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The theory of how solid metals conduct electricity has, until recently, been confined to crystalline metals in which the constituent atoms form regular arrays. The discovery of how to make solid amorphous metallic alloys (often called metallic glasses), in which the atoms are no longer ordered, led to an explosion of measurements on these new materials. A whole range of new and unexpected behaviour was found, particularly at low temperatures and in a magnetic field. At the same time theories to explain the electrical properties of disordered metals began to emerge.

To understand this new behaviour, conventional Boltzmann theory, which assumes that the free path of the conduction electrons is long and only occasionally interrupted by scattering, has to be extended and modified when the mean free path becomes comparable with the wavelength of the electrons and with the distance between neighbouring atoms.

The theory is explained in physical terms and the results are compared to experimental results on metallic glasses.

The book is designed to be self-contained and to appeal to non-specialist physicists, metallurgists and chemists with an interest in disordered metals or to students beginning to study these materials.

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THE ELECTRICAL PROPERTIES OF  
DISORDERED METALS

**Cambridge Solid State Science Series**

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# THE ELECTRICAL PROPERTIES OF DISORDERED METALS

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To my wife

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## Preface

The purpose of this book is to explain in physical terms the many striking electrical properties of disordered metals or alloys, in particular metallic glasses. The main theme is that one central idea can explain many of the otherwise puzzling behaviour of these metals, particularly at low temperatures and in a magnetic field. That idea is that electrons in such metals do not travel ballistically between comparatively rare scattering events but diffuse through the metal. These new effects are not large but they are so universal in high-resistivity metals, so diverse and qualitatively so different from anything to be expected in metals where the electrons have a long mean free path, that they cry out for an explanation.

The book is not a critical research review; the motivation is mainly to explain. In interpreting theory there are always the dangers of overinterpretation, misinterpretation and failure to interpret and I do not expect to have escaped these completely. Nonetheless, our new understanding of disordered metals and alloys constitutes a substantial addition to conventional Boltzmann theory and deserves to be more widely known and appreciated.

The book is aimed at those who know little of the subject such as students starting work in this field or those outside the field who wish to know of developments in it. There is no attempt at rigorous derivations; the aim is to present the physics as clearly as possible so that readers can think about the subject for themselves and be able to apply their thinking in new contexts.

For those whose knowledge of electron transport properties is limited to what they learned in undergraduate courses I outline briefly the main points of conventional theory in the first part of the book. This is not

meant to be an exhaustive treatment but a reminder of ideas already encountered and here put in the context of what is to follow.

I am greatly indebted to many friends and colleagues for discussions, reading the manuscript or parts of it and help in understanding the subject. I cannot attempt to mention them all but I am particularly grateful to the following: Denis Greig (who introduced me to the subject), Bryan Gallagher, Bryan Hickey, Mark Howson, Jim Morgan and Davor Pavuna, with all of whom I had the pleasure of working on aspects of this subject. I found the thesis of Dr A. Sahnoune most clear and helpful. I am indebted to Dr Moshe Kaveh for reading the manuscript and to Nathan Wisner for helpful discussions. I must also record my deep gratitude to the editor, Ian Ward, for much help and encouragement; to Dr B. M. Watts, who as copy editor did so much to improve the manuscript; and to Mrs Mary Edmundson for her tireless help in preparing drafts and organising the related correspondence.

To all these and to many others, my sincere thanks.

J. S. Dugdale  
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