

Index

SYMBOLS

- $A \otimes B$ product sigma-field, 82
 $A \setminus B$ difference of sets, 7
 $A \Delta B$ symmetric difference of sets, 7
 $BL(X)$ bounded Lipschitz functions, 170
 $\mathcal{B}(X)$ Borel sigma-field on X , 22
 $\mathcal{C}^k(\mathbb{R})$ smoothness class of functions, 177
 \mathbb{C} complex plane
 $\mathcal{C}(T)$ continuous, real functions on T , 215
 $D_+(x)$ right-hand derivative, 309
 $\Delta(x) := 2(e^x - 1 - x)/x^2$, 265
 $dv/d\mu$ density, 53
 $D(P\|Q)$ relative entropy, 61
 \mathcal{E}_δ (\mathcal{E}_σ) countable intersections (unions) of sets from a class \mathcal{E} , 20
 $:=$ equality, by definition
 $\overset{\circ}{f}, \bar{f}$ semicontinuous envelope, 173
 \mathcal{F}_τ pre- τ sigma-field, 142
 $H(P, Q)$ Hellinger distance, 61
 \Rightarrow NOT weak convergence, 171
 \mathbb{I}_A indicator function of a set
 $\langle \cdot, \cdot \rangle$ inner product, *see* Appendix B
 $\mathcal{L}^1(\mu)$ functions integrable wrt μ , 28
 L^1 equivalence classes of \mathcal{L}^1 , 35
 L^∞ , 49
 \mathcal{L}^p, L^p , 36
 $L(x) := (2x \log \log x)^{1/2}$, 261
 \vee maximum (pointwise, of functions)
 \wedge minimum (pointwise, of functions)
 m Lebesgue measure (usually), 29
 $\mathcal{M}^+(\mathcal{X}, \mathcal{A})$ cone of nonnegative, \mathcal{A} -measurable functions on a set \mathcal{X} , 24
 $\mathcal{M}_{\text{bdd}}^+$ bounded members of \mathcal{M}^+ , 59
 $\mu^x \lambda_x^y$ iterated integral, 84
 μf same as $\int f(x)\mu(dx)$, 27
 $(\mu_1 - \mu_2)^+$, 73
 $\mu_1 \wedge \mu_2$ minimum of measures, 61
 $\mu \star \nu$ convolution, 91
 $\mu \otimes \lambda$ product measure, 88
 $\mu \otimes \Lambda$ product of measure and kernel, 86
 \mathbb{N} natural numbers $(1, 2, \dots)$
 $\bar{\mathbb{N}}_0 := \{0\} \cup \mathbb{N}$
 $\bar{\mathbb{N}} := \mathbb{N} \cup \{\infty\}$
 $\bar{\bar{\mathbb{N}}}_0 := \{0\} \cup \mathbb{N} \cup \{\infty\}$
 $\nu \perp \mu$ mutual singularity, 57
 $O_p(\cdot), o_p(\cdot)$ stochastic order symbols, 183
 $\phi(x)$ standard normal density, 317
 $\bar{\Phi}(x) := \mathbb{P}\{N(0, 1) > x\}$, 317
 $P_n \rightsquigarrow P$ weak convergence, 171
 $\mathbb{P}(X | T = t)$ conditional expectation, 125
 $\mathbb{P}(X | \mathcal{G})$ conditional expectation, 126
 $\psi(x) := 2 \left((1+x) \log(1+x) - x \right) / x^2$, 264
 $\bar{\mathbb{R}}$ extended real line, $\mathbb{R} \cup \{-\infty, \infty\}$
 \mathbb{R} real line
 $\rho(x) := \phi(x)/\bar{\Phi}(x)$, 317
 $\sigma(\mathcal{E})$ sigma-field generated by a class of sets \mathcal{E} , 19
 $\sigma(\mathcal{H})$ sigma-field generated by a class of functions \mathcal{H} , 23
 $T\mu$ image measure, 40
 2^{\aleph_0} a particular cardinality, 103
 \mathbb{W} Wiener measure, 215
 $X_n \rightsquigarrow P$ convergence in distribution, 171
 $\mathcal{X} \times \mathcal{Y}$ product space, 82
 absolute continuity
 functions, 65
 measures, 55
 adapted, 138
 almost surely/everywhere, 33
 Andersen, E. S. and Jessen, B., 109
 Bayes theory, 119, 131
 consistent estimator, 155
 bet red, 144
 blocking, 95, 262
 Boolean algebra, 7
 Borel paradox, 122
 Borel-Cantelli lemma, 34, 46, 102, 150
 Lévy extension, 152
 used for LIL, 263
 Borell, C., 278
 bounded variation, 67
 branching process, 149
 Bretagnolle, J., 258
 Brownian
 Bridge, 252
 filtration, 213, 336
 isonormal process, 216
 Markov property, 220, 336
 modulus of continuity, 219

- motion, standard, 215
 - motion, started at x , 213
- bunyip, 149
- Burkholder, D. L., 135
- cadlag, 329
- Cantor
 - diagonalization, 185
 - set, 55
- Carathéodory splitting, 290
- Carter, A., 328
- central limit theorem, 169
 - Lindeberg, 181
 - martingale, 200
 - multivariate, 182
 - real random variables, 176
 - second moments, iid, 180, 199
 - third moments, 179, 244
- characteristic function, *see* Fourier transform
- CLT, *see* central limit theorem
- cluster point, 271
- complete
 - L^1 , 48
 - L^p , 49
- completion, *see* sigma-field
- conditional
 - density, 118
 - distribution, 113–117
 - expectation, as contingent bet, 13
- contingent bet, 12
- Continuous Mapping Theorem, 175
- convergence
 - almost surely/everywhere, 34
 - in \mathcal{L}^1 , 39, 154
 - in distribution, 171
 - in probability, 37
 - relationship between modes, 38
 - weak, 171
- convolution, 91
 - smoothing, 247
- Cramér-Wold device, 202
- Crofton's theorem, 116
- Csörgő, M. and Révész, P., 255
- Daniell, P. J., 6
 - infinite products of measures, 100
- de Finetti, B.
 - exchangeability, 160
 - measures as functionals, 10
 - sets as indicators, 7
- DeAcosta, A., 269
- Dellacherie, C. and Meyer, P. A., 338
- delta method, 184
- density
 - as derivative, 65
 - of measure, 53
 - uniqueness, 71
- Desirable Frog Condition, 257
- differentiation under integral, 32
- disintegration, 117, 342
- distribution, *see also* measure/image
 - Binomial and beta, 322
 - finite dimensional, 99
 - function, 40, 82, 174
 - joint, 90
 - normal, *see* normal distribution
 - regular conditional, 113
 - uniform, 115
 - uniform on sphere, 122
- Dominated Convergence, 31, 35, 132, 180
 - differentiation under integral sign, 32
 - truncation arguments, 98
- dominated measure, 53, 133
- Doob, J. L.
 - consistent Bayes estimator, 155
 - infinite products of measures, 109
 - martingales, 166
 - martingales in continuous time, 336
- Dudley, R. M.
 - almost sure representation, 241
 - coupling, 246
 - empirical process, 171
 - Strassen's theorem, 243
- Dynkin class, *see* λ -system
- Ehrhard, A., 280
- EM-algorithm, 120
- empirical measure, 134, 157, 249
- envelope, 158
- epigraph, 307
- essential supremum, 49
- Etemadi, N., 109
- event, *see* measurable set; sigma-field
- exchangeable, 159
- expectation
 - as fair price, 11
 - as linear functional, 5, 10
 - iterated conditional, 128
- fair price, 11
- Fatou's Lemma, 31, 132
- Feller, W., 272
- fidi, *see* finite dimensional distributions
- field, 47, 289
- filtration
 - completion, 212
 - decreasing, 156
 - definition, 138
 - natural, 213
 - standard, 330
- finite dimensional distributions, 99, 212
- Finkelstein, H., 271
- first hitting time, 142, 331
- Fourier transform
 - Brownian motion, 213
 - Continuity Theorem, 199
 - inversion formula, 276
 - lattice distribution, 194
 - uniqueness, 196
- Fréchet, M., 92, 109
- Fubini theorem, 89
- Fundamental Theorem of Calculus, 65

- Gaussian process
 centered, 212
 concentration of supremum, 280
 coupling, 247
 series expansion, 249
 generating class
 functions, 43
 sets, 41
 graph of a map, 117, 343, 344
 Haar basis, 216, 250, 304
 Hahn-Banach theorem, 314
 Hartman, P. and Wintner, A., 268
 Hellinger distance, 61, 105, 120, 132
 Hewitt-Savage zero-one law, 160
 Hilbert space, 226
 Hoeffding, W., 272, 316
 Hoffmann-Jørgensen, J., 171
 Huygens, C., 6
 iid (independent, identically distributed), 98
 inclusion/exclusion method, 10
 independent
 random variables, 81, 83
 sigma-fields, 80
 indicator function of a set, 7
 inequality
 Bennett, 264
 Bernstein, 265
 Binomial tail, 271
 Birnbaum-Marshall, 163
 Borell, 279
 Cauchy-Schwarz, 37, 302
 chi squared tail, 258
 concentration, 280
 DeAcosta, 269
 Doob, 163
 Dubins, 148
 Fernique, 275
 Fourier coupling, 251
 Gaussian tail, 279
 Hájek-Rényi, 146
 Hoeffding, 316
 Hölder, 48
 isoperimetric, 278
 Jensen, 29, 133, 140
 Kolmogorov, 145, 163, 266
 maximal, 50, 95, 109
 maximum of normals, 275, 286
 Minkowski, 48
 Poisson tail, 272
 Slepian, 287
 Sudakov, 275
 Tusnády, 249, 324
 upcrossing, 148, 164
 inner measure, 290
 inner product $\langle \cdot, \cdot \rangle$, *see* Appendix B, 36
 integrable
 function, 28
 uniformly, 37, 154
 integral
 as linear functional, 26
 by parts, 105
 iterated, 86
 Ionescu Tulcea, C. T.
 infinite products of measures, 100
 isonormal process, 216, 305
 \mathcal{K}_0 -regularity, 290
 \mathcal{K}_0 -tightness, 291
 kernel, 84, 113
 coupling, 240
 sigma-finite, 87, 339
 Khinchin, A. Ya., 272
 Kim, J., 242
 KMT
 = Komlós, J., Major, P., Tusnády, G., 249
 coupling, 252
 Knuth, D. E., 9
 Kolmogorov, A. N.
 Borel paradox, 122
 conditional expectation, 123
Grundbegriffe, 1, 5
 infinite products of measures, 100
 LIL, 266
 maximal inequality, 145, 163
 SLLN, 78, 96
 zero-one law, 81
 Koltchinskii, V. I., 258
 Krickeberg decomposition, 151, 335
 Kronecker's lemma, 105
 Kuelbs, J., 273
 Kullback-Leibler distance, 61
 λ -system of sets, 42
 λ -cone of functions, 44, 85
 lattice
 cone, 296
 of subsets, 290
 Lebesgue, H.
 Fubini theorem, 108
 Fundamental Theorem of Calculus, 65
 measure/integral, 22, 29, 295
 thesis, 4, 14
 Le Cam, L., 135, 174
 weakly convergent subsequences, 185
 Yurinskii's theorem, 245
 Lévy, P.
 Brownian modulus of continuity, 218
 Brownian motion as martingale, 223
 central limit theorem, 190
 martingales, 166, 200
 normal distribution, 205
 lifting theorem, 340
 LIL (law of the iterated logarithm), 261
 Lindeberg, J. W., 176, 200, 245
 linear functional
 as integral, 27
 integral representation, 297
 iterated, 84
 sigma-smooth, 100
 tight, 184

- linear isometry, 305
- Lipschitz condition, 170
- LSC, *see* semicontinuity
- Major, P., 246
- Marczewski, E., 294
- Marriage Lemma, 243, 256
- martingale
 - at stopping times, 333
 - Brownian motion, 223
 - closed on the right, 155
 - continuous time, 224
 - differences, 141
 - quadratic variation/compensator, 201
- Massart, P., 258
- maximum likelihood, 79, 120
- McLeish, D. L., 200
- measurability, 23
 - diagonal, 103
 - failure for product measure, 94
 - \mathcal{F}_t , 143
 - first hitting time, 332
 - stability properties, 24
- measurable
 - function, 22
 - progressively, 331
 - rectangle, 82
 - set, 22, *see also* sigma-field
- measure
 - absolutely continuous, 55
 - affinity, 60
 - consistent family, 99
 - convolution, 91
 - dominated, 53
 - \mathcal{L}^1 distance, 60
 - finitely additive, 289
 - image, 39
 - infinite products, 99
 - inner, 290
 - Lebesgue decomposition, 56
 - marginal, 84
 - product, 88
 - Radon, 290, 342
 - sigma-finite, 54
 - signed, 59
 - singular, 57
 - space, 17
 - total variation distance, 59
 - Wiener, 215
- median, 279
- Métivier, M., 330
- Monotone Convergence, 26
- μ -negligible, 33
- negligible set, 33
- Neyman-Pearson Lemma, 73
- norm
 - bounded Lipschitz, 170
 - \mathcal{L}^1 , 29
 - L^1 , 35
 - \mathcal{L}^p, L^p , 36
 - Orlicz, 50, 93
- normal distribution
 - convolution, 91
 - correlated bivariate, 88
 - Fourier transform, 195
 - Lévy-Cramér theorem, 205
 - maximum, 212
 - multivariate, 202
 - quantile coupling with Binomial, 320
 - $\sqrt{2\pi}$, 89
 - symmetric bivariate, 121
 - tail bounds, 317
- notation, 27
- orthogonal, 301
- Parseval's identity, 215, 303
- partition of unity, 185
- paving, 290
 - compact, 294, 345
- P -continuity set, 174
- permutation, 159
- Philipp, W., 246
- π - λ theorem, 42
- Portmanteau theorem, 174
- predictable
 - process, 227
 - sequence, 141
- probability space, 18
- product space, 82
- Prohorov, Yu. V.
 - distance, 242
 - weak convergence, 174
 - weakly convergent subsequences, 185
- projection
 - conditional expectation, 128
 - in Hilbert space, 302
- Pythagoras, 132
- quantile function, 41, 92, 238
 - coupling, 249, 320
- Radon measure, 290
- Radon-Nikodym Theorem, 56, 340
- random element, 170
- recursion, 251
- relative entropy, 61
- relative interior, 313
- Riesz-Fréchet Theorem, 303
- rotational symmetry, 121
- sample path, 212, 214
 - cadlag, 334
 - continuous, 223, 337
- Scheffé's lemma, 57, 157, 198
- semicontinuity, 172
- separation of convex sets, 308
- sets
 - cylinder, 215
 - inclusion/exclusion, 10
 - indicator functions, 7
 - inner/outer regular, 47
 - limsup, 8
 - measurable, 22
 - negligible, 33

- paving, 290
- symmetric difference, 7
- Shorack, G. R., 272
- Sierpiński class, *see* λ -system
- sigma-field, 18
 - atoms, 20
 - Borel, 20
 - completion, 34
 - conditional expectation, 126
 - countably generated, 163
 - event, 22
 - fidi/cylinder, 215
 - generated, 19
 - independence, 80
 - Lebesgue, 34
 - pre- τ , 142
 - product, 82
 - symmetric, 158, 159
 - tail, 81
 - trace, 107
- sigma-ring, 94
- σ -smooth, 100, 292
- simple function, 25
- Skorohod, A. V., 258
- SLLN, *see* strong law of large numbers
- Slutsky's theorem, 175
- Stirling's formula, 325
- stochastic integral, 225
- stochastic process, 212
 - continuous time, 329
 - version, 214, 330
- stock price, 228
- Stone's condition, 297
- stopping time, 142
- Stopping Time Lemma, 145, 332
- Strassen, V., 242
- strong law of large numbers, 78
 - Etemadi, N., 106
 - first moment, 97
 - fourth moments, 79
 - LIL, as rate, 262
 - second moments, 96
 - uniform, 158
- sub/super martingale, 139
 - as measures, 152
 - convergence, 147, 151
 - Doob-Meyer decomposition, 142
 - reversed, 156
 - uniform integrability, 333
- τ -smooth, 300
- Tchebychev's inequality, 10
- tight, 184, 344
- Tonelli theorem, 88
- topology, countably generated, 103
- Topsøe, F., 174, 300
- total variation, 59
- triangular array, 179
- truncation, 98, 180, 191, 269
- uniformly tight, 184
- upcrossing, 147, 334
- USC, *see* semicontinuity
- usual conditions, 332
- Varadarajan, V. S., 174
- Vitali covering, 68
- Wald, A. (maximum likelihood), 80
- Walther, G., 203
- weak convergence
 - (Fourier) Continuity Theorem, 199
 - almost sure representation, 239
 - equivalences, 175
 - multivariate, 182
- Weierstrass approximation theorem, 51
- Wellner, J. A., 272
- Whittle, P., 6
- Wichura, M. J., 258
- Wiener, N., 6, 234
- Yurinskii, V. V., 245