Bayesian Methods

A Bayesian "posterior distribution" or "predictive distribution" summarizes everything you need to know about an unknown parameter, or future observations. This unique book shows how to use Bayesian statistical techniques in a sound and practically relevant manner. It will guide the reader on inferring scientific, medical, and social conclusions from numerical data. The authors explain the subtle assumptions needed for Bayesian methodology and show how to use them to obtain good-quality conclusions. The methods also perform remarkably well in terms of computer-simulated frequency properties.

The lively introductory chapter on Fisherian methods (the frequency approach), together with a strong overall emphasis on likelihood, makes the text suitable for mainstream statistics courses whose instructors wish to follow mixed or comparative philosophies. A chapter on advances in utility theory, and several sections on time series and forecasting, makes the text also suitable for quantitative economics students. The other chapters contain material on the linear model, categorical data analysis, survival analysis, random effects models, and nonlinear smoothing.

The book contains numerous worked examples, self-study exercises, and practical applications. It provides essential reading for final-year undergraduates, Master's-degree and graduate students, statisticians, and other interdisciplinary researchers wishing to develop good-quality conclusions from their data and to pursue the notion of scientific truth.

Thomas Leonard was appointed to the Chair of Statistics at the University of Edinburgh in 1995. He was previously Professor of Statistics at the University of Wisconsin–Madison. His early career, at the University of Warwick, followed his Ph.D. in statistics, obtained from the University of London in 1973. During the 1980s, Leonard pioneered the introduction of Laplacian methods into Bayesian methodology. He has published numerous papers on applications of statistics and has appeared as statistical expert in numerous U.S. legal cases.

John S. J. Hsu is Associate Professor of Statistics and Applied Probability at the University of California, Santa Barbara. He also holds an honorary appointment at the University of Edinburgh, which cites his contributions to the analysis of log-linear models. He obtained his Ph.D. in statistics from the University of Wisconsin–Madison in 1990 under the supervision of Thomas Leonard and Kam-Wah Tsui. At Santa Barbara, Hsu has worked on applied problems, as Director of the Statistical Laboratory, and has also developed his own Bayesian theoretical research program.

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Bayesian Methods

An Analysis for Statisticians and Interdisciplinary Researchers

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JOHN S. J. HSU

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Preface

Statistics uses theoretical models and techniques to help applied researchers to extract, and infer, real-life, scientific, medical, and social conclusions from numerical data, which are subject to random uncertainty. For any particular study, it is important to combine theoretical and computational resources, together with applied skills, and an ability to interact with experts with knowledge relating to the background and usefulness of the data.

Many studies and data sets are nonstandard, and it is not always possible to provide a completely convincing analysis based upon preexisting techniques. Therefore, statisticians frequently need to develop new techniques, on line, for a particular practical study. Furthermore, the statistical state of the art is continuously evolving, and it is therefore important for researchers to continue to develop the available statistical methodology. Finally, when existing methodology is available, it is important that this should be applied with specific knowledge of the subtleties of the assumptions involved, together with their consequences.

There are nowadays two main streams of statistical thought. We will refer to these as the "Fisherian" and the "Bayesian" philosophies. The Fisherian philosophy is named after Sir Ronald Fisher and combines the "frequency approach" (unbiased estimators, hypothesis tests, and confidence intervals) with likelihood methods. The Fisherian philosophy also includes the "fiducial approach," an incomplete method, suggested by Fisher, which attempts to achieve some of the advantages of the Bayesian approach (e.g., good conditional inference, given the observed values of the data, combined with appealing frequency properties when repeating the experiment a number of times under identical conditions), but without the assumption of a "prior distribution."

The Bayesian philosophy is named after the Reverend Thomas Bayes and refers to such concepts as "prior and posterior knowledge," "prior, posterior, and predictive distributions," and "Bayes decision rules and estimators." The Bayesian approach possesses many advantages, even when viewed from a Fisherian viewpoint, in particular its inherent long-run frequency properties. In practical terms, this means that if computer simulations are used to compare the mean squared error, prediction error, coverage probability, or power of different procedures, then Bayesian methods can perform remarkably well. This validation is an essential ingredient, when combined with the construction of statistical techniques, and provides just one substantial justification of the Bayesian paradigm. Other advantages are summarized by Berger (1985) and Bernardo and Smith (1994), and in our introductions to Chapters 2, 3, 5, and 6 of the current text.

Chapter 1 describes a number of Fisherian procedures, which comprise important background to the Bayesian approach. It is, for example, essential for the reader to be able to construct and understand likelihood functions before attempting Bayesian techniques. The reader should also understand basic data analysis.

Chapter 2 provides an easy introduction to Bayesian ideas and utilizes easy forms of Bayes' theorem when the parameter space is discrete. These are of particular importance in medical and legal applications.

Chapter 3 develops the Bayesian paradigm when there is a single unknown parameter. In such cases, a univariate probability distribution readily summarizes the posterior information. Frequency properties of related estimators and decision rules are developed.

Chapter 4 provides a break to some of the technicalities and considers the "expected utility hypothesis" and its role in financial decision making. Some extensions to the expected utility hypothesis are considered.

Chapter 5 extends the ideas of Chapter 3 to statistical models with several parameters. Approaches to the linear statistical model, categorical data analysis, and time-series analysis are included.

Chapter 6 provides advanced studies of prior structures, posterior smoothing, and Bayes–Stein estimation. Many of the techniques again achieve appealing frequency properties. Computational techniques, already mentioned in Chapter 5, for approximating or simulating high-dimensional numerical integrations, for example, for providing adequate finite sample size analyses of nonlinear models, are developed further. These include Laplacian methods, importance sampling, and Markov Chain Monte Carlo Methods (MCMC).

The text contains 49 worked examples and 148 self-study exercises, which relate to special cases of methodology more broadly explained in the main body of the text. The reader is thereby provided with layers of knowledge, which can be studied at different levels. The volume progressively develops a number of special themes in a possibly unique manner. A large number of further practical examples are described throughout the text.

The bibliography integrates Bayesian statistics with other statistical methodologies and with interdisciplinary research. While the Bayesian references represent the last four decades of research, they do not provide an exhaustive reference list for the Bayesian literature.

Much of the material in this text has been previously taught to graduate students in statistics, economics, and business attending a Bayesian Decisions course at the University of Wisconsin–Madison, and to graduate students attending a Bayesian Inference course at the University of California at Santa Barbara. The text is also appropriate for the following readerships:

- Students attending a statistics course with a mixture of Fisherian and Bayesian philosophies, at final-year undergraduate or at Master's-degree level. In this case, the instructors should concentrate on the easier parts of Chapter 1, together with Chapters 2 and 3, and the easier parts of Chapter 5. If the course is taught within an economics graduate program, then Chapter 4 and Sections 5.3–5.7 will also be of interest, together with the simulation procedures of Chapter 6.

- Interdisciplinary research specialists wishing to develop statistical models and analyses relating to their own area. We have previously used techniques described in this text for interdisciplinary research in many areas, including geology, psychometrics, medicine, animal science, genetics, biology, archaeology, forensic science, civil engineering, plant science, pathology, and physics. We have been involved in many practical collaborations, as directors of statistical laboratories at the University of Edinburgh and The University of California, with these objectives in mind.
- Doctoral students, and other researchers, in statistics. For example, Chapters 5 and 6 will help you to achieve the research frontiers in Bayesian statistics. Chapters 3, 5, and 6 would provide useful material for an advanced graduate course in statistics.

The first co-author wishes to acknowledge his mentors Anne F. S. Mitchell, Dennis V. Lindley, and A. Philip Dawid for teaching him Bayesian statistics at Imperial College and University College, London. His early Bayesian ideas, also frequently employed in this volume, were further influenced by James M. Dickey, Irving Jack Good, Adrian F. M. Smith, Tony O'Hagan, Jim Q. Smith, Patricia M. E. Altham, and P. Jeffrey Harrison. The second co-author wishes to acknowledge David V. Hinkley and Raisa Feldman for their encouragement. Both co-authors are indebted to Arnold Zellner, George Tiao, and Kam-Wah Tsui for their outstanding help and encouragement. They would also like to thank George E. P. Box, Jeff C. F. Wu, Irwin Guttman, Colin G. Aitken, Grace Wahba, Nan Laird, Michael Newton, Greg Reinsel, Bob Miller, Douglas Bates, and Richard A. Johnson for their previous advice on Bayesian and other related methods contained in this volume. Peter Lee has provided very helpful information in relation to his own writings. Suggestions by Bob Barmisch, Robert McCullough, Peter Wakker, Derek Arthur, and John Searle are indicated in the text. Jerome Klotz has advised us on gambling with roulette. Orestis Papasouliotis collaborated on some of the recent methodological developments and prepared the mathematics and computer program for the graphs in the cover design (these are the posterior densities of the group means in an analysis of covariance model). Geoff McLachlan kindly provided us with a copy of his computer package for multivariate mixtures. Rod Leonard described valuable insights regarding the problem of spurious correlation in the context of the chemical industry.

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