1 Introduction: Maxwell and the history of physics

Reviewing James Clerk Maxwell's *Treatise on Electricity and Magnetism* on its publication in 1873, Peter Guthrie Tait described his friend as having 'a name which requires only the stamp of antiquity to raise it almost to the level of that of Newton'. At the time Tait's enthusiasm may have seemed to verge on hyperbole; yet such has been the judgement of posterity. Tait accurately highlighted as the cardinal features of the *Treatise* Maxwell's demonstration of 'the connection between radiation and electrical phenomena', and his achievement in having 'upset completely the notion of *action at a distance*'.[1]

Maxwell's theory of the electromagnetic 'field', expounded in the *Treatise*, supposes that electric and magnetic forces are mediated by the agency of the 'field', contiguous elements of the space in the neighbourhood of the electric or magnetic bodies, the 'field' being embodied by an ether. The impact of the *Treatise* was at first muted, and at the time Maxwell's reputation rested largely on his work on molecular physics and gases. But within a few years of Maxwell's death in 1879 his theory of the electromagnetic field shaped the work of 'Maxwellian' physicists (George Francis FitzGerald, Oliver Heaviside, Joseph John Thomson and others). Following Heinrich Hertz's production and detection of electromagnetic waves in 1888, Maxwell's field theory and electromagnetic theory of light was accepted, notably by the leading theorist of the 1890s Henrik Antoon Lorentz, and came to be regarded as one of the most fundamental of all physical theories. 'Maxwell's equations' were accorded the status of Newton's laws of motion; and the theory was basic to the new technology of electric power, telephony and radio.

Maxwell's field theory and molecular physics achieved pre-eminence in the 'classical' physics of the nineteenth century, and mark an epoch in the history of the science, establishing his special place in the history of physics alongside Isaac Newton and Albert Einstein. The revolution in the structure of physical theory which has occurred in the twentieth century has reinforced rather than qualified Maxwell's unique status. His contributions to fundamental physics – the theory of the physical field and the electromagnetic theory of light, and the description of the motions of gas molecules by a statistical function – stand as progenitors of the relativity and quantum theories.

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In his famous paper on the theory of light in 1905 Albert Einstein pointed to a 'profound formal distinction' between field theory, where continuous spatial functions specify the electromagnetic state of a space, and molecular theory, where the state of a body is specified by the positions and velocities of a finite number of particles.[2] Writing in 1931 on the centenary of Maxwell's birth, Einstein appealed to a 'programme which may suitably be called Maxwell's: the description of Physical Reality by fields which satisfy without singularity a set of partial differential equations', in support of his contention that classical field theory should serve as the starting-point from which quantum rules emerge.[3] Rendition of Maxwell's outlook in these terms was of course intended to evoke Einstein's own endeavours and aspirations, but underlines the gap (as he understood it) between the primacy of fields (Maxwell's electromagnetic theory) and quantum theory (the statistical physics of particles).

The historical literature has naturally placed special emphasis on Maxwell's canonical contributions to fundamental physics, his field theory and statistical physics. The representation of the conceptual structure of physics as a duality of fields and particles, of electromagnetism and statistical physics, seen as having its historical roots in Maxwell's work and its contemporary expression in general relativity and quantum theory, has fostered this focus on the twin glories of Maxwell's science. Viewing Maxwell's physics from the vantage point of fields and particles, the historical analysis of his science has been defined by two areas of physics, electromagnetism and the kinetic theory of gases.

Writing in 1856, at the outset of his career, Maxwell suggests that nature may not be analogous to a 'book', envisaged as an ordered unity, but that the appropriate metaphor is a 'magazine', implying a collection of disconnected parts and a disparity in theorising.

> Perhaps the 'book', as it has been called, of nature is regularly paged . . . but if it is not a 'book' at all, but a *magazine*, nothing is more foolish to suppose that one part can throw light on another.
>
> (*LP*, **1**: 382)

Maxwell's theory of the electromagnetic field and his statistical physics illustrate such a disjunction in physical science. But the customary rendition of his physics in terms of a duality of electromagnetism and the kinetic theory of gases has provided a too restricted basis for analysing the structure of his scientific worldview. To provide a more discriminating framework for historical analysis, responsive to the categories of his own evolving conceptualisations, I will discuss his physics under broad thematic headings, categories which transcend the customary duality – of field theory (electromagnetism) and molecular physics (gas theory) – which has been traditionally used to characterise his science.

This representation of the structure of physical theory, as a dualism of fields and particles, contrasts with the scope of physics as understood at the time Maxwell began his career, and attests to the enormous impact of Maxwellian physics in marking an epoch in the science. Between 1800 and 1850 the science of physics was developing into a recognisably modern form: the study of mechanics, optics, heat, electricity, and magnetism, employing a mathematical and experimental methodology.

Around 1850, when Maxwell began his career, thermodynamics was in its infancy, resting on the two newly established laws of thermodynamics (the law of the conservation of energy, and the directional flow of heat from hot to cold bodies), while only the first steps had been taken to impose a mathematical structure on Michael Faraday's innovations in the study of electricity and magnetism. In the 1850s the law of the conservation of energy, as a cardinal element of the mechanical worldview of particles of matter in motion, came to be seen as fundamental to physical explanation; it was basic to Maxwell's subsequent achievement. In 1854 William Thomson (later Lord Kelvin), at the time Maxwell's guide to current work in physics, declared that the statement of the energy principle was 'the greatest reform that physical science has experienced since the days of Newton'.[4] Around 1850 the science of physics came to be defined in terms of the unifying role of the concept of energy and the programme of mechanical explanation.[5] Quantification, the search for mathematical laws, and precision measurement, the attainment of accurate values in experimentation, came to be seen as normative in physical science.

Maxwell shaped physical theory into its Maxwellian form by building on the work of his immediate predecessors – Hermann Helmholtz and Thomson in energy physics, Faraday and Thomson in field theory, Thomson and Rudolf Clausius in thermodynamics, and Clausius in the theory of gases. Maxwell's great achievements – the unification of optics (the theory of the luminiferous ether) and electromagnetism in his electromagnetic theory of light, the application of particle mechanics to understand the properties of gases and the foundations of the science of thermodynamics – rested on understanding the analogies and unities between the disparate themes of contemporary physics.

He himself emphasised the value of the 'cross-fertilization of the sciences' (*SP*, **2**: 744), evoking the image of bees pollinating flowers; and from the outset, he stressed the creative value of grasping the 'physical analogies' between different phenomena. Fundamental to these analogies and unities was understanding the relation between the language of mathematics and the structure of physical reality, between mathematical abstraction and the data of physical experiment, the

> hidden and dimmer region where Thought weds Fact, where the mental operation of the mathematician and the physical action of the molecules are seen in their true relation. (*SP*, **2**: 216)

The mechanical or dynamical worldview, which dominated the programme of physical explanation in the nineteenth century, shaped Maxwell's scientific theorising. But his attitude to mechanical explanation was complex. There was a tension in his thought between physical and mathematical models of mechanical systems; and his reflections on the relationship between mechanical representations and physical reality shaped his evolving programme of explanation. His introduction of statistical reasoning in the theory of gas molecules, and discussion of the instability and unpredictability of mechanical systems, led him to qualify his commitment to mechanism. The role of mechanical principles in his physics is complex and variegated, and to provide a preliminary perspective on these issues I will outline some of the central elements of his physics.[6]

Writing to Thomson in February 1854, after graduating from Cambridge University, Maxwell declared his intention to attack the science of electricity. In the 'Preface' to his *Treatise on Electricity and Magnetism* (1873) he recalled that he had commenced his work by study of Michael Faraday's *Experimental Researches in Electricity* (1839–55). Faraday had explained magnetism in terms of lines of force traversing space, and electrostatics by the mediation of forces by the dielectric. In 1845, by drawing on the analogy between electrostatics and the conduction of heat, which opened up applications of potential theory, Thomson showed that Faraday's ideas were compatible with the mathematical theory of electrostatics based on direct action at a distance. Thomson went on to develop theorems which could be applied to Faraday's discoveries in magnetism.

Guided by Thomson, Maxwell advanced beyond the work of his mentor in grappling comprehensively with Faraday's concept of the 'magnetic field'. In his paper 'On Faraday's lines of force' (1856) he presented a 'geometrical model' of lines of force in space, a representation resting on potential theory and the geometry of orthogonal surfaces, given embodiment by the 'physical analogy' of the flow of an incompressible fluid (*SP*, **1**: 156–8). He formulated theorems of electromagnetism, expressing the relation between magnetic forces and electric currents.

The analogy of streamlines in a fluid was proposed as illustrative of the geometry of the field; but Maxwell sought a theory of the field grounded on the mechanics of a mediating ether. He found its basis in Thomson's proposal that the Faraday magneto-optical rotation could be explained by the rotation of vortices in an ether. Maxwell began to develop the idea of orienting molecular vortices along magnetic field lines, culminating in the publication of his paper 'On physical lines of force', published in four parts in 1861–2. His physical model of vortices and 'idle wheel' particles, an ether model which is the most famous image in nineteenth-century physics, provides mechanical correlates for electromagnetic quantities in his field equations. The angular velocity of the vortices corresponds to the magnetic field intensity, and the translational flow of the idle wheel particles to the flow of an electric current. But he emphasised that while the theory was 'mechanically conceivable', the model itself was hardly 'a mode of connexion existing in nature' (*SP*, **1**: 486).

During the summer of 1861, while modifying the ether model to encompass electrostatics, he obtained an unexpected consequence, the 'Electromagnetic Theory of Light', as he termed his theory in 1864 (*LP*, **1**: 194). He introduced a 'displacement' of electricity as an electromagnetic correlate of the elastic deformation of the vortices, an elastic property which allowed for the propagation of transverse shear waves. He established the close agreement between the velocity of propagation of waves in an electromagnetic medium (which he demonstrated to be given by the ratio of electrostatic and electromagnetic units, established experimentally), and the measured velocity of light. This led him to claim the unification of optics and electromagnetism: that light consists in the vibrations of the '*same medium which is the cause of electric and magnetic phenomena*' (*SP*, **1**: 500). He completed the theory by a quantitative account of the magneto-optic effect in terms of the rotation of molecular vortices.

He was, however, dissatisfied with the appeal to a mechanical model, and sought to base his theory on firmer theoretical ground and to confirm its experimental basis. In 1862 he joined the British Association committee on electrical standards; and in May and June 1863, with Fleeming Jenkin and Balfour Stewart, made an accurate measurement of electrical resistance in absolute units (of time, mass and space). As part of the committee's report in 1863, Maxwell and Jenkin wrote a paper introducing dimensional notation: for every electrical quantity there are two absolute units, the electrostatic and the electromagnetic, and the ratio of these units is a power of a constant with the dimensions of a velocity. As Maxwell had established, this ratio was the velocity of waves in an electromagnetic medium. In 1868 he obtained a new value for the ratio of units by an experiment balancing the (electrostatic) force between two oppositely charged discs against the (electromagnetic) repulsion between two current-carrying coils, providing support for his theory.

In 'A dynamical theory of the electromagnetic field' (1865) Maxwell achieved a more general and systematic presentation of his theory. The ether model was abandoned, yet he retained the mechanical foundations of his theory by grounding the eight sets of 'general equations of the electromagnetic field' (the forerunners of the four 'Maxwell equations', as reformulated in the 1880s by Heaviside and Hertz) on the Lagrangian formalism of abstract dynamics. But in detaching his theory from the model he altered the interpretation of the displacement current, leading to a loss of consistency, a problem resolved in the *Treatise* where he interprets the displacement current as manifested as electric charge emergent from the field.

In the *Treatise* Maxwell emphasises the expression of physical quantities free from direct representation by a mechanical model. He enlarges the physical geometry and mechanical foundations of his earlier papers, deploying four fundamental mathematical ideas: quaternions (vector concepts), integral theorems (Stokes' theorem), topological concepts, and the Lagrange–Hamilton method of analytical dynamics (as developed by Thomson and Tait in their *Treatise on Natural Philosophy* of 1867). Maxwell's distinctive theory becomes most explicit in the final part of the work, on electromagnetism: here he presents the general equations of the electromagnetic field, the electromagnetic theory of light, and the dynamical basis of his field theory. The work concludes with a rebuttal of contemporary theories deriving from the tradition of considering forces acting at a distance without the mediation of a 'field'. He argues that these theories cannot satisfactorily explain the transmission of energy, for 'there must be a medium or substance in which the energy exists'. Mediation by an ether, the seat of the electromagnetic field, was the keystone of his theory (*Treatise*, **2**: 438 (§866)).

Maxwell's gas theory also has its origins in work undertaken upon his graduation from Cambridge. In March 1855 the subject of the University's

Adams Prize for 1857 was advertised as a study of 'The Motions of Saturn's Rings'. On revising his prize-winning essay for publication, Maxwell concluded that the ring system consists of concentric rings of satellites; this formed the argument of his memoir *On the Stability of the Motion of Saturn's Rings* (1859). The problems generated by this investigation played a role in initiating his work on the kinetic theory of gases in 1859. In considering the rings as a system of particles he noted that he was unable to compute the trajectories of these particles 'with any distinctness' (*SP*, **1**, 354). The Saturn's rings problem alerted him to discuss the complex motions of gas particles, where he introduced a probabilistic argument.

On completing his work on Saturn's rings, Maxwell had drawn on data on gas viscosity to establish the effect of friction in disturbing the stability of the rings. Alerted to gas viscosity and particle collisions, in spring 1859 he became interested in a paper by Rudolf Clausius on the theory of gases considered as particles in motion. To explain the slow diffusion of gas molecules, Clausius had calculated the probability of a molecule travelling a given distance (the mean free path) without collision. Maxwell had been interested in probability theory as early as 1850; and he advanced on Clausius' procedure by introducing a statistical formula for the distribution of velocities among gas molecules, a function identical in form to the distribution formula in the theory of errors. Beginning as an 'exercise in mechanics' (*LP*, **1**: 610), his work generated results in molecular physics. He was able to calculate the mean free path of molecules, and established the unexpected result that the viscosity of gases was independent of their density.

He turned to investigate the viscosity of gases at different temperatures and pressures, by observing the decay in the oscillation of discs torsionally suspended in a container, experiments presented as the Royal Society's Bakerian Lecture in 1866. He found that gas viscosity was a linear function of the absolute temperature; and he suggested, in his major paper 'On the dynamical theory of gases' (1867), that gas molecules should be considered as centres of force subject to an inverse fifth-power law of repulsion, a result in agreement with this experimental finding. He presented a new derivation of the distribution law, demonstrating that the velocity distribution would maintain a state of equilibrium unchanged by collisions. In drafting this paper he found that his theory seemed to have the consequence that energy could be abstracted from a cooling gas, a result in conflict with the second law of thermodynamics, stated in the early 1850s by Clausius and Thomson as denoting the tendency of heat to pass from warmer to colder bodies. While he corrected his argument and resolved the difficulty, it is likely that reflection on the problem led him to consider the bearing of his theory of gases on the interpretation of the second law of thermodynamics.

He first formulated the famous 'demon' paradox (the term was later coined by Thomson) in December 1867. By suggesting how a hot body could take heat from a colder one he showed that the second law of thermodynamics is a statistical regularity. Because of the statistical distribution of molecular velocities in a gas at equilibrium there will be spontaneous fluctuations of molecules taking heat from a cold body to a hotter one. But it would require the action of Maxwell's 'finite being', as he termed it (*LP*, **2**: 332), to manipulate molecules so as to produce an observable flow of heat from a cold body to a hotter one, and violate the second law of thermodynamics; hence the law is statistical and applies only to systems of molecules.

Maxwell amplified his argument to highlight a disjunction between the laws of mechanics and the second law of thermodynamics: this law is time-directional, expressing the irreversibility of physical processes, while the laws of mechanics are time-reversible. He maintained that the second law of thermodynamics is a statistical expression, not a dynamical theorem. In the 1870s, notably in a major paper on statistical mechanics written in 1878 (where he introduced the concept of ensemble averaging), he strove to clarify the relations between the dynamical and statistical descriptions of physical systems.

This cursory summary of Maxwell's most famous and enduring contributions to fundamental physics does not of course do justice to the physical and mathematical arguments upon which his theories rest. But this summary does indicate the centrality in his physics of issues such as the nature of physical analogies and of mechanical models, the relation between these models and more general dynamical principles, and the relation between dynamical laws and statistical explanations. These foundational issues transcend the disjunction between Maxwell's field and particle theories, between electromagnetism and the theory of gases.

The issue of Maxwell's commitment to mechanical explanation has loomed large in the critical and historical literature. In his famous 'Lectures on Molecular Dynamics and the Wave Theory of Light', delivered at The Johns Hopkins University in Baltimore in October 1884, William Thomson expressed strong criticism of Maxwell's approach to mechanical explanation. He professed 'immense admiration' for Maxwell's 'mechanical model of electromagnetic induction' – the ether model of 'On physical lines of force' – regarding the model as 'immensely instructive' and 'a step towards a definite mechanical theory of electro-magnetism'. But he bemoaned Maxwell's retreat from this approach to mechanical modelling. He considered Maxwell's electromagnetic theory of light, in the form presented in the *Treatise*, to be 'rather a backward step' in renouncing the provision of a mechanical model as the basis of a dynamical theory of electromagnetism. He contrasted Maxwell's theory, which he believed to be insufficiently grounded on mechanical principles, with the 'absolutely definite mechanical notion that is put before us by Fresnel and his followers', who had developed elastic solid theories of the luminiferous ether. He set out his own canon of mechanical explanation in a famous statement: 'as long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electromagnetic theory [of light]'.[7]

Reviewing the status of mechanical explanation some twenty years later, Pierre Duhem poured scorn on Thomson's claim for mechanical models as the sole basis for physical intelligibility. Duhem included in his critique 'that model of electrical actions which Maxwell built and for which Thomson has constantly professed his admiration',[8] the ether model of 'On physical lines of force'. Duhem has not been alone in dismissing the rationale of this ether model.[9] But discussion of the status of mechanism in Maxwell's physics has faltered because of imprecision in defining the problem.

Even within his theory of the electromagnetic field, where his mechanical outlook seems most emphatic, Maxwell's papers elaborate mechanical ideas in different senses. His paper 'On physical lines of force' (1861–2) is based on an explicitly mechanical outlook, yet he was flexible in his commitment to different elements of his ether model of vortex cells and 'idle wheel' particles. In his later writings, notably in the *Treatise*, he adopted an abstract approach to mechanical representation, based on the Lagrangian theory of a connected dynamical system, abandoning the attempt to provide a description of the internal characteristics of the electromagnetic field. But he still makes reference to formulating a 'complete dynamical theory' of the field in which 'the whole intermediate mechanism and details of the motion' would be studied (*Treatise*, **2**: 202 (§574)).

Within Maxwell's statistical theory of gases and thermodynamics the issue is even more difficult. He consistently presents his theory of gases as a 'dynamical' theory, by which he means a theory of particles in motion regulated by laws of forces. But he distinguishes the knowledge generated by the 'statistical method' of his theory of gases, which rests on probabilities, from the certain predictions of the 'dynamical method' which could trace and predict the trajectories of individual particles. The burden of the 'demon' paradox – that the second law of thermodynamics is a statistical theorem, not a dynamical law – underscored the limits of a physics based on purely dynamical principles. While he continued to stress the link between the dynamical and statistical specifications of a system, there is some divergence between his statistical method and his expression of dynamical theory.

Moreover, Maxwell questioned some of the basic assumptions of mechanism. In arguing that there are limitations in the explanatory power of dynamics, he pointed to the instability of a mechanical system at a point of singularity, where its trajectory could not be predicted. He drew the implication that while the universe was regulated by causal dynamical laws, these laws were not wholly deterministic. There are different strata in Maxwell's exposition and critique of dynamical principles; his arguments must be considered in their full variety and complexity. Maxwell's construal of mechanical explanation cannot be understood if the problem is viewed in narrow focus, limited to analysis of strands of his argument such as the role of mechanical models in electromagnetism.

It is the aim of this book to describe the structure of Maxwell's physical worldview, based on fields and statistical physics, and to elucidate its architectonic by tracing the motifs which thread their way through his natural philosophy. The term 'natural philosophy', which was becoming obsolete by Maxwell's time, seemingly at odds with the norms of the emerging community of 'physicists', is aptly descriptive of the ambition and scope of Maxwell's physics, which aimed, in traditional style, to lay the foundations of a scientific worldview. It was at the 1833 meeting of the recently formed British Association for the Advancement of Science that William Whewell, responding to the poet Coleridge's complaint that the term 'philosopher' was 'too wide and too lofty' for contemporary students of natural knowledge, proposed the term 'scientist'; this neologism served to demarcate 'science' (natural philosophy) from 'philosophy' (moral and metaphysical), and to emphasise the communality of the scientific enterprise. Whewell gave the term currency in his *Philosophy of the Inductive Sciences* (1840), where he also coined the neologism 'physicist' to describe the student of physics, investigating 'force, matter, and the properties of matter'.[10]

This shift in terminology matched a transformation within science itself: