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978-0-521-00601-9 - A Course in Combinatorics, Second Edition

J. H. van Lint and R. M. Wilson

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A course in combinatorics

This is the second edition of a popular book on combinatorics, a subject dealing with ways of arranging and distributing objects, and which involves ideas from geometry, algebra and analysis. The breadth of the theory is matched by that of its applications, which include topics as diverse as codes, circuit design and algorithm complexity. It has thus become essential for workers in many scientific fields to have some familiarity with the subject. The authors have tried to be as comprehensive as possible, dealing in a unified manner with, for example, graph theory, extremal problems, designs, colorings and codes. The depth and breadth of the coverage make the book a unique guide to the whole of the subject. The book is ideal for courses on combinatorial mathematics at the advanced undergraduate or beginning graduate level. Working mathematicians and scientists will also find it a valuable introduction and reference.

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A Course in
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SECOND EDITION

J. H. van Lint

Technical University of Eindhoven

and

R. M. Wilson

California Institute of Technology



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Preface to the first edition

One of the most popular upper level mathematics courses taught at Caltech for very many years was H. J. Ryser's course *Combinatorial Analysis*, Math 121. One of Ryser's main goals was to show elegance and simplicity. Furthermore, in this course that he taught so well, he sought to demonstrate coherence of the subject of combinatorics. We dedicate this book to the memory of Herb Ryser, our friend whom we admired and from whom we learned much.

Work on the present book was started during the academic year 1988–89 when the two authors taught the course Math 121 together. Our aim was not only to continue in the style of Ryser by showing many links between areas of combinatorics that seem unrelated, but also to try to more-or-less survey the subject. We had in mind that after a course like this, students who subsequently attend a conference on “Combinatorics” would hear no talks where they are completely lost because of unfamiliarity with the topic. Well, at least they should have heard many of the words before. We strongly believe that a student studying combinatorics should see as many of its branches as possible.

Of course, none of the chapters could possibly give a complete treatment of the subject indicated in their titles. Instead, we cover some highlights—but we insist on doing something substantial or nontrivial with each topic. It is our opinion that a good way to learn combinatorics is to see subjects repeated at intervals. For this reason, several areas are covered in more than one part of the book. For example, partially ordered sets and codes appear several times. Enumeration problems and graph theory occur throughout

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the book. A few topics are treated in more detail (because we like them) and some material, like our proof of the Van der Waerden permanent conjecture, appears here in a text book for the first time.

A course in modern algebra is sufficient background for this book, but is not absolutely necessary; a great deal can be understood with only a certain level of maturity. Indeed, combinatorics is well known for being “accessible”. But readers should find this book challenging and will be expected to fill in details (that we hope are instructive and not too difficult). We mention in passing that we believe there is no substitute for a human teacher when trying to learn a subject. An acquaintance with calculus, groups, finite fields, elementary number theory, and especially linear algebra will be necessary for some topics. Both undergraduates and graduate students take the course at Caltech. The material in every chapter has been presented in class, but we have never managed to do all the chapters in one year.

The notes at the end of chapters often include biographical remarks on mathematicians. We have chosen to refrain from any mention of living mathematicians unless they have retired (with the exception of P. Erdős).

Exercises vary in difficulty. For some it may be necessary to consult the hints in Appendix 1. We include a short discussion of formal power series in Appendix 2.

This manuscript was typeset by the authors in $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{T}\mathcal{E}\mathcal{X}$.

J. H. v. L., R. M. W.

Eindhoven and Pasadena, 1992

Preface to the 2nd edition

The favorable reception of our book and its use for a variety of courses on combinatorial mathematics at numerous colleges and universities has encouraged us to prepare this second edition. We have added new material and have updated references for this version. A number of typographical and other errors have been corrected. We had to change “this century” to “the last century” in several places.

The new material has, for the most part, been inserted into the chapters with the same titles as in the first edition. An exception is that the material of the later chapters on graph theory has been reorganized into four chapters rather than two. The added material includes, for example, discussion of the Lovász sieve, associative block designs, and list colorings of graphs.

Many new problems have been added, and we hope that this last change, in particular, will increase the value of the book as a text. We have decided not to attempt to indicate in the book the level of difficulty of the various problems, but remark again that this can vary greatly. The difficulty will often depend on the experience and background of the reader, and an instructor will need to decide which exercises are appropriate for his or her students. We like the idea of stating problems at the point in the text where they are most relevant, but have also added some problems at the end of the chapters. It is not true that the problems appearing later are necessarily more difficult than those at the beginning of a chapter. A number of the hints and comments in Appendix 1 have been improved.

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Preparation of the second edition was done during a six-month visit to the California Institute of Technology by the first author as Moore Distinguished Scholar. He gratefully acknowledges the support of the Moore Foundation.