This volume includes contributions by leading workers in the field given at the workshop on Numerical Relativity held in Southampton in December 1991. Numerical relativity, or the numerical solution of astrophysical problems using powerful computers to solve Einstein's equations, has grown rapidly over the last 15 years. It is now an important route to understanding the structure of the universe, and is the only route currently available for approaching certain important astrophysical scenarios. The Southampton meeting was directed at providing a dialogue between theoreticians in general relativity and practitioners in numerical relativity. It was also notable for the first full report of the new 2+2 approach and the related null or characteristic approaches, as well as for updates on the established 3+1 approach, including both Newtonian and fully relativistic codes. The contributions range from theoretical (formalisms, existence theorems) to the computational (moving grids, multiquadrics and spectral methods).

This book will be of value to relativists, cosmologists, astrophysicists and applied mathematicians interested in the increasing contribution made to this subject using numerical techniques.

Approaches to Numerical Relativity

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Proceedings of the International Workshop on Numerical Relativity, Southampton, December 1991

Edited by
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INTRODUCTION

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This volume derives from a workshop entitled "Approaches to Numerical Relativity" which was held in the week 16-20th December, 1991, in the Faculty of Mathematical Studies at Southampton University, England. It was held principally because it was thought that the time was opportune to begin a dialogue between theorists in classical general relativity and practitioners in numerical relativity. Numerical relativity - the numerical solution of Einstein's equations by computer - is a young field, being possibly only some fifteen years old, and yet it has already established an impressive track record, despite the relatively small number of people working in the field. Part of this dialogue involved bringing participants up to date with the most recent advances. To this end, international experts in the field were invited to attend and give presentations, including Joan Centrella, Matt Choptiuk, John Miller, Ken-Ichi Oohara, Paul Shellard and Jeff Winicour. In addition, a significant number of European scientists, both theoreticians and practitioners in numerical relativity, were invited, the majority of whom attended. In the event, there were some 35 participants, most of whom gave presentations. This volume is largely comprised of the written versions of these presentations (their length being roughly proportional to the time requested by the authors for their presentations).

In an attempt to highlight the distinctive nature of the workshop, I have divided the contributions into Part A, Theoretical Approaches and Part B, Practical Approaches. This is to a large extent somewhat arbitrary, since several of the theoretical contributions involve a significant element of computing, and all of the practical contributions involve theoretical aspects. Again, where possible, I have tried to locate related contributions together. I hope that my arbitrary division and ordering will not offend anyone, but rather help to point out the dialogue nature of the workshop. Indeed, this is exemplified further by the inclusion of an edited version of the final Panel Discussion.

Most numerical relativity makes considerable demands on computer processing power. As a consequence, much of the work has been carried out in the US and Japan, although there have been significant contributions from Europe, especially in Germany, France, Italy and Spain. More surprising, perhaps, is the fact that the field has attracted a significant number of British scientists, and there are a growing number beginning to take an interest in it. Thus, an additional reason for holding the workshop was to hold it in the UK as a mark of this involvement and growing interest and, in so doing, to announce the setting up of three centres for numerical relativity, namely at Cambridge (under John Stewart), Cardiff (under Bernie Schutz) and Southampton (under Chris Clarke).

There was also a notable difference in the emphasis of this workshop as compared with other recent meetings in numerical relativity. Previously, most attention has been paid to the 3+1 approach and, to a lesser extent, the Regge calculus approach. This workshop, for the first time, gave greater attention to the newer null or characteristic approach and the related 2+2 approach. As someone who was involved in developing the 2+2 approach, I have a particular interest in seeing its application in numerical relativity. And so, as editor of this volume, I decided to reflect this change in emphasis by including theses approaches first in each part, and would wish to draw particular attention to the pioneering work of the groups of Jeff Winicour and John Stewart. Part A starts off with my own contribution, partly because it was the first presentation at the workshop, and partly because it includes a short (personally conceived) introduction to numerical relativity.

I would like to express my thanks for their co-operation to the authors of the articles and, in particular, to the head of our group, Chris Clarke, for agreeing to add a preface. I would like to thank Chris Clarke additionally, together with James Vickers, for their considerable help in the preparation of this volume. Finally, the three of us as co-organisers, would wish to express our gratitude for financial support for the workshop from the Science and Engineering Research Council, the London Mathematical Society and the Institute of Physics.

PREFACE

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General relativity was for too long the ugly duckling of science. In the 50s and 60s the dominant impression was of the difficulty of the equations, solvable only by arcane techniques inapplicable elsewhere; of the scarcety of significant experimental tests; of the prohibitive cost of computational solutions, compounded by a lack of rigorous approximation techniques; and of the isolation of the subject from the physics of the other fundamental forces. This led to a situation where, even in the 70s, much theoretical work was becoming increasing irrelevant to physics. Exact solutions proliferated but (with the exception of cosmology) attempts at physical interpretation were few and unconvincing. Mathematical investigations in the wake of the singularity theorems became increasingly sophisticated, but few were applied to actual physical models. In the 70s and 80s, however, all this changed, with the growth of experimental relativity, the trend to geometrical methods in high energy physics, and the inception of numerical relativity. The workshop reported in this book marks the complete clearing of this last hurdle, as reliable and practical computational techniques are established.

It brought together numerical and classical relativists, and showed that the cultural gap between them was closing fast. Dramatically increased standards of reliability and accuracy had been set, and were being achieved in many cases, so that numerical work can no longer be seen merely as providing a rough indication for the 'proper' work of analysis. In increasingly many areas numerical simulation will clearly be the decisive component in answering the questions being posed by the theorists. On the astronomical side, the sophistication of hydrodynamic codes is now enabling them to cope with real astrophysical problems.

A precise account of some of the main technical points is contained in the first review section of d'Inverno's paper, to which I refer the reader for an expansion of what follows. Here I shall try to give an overview of the place of the workshop papers in the subject as a whole. The papers cover the two main classes of numerical techniques: those that use Cauchy surfaces and those that use null surfaces. (This is in itself interesting. As Miller points out in the concluding discussion, the field is no longer totally dominated by the former method.) They include expositions of numerical techniques, both generally applicable ones and those specific to relativity, discussions of different formulations of relativity appropriate to numerical work, presentations of new physically interesting results, and reviews of areas where problems seem ripe for a numerical approach. To give a flavour of the many dialogues that enlivened the workshop, and to indicate the main directions opened up for future work, we include at the end an edited transcript of the concluding round-table discussion.

Einstein's equations are perhaps unique among physical models in the variety of different choices of coordinates and representations that can be made, all having some claim to simplicity or efficiency in some circumstances. The main subdivision, already referred to, is between those in which evolution is governed by a coordinate t with the surfaces t = constant spacelike (3+1 or Cauchy approaches) and those which instead use a u with u = constant null (2+2 or characteristic approaches). d'Inverno describes the geometric features of the methods, while Winicour's article begins with a useful survey of the relative merits of the two, concluding that Cauchy approaches are best suited to matter-dominated regions where trapped surfaces may be forming, while characteristic methods are suited to the intermediate zones round a system emitting radiation, where they can efficently handle the self-interaction and back-scattering of radiation, which, as his article shows, could crucially effect the appearance of the object to gravitational detectors.

While most papers used a traditional approach to the characteristic method, an illuminating alternative was provided by Hayward, who showed that, just as the Cauchy formalism can be seen as a Hamiltonian dynamical system, so the characteristic formalism can be seen as generalised '2-time' Hamiltonian system.

Within the characteristic methods there is further subdivision into those based on the use of tetrads and the Newman-Penrose or Geroch-Held-Penrose formalisms (discussed by Frauendiener, Vickers and Stewart) and those based directly on the metric (discussed by Winicour, Bishop and d'Inverno). The great strength of all these methods is the simplification of the equations: equations lying in the surfaces u = const. become ordinary differential equations in contrast to the elliptic equations specifying the constraints in the Cauchy approach. In addition, some of the methods give a dramatic reduction in the total size of the equation set. These together can lead to very fast and efficient numerical codes.

The majority of the papers use the Cauchy approach, however. Here also there are many different choices of coordinates, particularly since the choice of the hypersurfaces $t =$ const. (the slicing condition) now introduces much more freedom. There seems to be a lull in the debate as to the best approach here, with different groups being content to be judged by their fruits. The issue is far from dead, however, as witnessed by the recent controversy over whether or not a naked singularity can be produced by a collapsing cloud of particles (Wald and Iyer, 1991). The range of work on Cauchy methods, and particularly the elegant work of Bona's group, certainly shows that there is as much scope as in the characteristic methods for judicious simplification of the structure of equations.

Not surprisingly, in view of the complementarity of the two approaches there is interest in combining them. Thus Bishop presents a well articulated (but as yet untested) numerical scheme for joining together the two methods in different regions. Friedrich, on the other hand, shows that it is possible to carry out an intriguing scheme using hyperboloidal slices that have all the advantages of the Cauchy method in the interior, but which asymptote to null hypersurfaces, enabling one to compactify the entire space-time just as for the characteristic method.

In addition to the two above divisions of the full equations of relativity, there is a broad class of approaches which first approximate the equations and then solve numerically – typically used, as by Mark, Bonazzola and Nakamura, in collapsing stellar problems where the full equations are very complex. The status of approximation methods in relativity is an uncertain area; there is no approximation scheme known to be convergent and no effective estimates on the errors associated with existing schemes. It is therefore all the more interesting to see both exact and approximated methods developing numerically so that more can be learnt about the validity of these methods.

The most striking feature of the workshop, for a classical relativist like myself, was a repeated emphasis on the importance of discriminating fine structure. Underlying this is the fact that, even without general relativity, the hydrodynamics of a collapsing star is a difficult numerical problem: shock waves form, which, in the absence of high symmetry, can rapidly lead to great complexity. Relativity also has its tendency to generate fine structure, even with a system as simple as the spherically symmetric scalar field. Christodoulou had already demonstrated that the field evolved to a step function, and Choptuik's paper traces this evolution through the emergence of progressively finer structure. A repeated theme, here and with Lanza, for example, is the use of multi-grids for covering adequately the region with fine structure, while Centrella and others use adaptive grids.

It is clear from this that considerable sophistication is needed in the choice of numerical methods. Centrella, for example, uses a discretisation of advection that ensures local angular momentum conservation, while Ibañez and his group, in a widely applauded paper, used Godunov methods for hyperbolic equations to capture the shocks occuring in realistic models of white dwarfs with quite remarkable accuracy. Oohara, on the other hand, achieved good accuracy with a more classical TVD limiter approach. This well illustrates the power of modern numerical techniques, which at present are only applicable to the Cauchy implementation of general relativity.

At the level of numerical technique, several alternatives to standard finite differences were presented. Bonazzola and Marck used pseudo-spectral methods with great success, in combination with multi-domains to capture shocks; while Dubal successfully used multi-quadrics, an impressive technique, but one that seemed to require considerable artistry in its use. Regretably, the relativistic equivalent of finite elements – Regge calculus – was under-represented, with only one paper, by Barrett. This analysed the combinatorics of developing the triangulation of hypersurfaces, a subject with ramifications both for low dimensional topology (new invariants have recently been discovered by this method) and quantum theory, as well as for mesh-generation in Regge calculus. The direct use of the calculus for general relativity seems to have been somewhat sidelined, however.

A potentially vital numerical method, classical in essence but brought to the fore by the particular needs of relativity, was the ADI method of the Cardiff group (Allen and Alcubiere) adapted to grids moving with a super-characteristic velocity, as is needed for black hole problems. A simple idea with the benefit of hindsight, it was applied to achieve stability over a remarkably wide range.

A further feature of general relativity is the non-triviality of the problem of establishing initial data. In the Cauchy approach this involves solving the elliptic constraint equations, while in the characteristic approach, though it is numerically easier, there are subtle problems concerning the specification of 'no incoming radiation'. Several papers concerned the Cauchy problem here, including some on equilibrium configurations, such as Lanza's useful analysis of thin-disc equilibria. An interesting contribution by O'Murchadha showed how the multipole structure of the source was reflected in the details of the boundary conditions to be imposed on the constraint equations.

Perhaps the work most characteristic of the mood of the meeting was that by Choptuik, in both his own paper and in the closing discussion. While previous meetings had stressed the importance of calibration and test-bed calculations, comparing with known exact solutions (themes that were still present here), now the emphasis was on the construction of self-validating programmes: codes that could be run with a wide range of parameters governing step-sizes so as to produce estimates of their own error. This was controversial material, many participants worrying that such procedures could only check convergence to something, not necessarily convergence to Einstein's equations. But it was countered that, providing all the equations are monitored, properly designed code should be able to verify this, giving an estimate of the actual discrepancy from the exact result. This is clearly an area where valuable work could be done in collaboration with analytic theory. It is also an area where psychological issues are as important as mathematical ones, in convincing the relativistic community that numerical results can now be produced that are as firm as analytic results.

Though my own prejudices have placed mathematics first, the papers also marked a considerable advance in physical understanding. Miller argued powerfully that a careful analysis of numerical data could be used to develop our conceptual understanding of the role of angular momentum and so on in the relativistic regime, despite the difficulty of defining many of these concepts in the abstract. And the presentation of Oohara, on the interaction of neutron stars, showed distinct physical structures emerging in different parameter ranges – a valuable conceptual adjunct to the data.

The range of problems addressed covered the gamut of astrophysics and relativity. Particularly exciting was the number of 3 (spatial) dimensional calculations presented. Most used approximations, but Bona demonstrated an exact three dimensional vacuum code (available by e-mail, so we can all try it!) Cosmology was represented by Nakao's account of horizon formation in inflation, a new application of numerical techniques, as was the analysis of boson stars (possible candidates for dark matter) by Schunck, interestingly using catastrophe theory to explore genericity ideas that were also predominant in Choptuik's related paper. Cosmic strings were presented by Shellard, who not only showed that the subject was still very active, but also broadened the context of numerical relativity by linking it with numerical work in this other area.

The majority of numerical papers, however, were either directly concerned with gravitational wave generation, or (as with Centrella and Miller) studied related problems of stability in relativistic stellar physics – very appropriately so in view of the rapidly developing experimental situation. The central problem here is the efficiency of generation from likely astrophysical sources of gravitational radiation, exploring the conjecture that 3-dimensional configurations should be much more efficient than axisymmetric ones. This was well illustrated by Oohara's finding of a 30-fold increase in efficiency compared with axisymmetry, but this had to be qualified by doubts concerning more global relativistic effects in the wave-zone, whose importance was stressed by Winicour, and concerning the effect of red shifts on the radiation.

The interface between standard relativity problems and numerical work was also addressed, most provocatively in the paper by Rendal which bore on the work already referred to, by Shapiro and Teukolsky (1991), on naked singularities. Colliding waves, reviewed by Griffiths, also offered an arena where current techniques for 3-D vacuum codes should already be able to shed light on the outcome of generic interactions, where the occurrence or otherwise of singularities is very uncertain.

Though hardware was not singled out for discussion, some interesting trends emerged. Not all problems required the power of super-computers; the simplifications achieved by Stewart, for instance, enabled his code to run on a workstation. And, while high power was often necessary (for hydrodynamics in particular), this did not now involve prohibitive cost, as illustrated by the use of transputer networks by the Southampton group (d'Inverno and Bishop). Thus not only was numerical relativity important, but it was also accessible.

The conclusion of the meeting was that numerical work had emerged as focally important in general relativity as well as astrophysics, with almost no topic that was not decisively affected by it. General relativity now emerges as no longer the ugly duckling, but a swan-like subject combining both elegance and power.

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