Crispin Nash-Williams

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Abstract

This tribute consists of an appreciation from 1996, some further thoughts, and a list of Nash-Williams' publications.

1 An appreciation written on his retirement in 1996

I arrived in Aberdeen in 1965 to start my academic career as an assistant lecturer. I had become interested in graph theory and in particular in a series of papers with such resonant titles as

On well-quasi-ordering infinite trees
By C. ST. J.A. NASH-WILLIAMS
King's College, Aberdeen

So it is with pleasure that I am writing this appreciation of Professor Nash-Williams in the Quincentennial Year of the University of Aberdeen.

In 1967, after some ten years at Aberdeen, Nash-Williams moved to the University of Waterloo, returning to Aberdeen as Professor of Mathematics in 1972. In 1975 he took up a Professorship of Mathematics at the University of Reading where he joined a flourishing group of combinatorialists which included Richard Rado (then recently retired), David Daykin and Anthony Hilton. He has remained at Reading ever since, apart from a year in West Virginia and frequent visits to Waterloo.

It is not my intention to give a full appreciation of Nash-Williams' contribution to graph theory. How could I? This will, I hope, be done elsewhere with a complete edition of his papers. I shall content myself with a few random remarks on his work.

Nash-Williams is a graph theorist. Amongst combinatorialists this is an unusually positive statement. Though to call oneself simply a combinatorialist is too unspecific; after all, what mathematician is not to some extent a combinatorialist? Not only is he a graph theorist but even more unusual for a combinatorialist he claims [3] it as his first interest, and I quote:

> ... This cannot but provoke one's own early recollections, a favourite topic of conversation among graph-theorists being what first drew one's attention to graph theory. My own answer is 'Nothing: I just invented it'. In other words, when starting work as a research student, also at Cambridge, I felt that there ought to be a branch of mathematics dealing with this kind of thing, and, if there was not, I would create it. (References to binary relations in algebra courses might have helped to foster this idea.) it is a measure

> of the little known state of graph theory at that time that it took me some weeks to discover that I was not its first inventor and to hear of the one existing textbook on the subject Koenig's *Theorie der endlichen und unendlichen Graphen*, published eighteen years earlier in 1936.

The areas of graph theory in which Nash-Williams has made central contributions include: well-quasi-ordering of graphs – both finite and infinite – relative to subdivisions; Menger's Theorem; the theory of transversals – both finite and infinite; orientations of graphs and the theory of infinite graphs in general. In recent years he has published a series of papers on detachments of graphs, the theory of which in a certain sense generalizes Eulerian graph theory.

Nash-Williams' work has applications outside graph theory, for example in the theory of well-quasi-ordering transfinite sets, and most recently he, together with David White, have become interested in the interaction of graph theory with the rearrangements of conditionally convergent real series.

One senses, however, that throughout his career, Nash-Williams has been infected more than most by two of the three graphical diseases [2]: Hamiltonian circuits and the Reconstruction Problem. These are interests to which he continually returns. He appears, at least up to now, to have developed an immunity to the third disease - the Four Colour Theorem.

The really surprising thing is that he has any time at all for research. During the term he has always devoted the majority of his time to teaching. This is surely one of his most valuable contributions; there are countless students who have given testimony to the clarity of his teaching and the generosity with which he makes time for them. A particular characteristic is his willingness to help the weakest as well as the more able students. All of us appreciate the excellence of his lectures at conferences; this excellence is repeated day in and day out during the term.

Nash-Williams is also much in demand to serve on committees and seems not to be able to say no. He has served on the British Combinatorial Committee for many years and was its Chairman from 1987 to 1992.

As an example of the variety of his activities he has always been an active member of the Association of University Teachers and was indeed the President of the Reading Branch of the Association in 1987-89. This was a somewhat troubled time involving threats of industrial action. I am sure that few would believe that he was described by someone in high office at this time as an 'out and out militant'.

Finally, no tribute would be complete without mentioning the time and energy Nash-Williams spends in refereeing: many of us have been grateful recipients of his constructive critiques which are sometimes as lengthy and certainly more meticulous than the original articles. It is probably no secret that many of a long series of papers concerning well-quasi-ordering of graphs by relationship of one graph being a minor of another, which have been appearing in the Journal of Combinatorial Theory in the last decade, have been refereed by Nash-Williams. The amount of selfless work this has involved is probably not universally appreciated; graph theorists owe him an inestimable debt for this alone.

Professor Tutte, when also nearing retirement, remarked [4]:

> What is Mathematics? You seem to have three choices. Mathematics is the Humanity that hymns eternal logic. It is the Science that studies the phenomenon called logic. It is the Art that fashions structures of ethereal beauty out of the raw material called logic. It is all of these and more. Much more, I can assure you, for Mathematics is Fun.

Crispin St John Alvah, and I leave it until now to use his full name, has contributed in great measure to this fun and I finish with an anecdote from Blanche Descartes [1]:

> I think of the occasion at a crowded conference when Crispin was standing, quite happily it seemed, in a stream of hot air from a grill on the floor. He was urged to come away on the grounds that he was 'crispin' and singein'.

2 Further thoughts

Professor C St-J A Nash-Williams died on the 20th of January 2001. The stereotypical 'English Gentleman' is oft maligned. Crispin was an English Gentleman in the good old sense of this phrase. He was a gentle, unassuming and kind man with very strong principles which he defended courageously. As a teacher he is remembered and loved for his consideration for the weak students as well as the strong. I remember one such student in particular – necessarily nameless – who even by today's standard would be considered at risk. He slavishly struggled to get him to pass and eventually, after several resits, indeed he did. I have met very few teachers, if any, who could have risen to this particular challenge. It was all achieved with unflappable patience and I was never once aware of him mentioning it. A truly modest man.

Crispin retired in 1996 and since then he seemed to be at his happiest. He had been out of sympathy with many of the recent developments in University Education. With no administrative and teaching chores he was able to pursue his research interests which he enjoyed so much. He continued to collaborate with his colleagues in the department, in particular including David White and Anthony Hilton, until almost the very end. Professor Crispin St-J Alvah Nash-Williams was a good man: incapable of pettiness, generous and noble of spirit.

Our Combinatorial Community will always remember him with affection and pride that he took such a delight in our subject and our company.

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The Penrose polynomial of graphs and matroids

Martin Aigner

1 Introduction

In 1971 Roger Penrose published in a Conference Proceedings a paper entitled "Applications of negative-dimensional tensors". Perhaps it was this somewhat cryptic title that prevented the paper from becoming widely known at first. Still, as is to be expected from Penrose, it contained a wealth of original ideas on the theory of plane graphs. If one studies the paper more closely one finds that the main object is the enumeration of certain colourings of plane graphs. And pretty soon one surmises that Penrose wanted, in fact, to solve the 4-colour conjecture. Remember that in 1971 it was still a problem; it was only 1976 that it became the 4-colour theorem.

Penrose implicitly defines in his paper a polynomial for 3-regular plane graphs, now called the *Penrose polynomial*, and deduces four equivalent formulations of the 4-colour theorem. His ideas were taken up in the eighties by several people, foremost by the late François Jaeger, and generalized in various ways. In this article we survey at a leisurely pace how the Penrose polynomial is connected to some famous conjectures in graph theory, to binary spaces, Hopf algebras and polynomial invariants in knot theory.

2 The Penrose polynomial

Let $G = (V, E, F)$ be a plane connected graph with vertex-set V, edge-set E, and face-set F. To avoid trivialities we always assume that G contains at least one edge.

Look at the plane graph G in Figure 1:

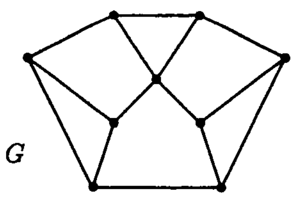

Figure 1

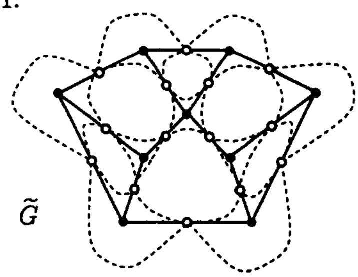

Figure 2

In every face (including the outer face) we draw a closed curve near the