, 222, 223  
 Kripke, 182, 227, 291, 294, 295  
 Krivine, 222, 223  
 Kunen, 98, 103, 106

**Q**

Quine, 4, 182, 215, 292

**L**

Leibniz, 2, 53, 55, 71, 85, 159, 173, 294, 309  
 Lemmon, 291  
 Lewis, 291, 292, 293, 295  
 Lindenbaum, 245  
 Lindström, 224  
 Löwenheim, 5, 61, 112, 153, 159, 222, 224, 246, 256  
 Łos, 24  
 Łukasiewicz, 291, 294

**R**

Ramsey, 182, 214  
 Rasiowa, 294  
 Reiterman, 225  
 Robbin, 78, 128, 165  
 Rogers, 182  
 Rónyai, 225  
 Russell, 1, 181, 182, 186

**S****M**

Mahr, 225  
 Makowsky, 219  
 Manzano, 165, 197  
 Márkus, 222, 224, 226  
 Medieval logicians, 281  
 Megarians, 291, 292  
 Meinke, 154, 224, 225  
 Meseguer, 225, 226  
 Monk, 221, 222, 225, 294, 336  
 Montague, 225, 295  
 Mortimer, 295

Sain, 5, 97, 107, 225  
 Sainsbury, 182  
 Sahlqvist, 264  
 Segerberg, 291, 293, 295  
 Schauß, 226  
 Schmidt, 221, 226  
 Scott, 291  
 Shoenfield, 59  
 Schönfinkel, 187  
 Skolem, 5, 53, 112, 121, 153, 159, 222, 224, 246, 256  
 Socrates, 291, 293  
 Stalnaker, 295  
 Stoics, 291, 292  
 Suskind, 70  
 Szöts, 225

**N**

Németi, 5, 225, 264, 336  
 Neumann, 98, 102, 103, 182

388

---

T

Tarski, 50, 97, 181, 182, 183, 214, 225, 294,  
336  
Thomas, 4, 5, 49, 75, 97  
Thomason, 264, 291  
Trahtenbrot, 5, 97  
Trnková, 225  
Tucker, 154, 224, 225  
Turing, 181

V

van Benthem, 4, 60, 263, 264, 291  
van Dalen, 221  
Venema, 336  
von Newman, 177

Z

Zeno of Citium, 291  
Zermelo, 47, 59, 98, 102, 152, 182  
Zucker, 223, 225

W

Walther, 224, 226  
Wang, 221, 222  
Whitehead, 181  
Wittgenstein, 215, 294