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Preface

Hopf algebras or 'quantum groups' are natural generalisations of groups. They have many remarkable properties and, nowadays, they come with a wealth of examples and applications in pure mathematics and mathematical physics.

Most important are the quantum groups $U_q(\mathfrak{g})$ modelled on, and in some ways more natural than, the enveloping algebras $U(\mathfrak{g})$ of simple Lie algebras $\mathfrak{g}$. They provide a natural extension of Lie theory. There are also finite-dimensional quantum groups such as bicrossproduct quantum groups associated to the factorisation of finite groups. Moreover, quantum groups are clearly indicative of a more general 'noncommutative geometry' in which coordinate rings are allowed to be noncommutative algebras.

This is a self-contained first introduction to quantum groups as algebraic objects. It should also be useful to someone primarily interested in algebraic groups, knot theory or (on the mathematical physics side) q-deformed physics, integrable systems, or conformal field theory. The only prerequisites are basic algebra and linear algebra. Some exposure to semisimple Lie algebras will also be useful.

The approach is basically that taken in my 1995 textbook, to which the present work can be viewed as a companion 'primer' for pure mathematicians. As such it should be a useful complement to that much longer text (which was written for a wide audience including theoretical physicists). In addition, I have included more advanced topics taken from my review on Hopf algebras in braided categories and subsequent research papers given in the Bibliography, notably the 'braided geometry' of $U_q(\mathfrak{g})$. This is material which may eventually be developed in a sequel volume to the 1995 text.

In particular, our approach differs significantly from that in other

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textbooks on quantum groups in that we do not define $U_q(\mathfrak{g})$ by means of generators and relations 'pulled out of a hat' but rather we deduce these from a more conceptual braided-categorical construction. Among the benefits of this approach is an inductive definition of $U_q(\mathfrak{g})$ as given by the repeated adjunction of 'quantum planes'. The latter, as well as the subalgebras $U_q(n_+)$, are constructed in our approach as braided groups, which can be viewed as a modern braided-categorical setting for the first (easy) part of Lusztig's text.

The book itself is the verbatim text of a course of 24 lectures on *Quantum Groups* given in the Department of Pure Mathematics and Mathematical Statistics at the University of Cambridge in the Spring of 1998. The course was at the *Part III* diploma level of the mathematics tripos, which is approximately the level of a first year graduate course at an American university, perhaps a bit less advanced. Accordingly, it should be possible to base a similar course on this book, for which purpose I have retained the original lecture numbering. The first 1/3 of the lectures cover the basic algebraic structure, the second 1/3 the representation theory and the last 1/3 more advanced topics. There were also three useful problem sets distributed during the course, which I include at the end of the book.

I would like to thank the students who attended the course for their useful comments. Particularly, the lectures start off quite slowly with a lot of explicit computations and notations from the theory of Hopf algebras; depending on the wishes of the students, one could skip faster through these lectures by deferring the proofs as exercises – with solutions on handouts. Meanwhile, the last five lectures are an introduction to some miscellaneous topics; they are self-contained and could be omitted, depending on the time available. Finally, I want to thank Pembroke College in the University of Cambridge, where I was based at the time and during much of the period of writing.

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