

---

# Contents

---

<i>Preface</i>	xv
<b>1 Background on classical hydrodynamics</b>	<b>1</b>
1.1 Equations of motion, potential flow and vorticity	1
1.2 Creation of vorticity in classical viscous flow	8
1.3 Vortices	12
1.4 Rotating fluids	16
1.4.1 Equations of motion in a rotating frame, Rossby and Ekman numbers	16
1.4.2 Spin-up	17
1.5 Laboratory generation of thin line vortices	19
1.6 Dynamics of classical vortex rings and the localized induction approximation	22
1.7 Waves on vortex lines and rings: stability of vortices	28
1.7.1 Helical waves of constant amplitude	28
1.7.2 Solitary kink waves	31
1.7.3 Instability of parallel vortices	33
1.8 Bénard convection and Taylor–Couette flow	35
1.9 Brownian motion and escape over barriers	38
<b>2 Background on liquid helium II</b>	<b>42</b>
2.1 The two-fluid model	42
2.2 Elementary excitations in helium II	45
2.3 First ideas about quantized circulation and vortices	48
2.3.1 Onsager’s quantization of circulation	48
2.3.2 Feynman vortices	50
2.4 Early experimental evidence for quantized vortices and circulation	53
2.4.1 Minimization of free energy	53

x	<i>Contents</i>	
	2.4.2 Second sound and quantized vortices. Mutual friction	55
	2.4.3 Experiments on quantized circulation	58
	2.4.4 Vortex-free rotation of the superfluid	62
	2.5 Ions and vortices: evidence for quantized vortex rings and lines	63
	2.5.1 Experiments on quantized vortex rings	63
	2.5.2 Experiments on quantized vortex lines	65
	2.6 Rotation of an annulus	66
	2.7 Vortex waves	69
	2.8 Quantum mechanics and superfluidity. Phase slip	72
	2.8.1 The Bose gas	72
	2.8.2 The Bose condensate	73
	2.8.3 The Madelung transformation	75
	2.8.4 Ginzburg–Pitaevskii theory	82
	2.8.5 Phase slip	83
<b>3</b>	<b>Vortex dynamics and mutual friction</b>	86
	3.1 Vortex dynamics	86
	3.2 Uniformly rotating helium II	91
	3.3 Frequency and velocity dependence of $B$ and $B'$ . Temperature dependence of the coefficients of mutual friction	92
	3.4 Microscopic form of mutual friction and the Iordanskii force	93
	3.5 The lifetime and range of vortex rings	105
	3.6 Axial mutual friction, the vortex-free strip and metastability	108
<b>4</b>	<b>The structure of quantized vortices</b>	112
	4.1 Vortex rings in helium II	112
	4.2 Vortex rings in a Bose condensate	114
	4.3 The bound excitation model	118
	4.4 The Hills–Roberts theory	121
	4.5 Interactions of ions and of vortices	127
	4.6 The mobility of ions along vortices	132
	4.7 $^3\text{He}$ condensation onto vortex cores	135
<b>5</b>	<b>Vortex arrays</b>	143
	5.1 Vortex arrays in a rotating bucket	143
	5.2 Photographs of vortex arrays	145

<i>Contents</i>	xi
5.3 Vortex arrays in a rotating annulus, stability of the flow	147
5.3.1 Entry of vortices	147
5.3.2 Stability of the flow	154
5.4 Thermorotation effects	156
5.5 Vortex pinning	160
5.5.1 Neutron stars	160
5.5.2 Experiments on vortex pinning	164
5.5.3 Simulation studies of vortex pinning	169
5.6 Spin-up and the vortex–surface interaction	170
5.6.1 Spin-up in helium II	170
5.6.2 Experiments on the vortex–surface interaction	171
5.7 Remnant vortices in helium II	177
<b>6 Vortex waves</b>	<b>179</b>
6.1 Waves on isolated vortices	179
6.1.1 Helical waves of constant amplitude	179
6.1.2 Experimental determination of the dispersion for helical waves	182
6.1.3 Solitary waves; sideband instability of Kelvin waves	184
6.2 Collective effects of Tkachenko waves on infinite vortex arrays	187
6.2.1 Rajagopal’s calculations	187
6.2.2 A simple calculation of motion of vortices in a Tkachenko wave	189
6.2.3 Combined Tkachenko and bending waves	191
6.2.4 Macroscopic hydrodynamic equations	194
6.2.5 Equations of Baym and Chandler	196
6.2.6 Experimental investigation of Tkachenko waves	199
6.3 Collective effects – finite vortex arrays	204
6.4 A vortex instability	208
6.5 Thermally induced vortex waves	211
6.6 Effect of mutual friction on vortex waves	212
<b>7 Superfluid turbulence</b>	<b>215</b>
7.1 Background	215
7.2 Application of vortex dynamics to superfluid turbulence	217
7.2.1 Dimensionless form of vortex dynamics	217

Cambridge University Press

0521018145 - Quantized Vortices in Helium II

Russell J. Donnelly

Table of Contents

[More information](#)

xii	<i>Contents</i>	
	7.2.2 Dynamical similarity	218
	7.2.3 Anisotropy of the vortex tangle	220
	7.2.4 Applications of dynamical similarity	221
	7.3 Numerical simulation of a vortex tangle	225
	7.3.1 Vortex reconnections	225
	7.3.2 Computational considerations	227
	7.3.3 Results of numerical simulations	229
	7.4 Production, measurement and analysis of superfluid turbulence	230
	7.4.1 Devices for producing turbulence in the superfluid	230
	7.4.2 Second sound and temperature difference measurements in a wide channel	231
	7.4.3 Production of turbulence in narrow channels	236
	7.4.4 Analysis of counterflow experiments	236
	7.4.5 Shock wave apparatus	238
	7.4.6 Ion measurements in a wide channel	239
	7.4.7 The normal jet	240
	7.4.8 Ultrasonically generated turbulence	242
	7.4.9 Injected turbulence	244
	7.5 Observed properties of counterflow turbulence and their interpretation	244
	7.5.1 Classification of turbulent superfluid flows	244
	7.5.2 Anisotropy of the vortex tangle	249
	7.5.3 Intrinsic fluctuations	252
	7.5.4 Study of more general flows	254
<b>8</b>	<b>Thermal activation and nucleation of quantized vortices</b>	<b>255</b>
	8.1 Extrinsic and intrinsic nucleation processes	255
	8.2 The nucleation energy barrier	256
	8.3 Intrinsic thermal nucleation of vortex rings	260
	8.3.1 Homogeneous nucleation theory	260
	8.3.2 Decay of a superflow in a toroidal geometry	263
	8.3.3 Comparison of the homogeneous nucleation theory with experiment	266
	8.4 Vortex motion in porous media	269
	8.4.1 The competing barrier model	269
	8.4.2 Vortex nucleation in porous media	274
	8.5 Nucleation of vorticity by tunnelling	277
	8.5.1 Motion of ions in helium II	277

<i>Contents</i>	xiii
8.5.2 Calculations of critical velocities and energy barriers at $T = 0$	279
8.5.3 Equations of motion and quantum mechanics for vortices in a thin film	282
8.5.4 The nucleation of a vortex at the edge of a thin film by quantum tunnelling	285
8.5.5 Application to nucleation of vorticity by a moving ion	288
8.5.6 The experimental situation	289
8.5.7 Direct experiments on nucleation by tunnelling	292
8.6 The effect of dissolved $^3\text{He}$ on vortex nucleation from ions	295
8.6.1 Ions and $^3\text{He}$	295
8.6.2 The binding of a $^3\text{He}$ atom to the negative ion bubble	296
8.6.3 Binding of a $^3\text{He}$ atom to a vortex line	298
8.6.4 Vortex nucleation by a negative ion containing one or two adsorbed $^3\text{He}$ atoms	299
8.6.5 The predicted nucleation rates and a comparison with experiment	300
8.7 Vortex arrays and vortex dynamics in thin films	302
8.7.1 Two-dimensional superfluidity	302
8.7.2 Kosterlitz–Thouless theory	303
8.7.3 Linear dynamic theory	308
8.7.4 Finite amplitude effects	310
8.7.5 Vortex diffusivity in two dimensions	313
8.7.6 Experiments on thin films	314
<i>References</i>	323
<i>Index</i>	343