THE MATH BEHIND THE MUSIC

Mathematics has been used for centuries to describe, analyze, and create music. In this book, Leon Harkleroad explores the math-related aspects of music from its acoustical bases to compositional techniques to music criticism, touching on

- overtones, scales, and tuning systems
- the musical dice games attributed to Mozart and Haydn
- the several-hundred-year-old style of bell-playing known as ringing the changes
- the twelve-tone school of composition that strongly influenced music throughout the twentieth century

and many other topics involving mathematical ideas from probability theory to Fourier series to group theory. He also relates some cautionary tales of misguided attempts to mix music and mathematics.

Both the mathematical and the musical concepts are described in an elementary way, making the book accessible to general readers as well as to mathematicians and musicians of all levels. The book is accompanied by an audio CD of musical examples.

Leon Harkleroad gives frequent lectures on mathematics and music for the Mathematical Association of America and other groups. He has been published in many journals, and he has received the George Pólya Award from the MAA for one of his papers.

OUTLOOKS

PUBLISHED BY CAMBRIDGE UNIVERSITY PRESS AND THE MATHEMATICAL ASSOCIATION OF AMERICA

Mathematical content is not confined to mathematics. Eugene Wigner noted the unreasonable effectiveness of mathematics in the physical sciences. Deep mathematical structures also exist in areas as diverse as genetics and art, finance and music. The discovery of these mathematical structures has in turn inspired new questions within pure mathematics.

In the *Outlooks* series, the interplay between mathematics and other disciplines is explored. Authors reveal mathematical content, limitations, and new questions arising from this interplay, providing a provocative and novel view for mathematicians, and for others an advertisement for the mathematical outlook.

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THE MATH BEHIND
THE MUSIC

LEON HARKLEROAD

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For Cynthia

Nagyon szeretlek, legkedvesebbem!

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PREFACE

For centuries people have explored connections between mathematics and music. Yet, when I started writing this volume, very little in the way of books surveyed the range of these connections. You could find monographs devoted to one specific niche or another, as well as any number of journal articles. But you would have had a hard time locating a reputable source that gathered several of the important topics in one place and gave them a broadly accessible presentation. Fortunately, that situation has been changing recently. I hope that this book helps with such change. In Chapter 1, I give an overview of the contents of the subsequent chapters. I will just say here that I have tried for a blend of standard, core topics with less familiar ones. In particular, I have highlighted some recent work on math and music that may be new even to aficionados of the subject.

The potential readership for an interdisciplinary book like this encompasses a wide variety of backgrounds. Accordingly, I have aimed to presume as little as possible on your prior experience in either mathematics or music. The book, in part, grew out of lectures I gave in a course that had no prerequisites. So, the students ranged from math majors who were accomplished musicians to math-phobes who could not read or perform music at all. I don't start totally from scratch in these pages as I did in the course, but I have written with a fairly general audience in mind. Again, those

PREFACE

already familiar with math and/or music should still find much new material here. Despite the book's origins, I have not written it as a textbook. However, anyone inclined to take on the highly enjoyable task of teaching a math-and-music course could certainly make use of this in conjunction with the class.

ACKNOWLEDGMENTS

This book developed in tandem with three other math-and-music projects of mine: the course at Cornell University that I previously mentioned, a seminar that I organized at Cornell, and a workshop that I have offered under the auspices of the Mathematical Association of America. Thanks to Cornell and the MAA for letting me organize these activities and especially to all the participants for sharing an interest in math and music.

Particular thanks to Kevin Hamlen for allowing me to present his previously unpublished work here.

A huge thank you to concert musician and audio engineer Graybert Beacham for the time and skill he devoted to producing the CD. He did a masterful job of tracking down sources, recording my performances, playing the violin, and compiling it all.

Many thanks to everyone from Cambridge University Press and the MAA who helped bring this book into existence: Pooja Jain, Katie Hew, Lara Zoble, Caitlin Doggart, Roger Astley, Don Albers, John Barrow, and Don Zagier. I must single out Jessica Farris, who did some extraordinary work in obtaining reprint permissions; David Tranah, who proposed the book in the first place; and my editor Lauren Cowles, whose helpful advice and abundant patience I very much appreciate.

ACKNOWLEDGMENTS

Above all, my utmost gratitude to my wife Cynthia for proofreading, suggestions, encouragement, and support. This book owes a great deal to her – but she specifically disclaims responsibility for any of my puns contained in it.