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Henry E. Kyburg and Choh Man Teng
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1

Historical Background

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The theory of Induction is the despair of philosophy—and yet all our activities are based upon it.

Alfred North Whitehead: *Science and the Modern World*, p. 35.

1.1 Introduction

Ever since Adam and Eve ate from the tree of knowledge, and thereby earned exile from Paradise, human beings have had to rely on their knowledge of the world to survive and prosper. And whether or not ignorance was bliss in Paradise, it is rarely the case that ignorance promotes happiness in the more familiar world of our experience—a world of grumbling bellies, persistent tax collectors, and successful funeral homes. It is no cause for wonder, then, that we prize knowledge so highly, especially knowledge about the world. Nor should it be cause for surprise that philosophers have despaired and do despair over the theory of induction: For it is through inductive inferences, inferences that are uncertain, that we come to possess knowledge about the world we experience, and the lamentable fact is that we are far from consensus concerning the nature of induction.

But despair is hardly a fruitful state of mind, and, fortunately, the efforts over the past five hundred years or so of distinguished people working on the problems of induction have come to far more than nought (albeit far less than the success for which they strove). In this century, the debate concerning induction has clarified the central issues and resulted in the refinement of various approaches to treating the issues. To echo Brian Skyrms, a writer on the subject [Skyrms, 1966], contemporary inductive logicians are by no means wallowing in a sea of total ignorance and continued work promises to move us further forward.

1.2 Inference

In common parlance, an inference occurs when we make a judgment based on some evidence. We make inferences all the time: If we know that Adam had ten cents and later learn that he found another thirty cents, then we infer that he has a total of forty cents; if we know that all farmers depend on the weather for their livelihood, and we

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know that Salvatore is a farmer, then we infer that Salvatore depends on the weather for his livelihood; if we have won at the track when we have brought our lucky rabbit's foot in the past, we infer that we will win today, since we have our rabbit's foot; if we see Virginia drop her cup of coffee, we infer that it will fall to the floor and spill.

Some of the inferences we make are good ones, some bad. Logic is the science of good inferences, and for logicians, a correct inference occurs when we derive a statement, called a *conclusion*, from some set of statements, called *premises*, in accordance with some accepted rule of inference. In a given instance of inference, the set of statements constituting the premises and the conclusion, perhaps together with some intermediate statements, constitute an *argument*, and good arguments, like correct inferences, are those in which the conclusion is derived from the premises according to accepted rules of inference.

Traditionally, deductive logic is the branch of logic that studies inferences that are both sound (all the premises are true) and valid. If it is impossible for both the premises of an argument to be true and the conclusion of that argument to be false, then the inference from those premises to that conclusion is considered deductively valid. Valid arguments have the following three important features:

(1) All of the information contained in the conclusion of a valid argument must already be implicitly "contained in" the premises of that argument (such arguments are thereby termed *nonampliative*).
(2) The truth of the premises of a valid argument guarantees the truth of its conclusion (thereby making such arguments *truth preserving*).
(3) No additional information, in the form of premises, can undermine the validity of a valid argument (thus making such arguments *monotonic*).

For example, suppose that we have the following three claims:

(a) all arguments are tidy things;
(b) all tidy things are understandable; and
(c) all arguments are understandable.

The inference from claims (a) and (b) to the claim (c) is a valid inference: If (a) and (b) are true, then (c) must be, too. Notice that the information expressed in (c) concerning arguments and understandable things is in some sense already to be found in the conjunction of (a) and (b); we just made the connection explicit in (c). Also note that no further information concerning anything whatever will render (c) false if (a) and (b) are true.

Sound arguments are valid arguments with the following additional property: All of the premises of sound arguments are true. Valid arguments may have either true or false premises; validity is not indicative of the truth of the premises of an argument; nor does validity ensure the truth of the conclusion of an argument. Validity, rather, concerns the relationship that is obtained between the premises and the conclusion of an argument, regardless of their actual truth values, with one exception: If it is impossible for the conclusion of an argument to be false on the supposition that the premises are true, then the argument is valid; otherwise it is invalid. Soundness does concern the truth value of the premises and the conclusion. A sound argument is a valid argument with true premises, and since it is a valid argument, a sound argument also has a true conclusion.

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The main branches of deductive logic are well established, and the rules for deductive inference well known.[1] Not so for inductive logic. *Induction* is traditionally understood as the process of inferring from some collection of evidence, whether it be of a particular or a general nature, a conclusion that goes beyond the information contained in the evidence and that is either equal to or more general in scope than the evidence. For example, suppose an ethologist is interested in whether or not owls of the species *Bubo virginianus* will hunt lizards of the species *Sceloperous undulatus*. The scientist exposes some of the lizards to ten different owls of the species, and all of the owls respond by capturing and hungrily eating the lizards.

In setting up her eleventh experiment, the researcher is asked by her assistant whether or not she believes that the next owl will hunt and eat the released lizards. The animal behaviorist tells the assistant that, based on the prior tests, she believes the next owl will eat the lizards. Here the ethologist has used evidence about the particular owls that were tested to infer a conclusion which is equally general in scope—the conclusion is about a particular owl, the next one to be tested—but which goes beyond the information implicit in the premises. After having completed the eleventh test, the ethologist returns to her laboratory and opens her laboratory notebook.

She writes, "After having completed the eleven field studies and having a positive response in each case, I conclude that all owls of the species *Bubo virginianus* will eat lizards of the species *Sceloperous undulatus*." In this case, the scientist has for her evidence that the eleven owls hunted and ate the lizards. She concludes that all owls of that species will eat that species of lizard. Her evidence is about particular owls; her conclusion is a generalization about all owls of a certain species. The generality and the informational content of the conclusion are greater than those of the premises.

Again, suppose a market researcher does a demographic study to determine whether or not most people in America shop for food at night. He collects data from ten major American cities, ten suburban American towns of average size, and ten average size rural American towns. In each case, the data shows that most people shop for their food at night. The market researcher concludes that most Americans shop for their food at night. Here, the researcher's evidence consists of thirty generalizations concerning the shopping habits of most people in specific locations in America. The researcher's conclusion generalizes from this evidence to a claim about the shopping habits of most Americans in all locations of America.

Inductive logic is the branch of logic that inquires into the nature of inductive inferences. Many writers distinguish two general types of simple inductive inferences—those by enumeration and those by analogy—and we can divide each of these into two further kinds: particular enumerative inductions and general enumerative inductions; and particular analogical inductions and general analogical inductions.

A particular enumerative induction occurs when we assert that a particular individual A has the property of being a B on the basis of having observed a large number of other As also being Bs. For example, if each of the twenty dogs we have encountered

[1] Whereas this is true generally, there are extensions of deductive logic, for example, modal logic, concerning which there is less agreement. There are also arguments over the foundations of logic. See, for example, [Haack, 1996].

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has barked, and we infer from this that the next dog we encounter will also bark, we have performed a particular enumerative induction.

If we conclude from a number of observed As being Bs the more general claim that most As are Bs or that all As are Bs, then we have performed a general enumerative induction. Continuing our dog example, we might have inferred from our having observed twenty barking dogs that most or all dogs bark.

In analogical induction, we use the fact that a given A (which may be a particular thing or a class of things) possesses properties $P_1, \ldots, P_n$ in common with some C (another particular thing or class of things) to support the conclusion that A possesses some other property P_{n+1} that C also possesses. If our conclusion is about a particular A, then we have performed a particular analogical induction; if the conclusion is general, then we have performed a general analogical induction. Here the warrant for our conclusion about A possessing the property P_{n+1} is derived not from the number of As observed, but rather from the similarities found between A and something else.

For example, we know of geese that they are birds, that they are aquatic, and that they migrate. We also know that they mate for life. Ducks are birds, are aquatic, and migrate. We might infer, by general analogical induction, that they also mate for life.

Inductive inferences are notoriously uncertain, because in an inductive inference the conclusion we infer from our premises could be false even if the evidence is perfectly good. We can put this in terms of arguments by saying that inductive arguments are not truth preserving—the truth of the premises in an inductive argument does not guarantee the truth of the conclusion.

For example, suppose we know that a very small percentage of the U.S. quarter dollars in circulation are silver quarter dollars; it would be reasonable to infer from this that the next U.S. quarter dollar we receive will not be made of silver. Nevertheless, the next one *might* be made of silver, since there are still some silver quarter dollars in circulation. Our inference concerning the next U.S. quarter dollar we receive, while reasonable, is uncertain; there is room for error. Our inferred conclusion could be false, even though our premises are true. Such inferences are not truth preserving. Because the conclusion of an inductive argument could be false even if the premises are true, all inductive arguments are invalid and therefore unsound.

The reason that some, indeed most, of our inferences are uncertain is that often the information contained in our conclusions goes beyond the information contained in our premises. For instance, take the following as exhausting our premises: Most crows are black, and the bird in the box is a crow. These premises make it reasonable to conclude that the bird in the box is black. But, of course, the bird in the box might not be black. Our premises don't tell us enough about the particularities of the bird in the box to make it certain that it will be a black crow. Our premises present us with incomplete information relative to our conclusion, and as a result the truth of our conclusion is uncertain. Inductive arguments have been called *ampliative* to describe the fact that their conclusions contain more information than their premises.

Scientific inferences are ampliative in nature. For example, when a scientist collects data about some phenomenon, analyzes it, and then infers from this data some generalization about every instance of that phenomenon, the scientist is making an ampliative inference. Thus, suppose a scientist tests for the resistance to an antibiotic of some gonococcus he has prepared in a number of Petri dishes, analyzes the results of the experiment, and then (perhaps after a number of similar trials and controlling

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for error) infers from his results that all gonococci of the type under investigation are resistant to the antibiotic. His inferred conclusion concerns all instances of that type of gonococcus, but his evidence is only about a limited number of such instances.

In cases of uncertain inference, true premises make the truth of the conclusion more or less plausible. Whereas deductive arguments are monotonic, inductive arguments are *nonmonotonic* in character. If we gather more information and add it to our set of premises, our conclusion may become either more tenable or less tenable.

For example, suppose Gumshoe Flic, an able detective, is investigating a reported shooting. The victim was a successful, single, middle-aged woman named Sophia Logos who was a prosecuting attorney in the community. Sophia was found by her housecleaner dead in the living room some hours ago. She had been shot in the chest repeatedly. No gun was found near the body. Gumshoe, upon inspecting the body, comes to the conclusion that Sophia was murdered by someone who shot her. Later, while searching the house, Gumshoe finds an empty bottle of barbiturates on the counter in the bathroom and a small caliber handgun hidden in the basement. He has these sent to the forensic laboratory for fingerprints and ballistic testing.

Back at the office Gumshoe hears that, the day before the shooting, a criminal, Hunde Hubris, had escaped from the nearby prison. Hunde, Gumshoe knew, had been sent to prison for life six years before in a sensational murder case brought to court by prosecutor Sophia Logos. Gumshoe remembers that Hunde had vowed, as he left the courtroom, to kill Sophia if it was the last thing he did. Gumshoe tells his colleague over a coffee and doughnut that he thinks Hunde was most likely the murderer.

After leaving the office and upon questioning one of Sophia's closest friends, Gumshoe learns that, recently, Sophia had been suffering from severe depression and had mentioned to several friends her despair and her occasional thoughts of suicide. Gumshoe puts this information together with that of the empty bottle of barbiturates and pauses. He asks the friend if Sophia had been taking any prescription drugs. The friend replies that Sophia was notorious for refusing medication of any sort. "She practiced Eastern medicine. You know, acupuncture, Taoism, that kind of thing. She often said, usually as a joke of course, that she would only take drugs in order to kill herself." Gumshoe begins to wonder. On calling the forensic lab and inquiring about the blood analysis done by the coroner, it turns out that Sophia had died from an overdose of barbiturates two hours before she had been shot. Well, Gumshoe thinks to himself, that blows the theory that she died from the shooting, and it's beginning to look like Sophia committed suicide by taking barbiturates. But the gunshot wounds.... It still seems likely that she was murdered.

As he ponders, he gets a call on his cellular phone from a fellow detective at the station. Hunde has been apprehended in a nearby hotel. He had called the authorities and turned himself in. Under questioning, he said he was innocent of the shooting. Instead, Hunde claimed that he and Sophia had been in correspondence while he was in prison, that she had made him see the error in his ways, and that he had escaped from prison in order to help her after having received her last letter in which she mentioned her depression and growing fear of her housecleaner. Gumshoe takes this in and cynically responds, "And I'm Santa Claus." The other detective replies by explaining how he had checked with forensics and found out that the handgun found in the basement matched the weapon used in the killing and that, most surprisingly, the handgun failed to have Hunde's prints on it; but, also surprisingly, both the handgun

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Excerpt
More information

and the bottle of barbiturates had the fingerprints of the housecleaner all over them. "And get this, Hunde has the last letter he got from Sophia, and I'm telling you, Gum, it looks authentic."

After hanging up, Gumshoe revises his previous conclusions on the basis of his new evidence. It now seems less plausible that Hunde killed Sophia, though given the wacky story Hunde is telling, Hunde is still a prime suspect. The housecleaner now seems to be a likely suspect, and it looks like this is indeed a murder, not a suicide. If the housecleaner skips town, then there's more reason to think she did it. Maybe she heard that Hunde had escaped and was trying to make it look like Hunde did it by shooting Sophia. But what is the motive?

Note that as Gumshoe's evidence changes, so does the plausibility of the conclusions he previously inferred. Sometimes the new evidence undermines the plausibility that a previous conclusion is correct, sometimes it strengthens the plausibility of the conclusion. If a deductive conclusion is entailed by its premises, nothing can weaken that relation. The kind of inference we are looking at is thus nonmonotonic, and therefore not deductive. Attempting to provide canons for nonmonotonic reasoning is one of the major challenges for an inductive logic.

1.3 Roots in the Past

The first system of logic was presented to the Western world by Aristotle in the fourth century B.C. in his collected works on logic entitled *Organon* (meaning, roughly, "a system of investigation"). Aristotle's was a syllogistic deductive logic, taking the terms of a language to be the fundamental logical units. Whereas Aristotle emphasized deductive inferences, he did allow for syllogisms that had for premises generalizations that asserted what happened for the most part (i.e., generalizations of the form "Most *A*s are *B*s"), and Aristotle was clearly aware of both enumerative and analogical inductions. Nevertheless, Aristotle did not present a systematic study of inductive inferences.

During the Hellenistic period of Greek philosophy, the Stoics extended Aristotle's work and were the first to develop a propositional logic, according to which the propositions of a language are taken as the basic logic units. As Aristotle had before them, these later thinkers devoted their attention to deductive inference and did not develop a logic of uncertain inference. Likewise, during the Middle Ages such figures as Peter Abelard (1079–1142) and William of Ockham (ca. 1285–1349) made contributions to the work done by the ancients on deductive logic but did little work on the theory of uncertain inference.

From Aristotle to Newton and into the present, philosophers and scientists have modeled science upon mathematics. This way of analyzing science in terms of deductive demonstration may be called the *axiomatic approach* to science, and one of its primary characteristics will help to motivate our discussion of the historical development of inductive logic. The axiomatic approach to science can be characterized by its emphasis on the need to model science and scientific language upon the exact methods and languages of mathematics. Historically, this has meant presenting scientific theories in terms of axioms which express the fundamental laws and concepts of the science. The substantive content of the science is then embodied in theorems derived from these axioms.

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Euclidean geometry provides a prime example of this approach. Euclidean geometry is a theory expressed in a limited vocabulary (point, line, plane, lies on, between, congruent, etc.) and is based on a small number of statements accepted without proof. In many presentations, a distinction is made between *axioms* and *postulates*. For example, it is regarded as an axiom that things that are equal to the same thing are equal to each other, while it is a postulate peculiar to geometry that a straight line is determined by two points. Non-Euclidean geometry is based on postulates that are different from the standard Euclidean postulates. Euclidean geometry accepts the postulate that there is one and only one parallel to a given line through a given point. Lobachevskian geometry allows for more than one line parallel to a given line through a given point. Riemannian geometry denies the existence of parallel lines. All these geometries are internally perfectly consistent. But which is true?

An important component of any axiomatic account of science is how we come to know the axioms or first principles of a science. For example, which set of geometric postulates captures the truth about lines and planes? Here we discern a source of one of the great schisms in Western philosophy—that between the *rationalist* approach to scientific knowledge and the *empiricist* approach.

The rationalists, for example, Plato, Augustine, Descartes, and Leibniz, typically assume that our knowledge of the axioms of science is independent of any empirical evidence we may have for them. Different rationalist thinkers have proposed different views concerning how we come to know the axioms. Plato, in the *Phaedo*, asserted that we gained knowledge of the axioms before birth, forgot them at birth, and then rediscovered them during our lifetime. Augustine believed that it was through the presence of God's light in the mind that the axioms were rendered certain and known. Descartes held that the knowledge of the axioms of science was derived from self-evident truths, and Leibniz maintained that we possess knowledge of the axioms of science in virtue of certain conceptual truths about God. Reliance on some supersensible ground is characteristic of rationalist accounts of our knowledge of the axioms of science.

Empiricists, on the contrary, typically argue that all scientific knowledge is based, ultimately, on our sensible experience of the world—experience which is always of the particular and immediate. Given their assumption that all knowledge is ultimately based on experience, empiricists need to explain how we can achieve the generality of our scientific knowledge of phenomena given that our experience is always of particular instances of phenomena. Typically, but not always, induction will play an important part in any such empiricist explanation. To the extent that an empiricist who relies on induction as the source of our scientific knowledge cannot provide an account of the nature of induction, his view of science, and of our knowledge of the world, is incomplete. In other words, the empiricist is under a strong compulsion to provide an account of inductive inference.

1.4 Francis Bacon

The English statesman, philosopher, and moralist Francis Bacon (1561–1626) was well aware of the need for an inductive logic that would serve as a canon for scientific inference. To meet this need, Bacon presented the first explicit account of the methods of induction in his *Novum Organum* [Bacon, 1620], and with Bacon the story of inductive logic begins.

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Long before Bacon, ancient and medieval natural scientists had investigated and amassed data concerning the phenomena of nature, but Bacon for the first time attempted to codify the methods of gathering empirical data and making inferences based on it. Bacon believed that previous scientific and philosophical efforts had been marred by unswerving devotion to the Aristotelian approach to science laid out in the *Organon*. This approach stressed deduction from first principles as the main mode of scientific inquiry. Bacon believed that, while such deduction was appropriate at the stage where science was nearly, if not already, complete, scientists needed to free themselves from the Aristotelian paradigm during the developing stages of science and adopt a different and inductive approach to science. He thus proposed a new system of investigation in his *Novum Organum*.

Bacon sets the stage for his investigation by first explaining the available options and the current tendencies:

> There are and can be only two ways of searching into and discovering truth. The one flies from the sense and particulars to the most general axioms, and from these principles, the truth of which it takes for settled and immovable, proceeds to judgment and to the discovery and middle axioms. And this is the way now in fashion. The other derives axioms from the senses and particulars, rising by a gradual and unbroken ascent, so that it arrives at the most general axioms last of all. This is the true way, but as yet untried. [Bacon, 1620, Book I, Aphorism xix or Section xix.]

Bacon identifies the "true way" of searching into and discovering truth with induction, and by "induction" he means a method by which science "shall analyze experience and take it to pieces, and by a due process of exclusion and rejection lead to an inevitable conclusion" [Bacon, 1620, preface.] For Bacon, it is knowledge of causes that is true knowledge, and Bacon's is an Aristotelian conception of causes, including the material, efficient, formal, and final causes; but most important for science are the efficient causes. Thus, every individual in the world has, in Bacon's view, a *material cause* (the stuff of which it is constituted), an *efficient cause* (that which caused, in our sense of the term, the thing to occur), a *formal cause* (the essential properties of the thing), and a *final cause* (the purpose of the thing). Metaphysics has for its task the determination of the formal causes of things, and science the determination of the material and efficient causes.

Bacon's scientific method (not his inductive method) consists of three stages:

(1) the amassing of experimental data (termed "a natural and experimental history"),
(2) an ordered arrangement of the experimental data (termed "tables and arrangements of instances"), and
(3) the principled inference from the ordered data to more and more general axioms (termed "induction").

Let us follow Bacon, and suppose we are investigating the nature of heat. So, first, we gather a lot of data about heat. Once we complete this first stage of our investigation, Bacon tells us we are to order the experimental data we have collected into three tables:

(a) the *table of essence and presence* (wherein we list all those instances of things that possess the property of heat),

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(b) the *table of deviation, or of absence in proximity* (wherein we list all those instances of things that fail to possess the property of heat), and

(c) the *table of degrees or comparison* (wherein we list the observed changes in the property of heat as we vary another property).

Having completed the second stage and ordered our information, we can then apply Bacon's method of induction.

Bacon's method of induction involves three parts—a negative part, an affirmative part, and a corrective part. The negative part, termed "rejection or process of exclusion," is applied first and involves rejecting as distinct from the nature of heat anything that fails to always be correlated with the property of heat. So, for example, if there are cases in which stones are not hot, then we can exclude the property of being a stone from being a part of the nature of heat. If there are cases in which heat attends water, then we can discount the property of solidity as being a part of the nature of heat. Again, if there are cases of change in brightness without a change in heat, then we can reject brightness as being a part of the nature of heat.

Following this negative enterprise, or concomitant with it, we are to engage in an effort to determine which among the properties correlated with heat has a nature "of which Heat is a particular case" [Bacon, 1620, Book 2, Aphorism xx or Section xx]. That is to say, of which of these latter properties is heat a species? Bacon is optimistic that there will be a few instances of heat which are "much more conspicuous and evident" [Bacon, 1620, Book 2, Aphorism xx or Section xx] than others and that we can therefore, by choosing among these, eventually determine which is the correct one. Bacon is aware that this affirmative part of induction is prone to error, since we might initially pick the wrong general property, and he aptly terms it "indulgence of the understanding." (Bacon, in fact, proposed the property of motion as that property of which heat is a species.) But Bacon believes that in putting forward the hypothesis that heat is a species of some more general property and, as a consequence, focusing our attention on our data concerning heat and this other property, we shall be more efficient in determining whether or not heat is in fact a species of it.

Once we have completed the processes of exclusion and the indulgence of the understanding, we shall have arrived at a provisional definition of the phenomenon under investigation (Bacon terms this the *first vintage*), which will serve as the raw material for the next stage of the inductive process, that of *correction*. What we have termed the "corrective part" of Bacon's method was to involve ten distinct steps. Of these Bacon presents an exposition of only the the first, *prerogative instances*, having been unable to complete his entire project. This exposition amounts to twenty-seven different and special kinds of instances purported by Bacon to facilitate the refinement of the first vintage.

Bacon believed that if his methods were followed scientists would discover the laws that govern the phenomena of nature. Bacon clearly maintained the major tenets of the Aristotelian method with regard to completed science, but he recognized that completed science was a long way off and that, during the interim between incipient science and completed science, the mainstay of scientific activity would be inductive methods.

After the publication of Bacon's work in 1620, it was roughly two hundred years before further work on inductive logic was pursued again in a systematic fashion

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by John Herschell, William Whewell, and John Stuart Mill. During this interim there were developments in the mathematical treatment of chance and statistical approaches to observational evidence that play an important part in the history of uncertain inference.

1.5 The Development of Probability

Attempts to mathematize decisionmaking under uncertainty were not originally undertaken primarily with an eye to clarifying problems within natural science, although recent scholarship suggests that such problems were part of the motivating force. Rather, interest in a systematic understanding of probabilities is traditionally considered to have arisen in the seventeenth century in connection with gambling.

As history preserves the story of the development of the mathematical theory of probability, Antoine Gombaud (1607–1684), who is known as the Chevalier de Mere and who was an author, councillor, and prominent figure in the court of Louis XIV, proposed some questions to Blaise Pascal (1623–1662), the noted and brilliant French mathematician, concerning some problems surrounding games of chance. Pascal, in working out the solutions to these problems, corresponded with Pierre de Fermat (1601–1665), the preeminent mathematician of the time, and the work done by the two of them is usually viewed as the start of the mathematical theory of probability.

The two problems posed to Pascal by the Chevalier were concerned, respectively, with dice and with the division of stakes. These are of sufficient interest to repeat. The dice problem goes as follows: When one throws two dice, how many throws must one be allowed in order to have a better than even chance of getting two sixes at least once? The division problem (also known as the *problem of points*) involves the following question: How shall one divide equitably the prize money in a tournament in case the series, for some reason, is interrupted before it is completed? This problem reduces to the question: What are probabilities for each player to win the prize money, given that each player has an equal chance to win each point?

Dice problems such as the one above were well known by Pascal's time; Geralmo Cardano (1501–1576), around 1525, had discovered and presented in his *De Ludo Aleae* rules for solving the dice problem for one die. Problems similar to the *problem of points* had also been around for some time. An early example of the division problem can be found in Fra Luca Paciuoli's *Summa* (1494), and has been found in Italian manuscripts as early as 1380. Notwithstanding the fact that these problems were current before Pascal and Fermat attended to them, the mathematical techniques, such as the *arithmetical triangle*, marshaled by the two in solving them were novel. However, Pascal's life was a short one (he lived but 39 years), and neither Pascal nor Fermat felt the need to publish their mathematical results. All we possess of Pascal's and Fermat's efforts is found in their few surviving letters to each other.

The importance of Pascal's and Fermat's investigations far outstrips the apparently trivial concerns of dicing, for in their attempts to solve the difficult combinatorial problems posed to them, they developed techniques for analyzing situations in which a number of alternative outcomes are possible and in which knowledge of the probabilities of the various alternatives is important for practical decisions. After the work of Pascal and Fermat little progress was made on the mathematical theory of probability for about 50 years.

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