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Cambridge University Press
978-0-521-01040-5 — Quantum Groups and Lie Theory
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London Mathematical Society Lecture Note Series. 290

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Edited by

Andrew Pressley
King's College, University of London

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CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India
103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521010405

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First published 2001

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

Quantum groups and Lie theory / edited by Andrew Pressley.

p. cm.--(London Mathematical Society lecture note series; 290)

Includes bibliographical references and index.

ISBN 0-521-010403

1. Quantum groups-- Congresses. 2. Lie groups-- Congresses. 3. Mathematical physics -- Congresses. I. Pressley, Andrew. II. Series.

QC20.7.G76 Q82 2001

530.15'255--dc21

2001043214

ISBN 978-0-521-01040-5 Paperback

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Introduction

Since its genesis in the early 1980s, the subject of quantum groups has grown very rapidly. Although much of the groundwork was laid by V. G. Drinfeld in his remarkable talk at the 1986 International Congress of Mathematicians in Berkeley, a number of basic issues in the theory were not resolved until later. In addition, important new developments occurred in the late 1980s and early 1990s, such as the crystal and canonical bases, and the applications of quantum groups in low-dimensional topology. By the late 1990s, however, the theory had reached a stage in which most of the foundational issues had been resolved, and many of the outstanding problems clearly formulated. It was felt that this was an opportune moment to hold a meeting of experts representing all the main strands of the theory, to take stock of what had been achieved so far and to discuss the most fruitful directions for future research. The result was the LMS Durham Symposium on Quantum Groups, which was held at Grey College in the University of Durham from 19 July to 29 July, 1999, and organised by S. Donkin, A. Pressley and A. Sudbery. The present volume is a record of some of the lectures given at the Symposium.

Two lecture series are represented here which form excellent surveys of two important areas. Ariki's 'Lectures on Cyclotomic Hecke Algebras' describe his remarkable realisation of the canonical basis in terms of a Fock space that arises in the study of solvable lattice models in statistical mechanics. Also closely connected with Physics are Etinghof's 'Lectures on the dynamical Yang-Baxter equation'. The Yang-Baxter equation has played a central role in the theory of quantum groups from the beginning, and its dynamical version is a generalization which, like the ordinary Yang-Baxter equation, first appeared in the Physics literature, but has since found many applications, particularly to integrable systems and representation theory. The remaining articles relate to single lectures given at the Symposium, and they cover a wide variety of topics within quantum groups. Several treat the problem of constructing and classifying quantum groups or the associated solutions of the quantum Yang-Baxter equation, including those by Ding & Hodges, Musson and Parashar & McDermott. The papers of Drabant and

Wenzl deal with the tensor categories of representations of quantum groups. Those of Chari & Pressley and Gordon treat the representation theory directly, the first for infinite-dimensional quantum groups and generic q , the latter for finite-dimensional quantum groups and but q a root of unity. The paper of Carter & Marsh gives a new, and partly conjectural, parametrisation of the canonical basis. The papers of Goodearl and Majid take the function algebra approach to quantum groups, from the point of view of algebraic geometry and differential geometry, respectively. And Beggs' paper describes some new ideas which relate to the origins of quantum groups in the theory of integrable systems.

We hope that this volume provides a picture of the state of the theory of quantum groups towards the end of the second millenium and that it also indicates some directions in which the theory can be expected to develop in the next.