AN INTRODUCTION TO ECONOPHYSICS
Correlations and Complexity in Finance

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CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge, CB2 2RU, UK http://www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA http://www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain

First published 2000
Reprinted 2000

Printed in the United Kingdom by Biddles Ltd, Guildford & King's Lynn

Typeface Times 11/14pt *System* LaTeX [UPH]

A catalogue record of this book is available from the British Library

Library of Congress Cataloguing in Publication data
Mantegna, Rosario N. (Rosario Nunzio), 1960–
An introduction to econophysics: correlations and complexity in
finance / Rosario N. Mantegna, H. Eugene Stanley.
p. cm.
ISBN 0 521 62008 2 (hardbound)
1. Finance–Statistical methods. 2. Finance–Mathematical models.
3. Statistical physics. I. Stanley, H. Eugene (Harry Eugene),
1941– . II. Title
HG176.5.M365 1999
332′.01′5195–dc21 99-28047 CIP

ISBN 0 521 62008 2 hardback

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Introduction

1.1 Motivation

Since the 1970s, a series of significant changes has taken place in the world of finance. One key year was 1973, when currencies began to be traded in financial markets and their values determined by the foreign exchange market, a financial market active 24 hours a day all over the world. During that same year, Black and Scholes [18] published the first paper that presented a rational option-pricing formula.

Since that time, the volume of foreign exchange trading has been growing at an impressive rate. The transaction volume in 1995 was 80 times what it was in 1973. An even more impressive growth has taken place in the field of derivative products. The total value of financial derivative market contracts issued in 1996 was 35 trillion US dollars. Contracts totaling approximately 25 trillion USD were negotiated in the over-the-counter market (i.e., directly between firms or financial institutions), and the rest (approximately 10 trillion USD) in specialized exchanges that deal only in derivative contracts. Today, financial markets facilitate the trading of huge amounts of money, assets, and goods in a competitive global environment.

A second revolution began in the 1980s when electronic trading, already a part of the environment of the major stock exchanges, was adapted to the foreign exchange market. The electronic storing of data relating to financial contracts – or to prices at which traders are willing to buy (bid quotes) or sell (ask quotes) a financial asset – was put in place at about the same time that electronic trading became widespread. One result is that today a huge amount of electronically stored financial data is readily available. These data are characterized by the property of being high-frequency data – the average time delay between two records can be as short as a few seconds. The enormous expansion of financial markets requires strong investments in money and human resources to achieve reliable quantification and minimization of risk for the financial institutions involved.

1.2 Pioneering approaches

In this book we discuss the application to financial markets of such concepts as power-law distributions, correlations, scaling, unpredictable time series, and random processes. During the past 30 years, physicists have achieved important results in the field of phase transitions, statistical mechanics, nonlinear dynamics, and disordered systems. In these fields, power laws, scaling, and unpredictable (stochastic or deterministic) time series are present and the current interpretation of the underlying physics is often obtained using these concepts.

With this background in mind, it may surprise scholars trained in the natural sciences to learn that the first use of a power-law distribution – and the first mathematical formalization of a random walk – took place in the social sciences. Almost exactly 100 years ago, the Italian social economist Pareto investigated the statistical character of the wealth of individuals in a stable economy by modeling them using the distribution

$$y \sim x^{-\nu}, \tag{1.1}$$

where y is the number of people having income x or greater than x and ν is an exponent that Pareto estimated to be 1.5 [132]. Pareto noticed that his result was quite general and applicable to nations 'as different as those of England, of Ireland, of Germany, of the Italian cities, and even of Peru'.

It should be fully appreciated that the concept of a power-law distribution is counterintuitive, because it may lack any characteristic scale. This property prevented the use of power-law distributions in the natural sciences until the recent emergence of new paradigms (i) in probability theory, thanks to the work of Lévy [92] and thanks to the application of power-law distributions to several problems pursued by Mandelbrot [103]; and (ii) in the study of phase transitions, which introduced the concepts of scaling for thermodynamic functions and correlation functions [147].

Another concept ubiquitous in the natural sciences is the random walk. The first theoretical description of a random walk in the natural sciences was performed in 1905 by Einstein [48] in his famous paper dealing with the determination of the Avogadro number. In subsequent years, the mathematics of the random walk was made more rigorous by Wiener [158], and now the random walk concept has spread across almost all research areas in the natural sciences.

The first formalization of a random walk was not in a publication by Einstein, but in a doctoral thesis by Bachelier [8]. Bachelier, a French mathematician, presented his thesis to the faculty of sciences at the Academy of Paris on 29 March 1900, for the degree of *Docteur en Sciences Mathématiques*. His advisor was Poincaré, one of the greatest mathematicians of his time. The thesis, entitled *Théorie de la spéculation*, is surprising in several respects. It deals with the pricing of options in speculative markets, an activity that today is extremely important in financial markets where derivative securities – those whose value depends on the values of other more basic underlying variables – are regularly traded on many different exchanges. To complete this task, Bachelier determined the probability of price changes by writing down what is now called the Chapman–Kolmogorov equation and recognizing that what is now called a Wiener process satisfies the diffusion equation (this point was rediscovered by Einstein in his 1905 paper on Brownian motion). Retrospectively analyzed, Bachelier's thesis lacks rigor in some of its mathematical and economic points. Specifically, the determination of a Gaussian distribution for the price changes was – mathematically speaking – not sufficiently motivated. On the economic side, Bachelier investigated price changes, whereas economists are mainly dealing with changes in the logarithm of price. However, these limitations do not diminish the value of Bachelier's pioneering work.

To put Bachelier's work into perspective, the Black & Scholes option-pricing model – considered the milestone in option-pricing theory – was published in 1973, almost three-quarters of a century after the publication of his thesis. Moreover, theorists and practitioners are aware that the Black & Scholes model needs correction in its application, meaning that the problem of which stochastic process describes the changes in the logarithm of prices in a financial market is still an open one.

The problem of the distribution of price changes has been considered by several authors since the 1950s, which was the period when mathematicians began to show interest in the modeling of stock market prices. Bachelier's original proposal of Gaussian distributed price changes was soon replaced by a model in which stock prices are log-normal distributed, i.e., stock prices are performing a geometric Brownian motion. In a geometric Brownian motion, the differences of the logarithms of prices are Gaussian distributed. This model is known to provide only a first approximation of what is observed in real data. For this reason, a number of alternative models have been proposed with the aim of explaining

(i) the empirical evidence that the tails of measured distributions are fatter than expected for a geometric Brownian motion; and
(ii) the time fluctuations of the second moment of price changes.

Among the alternative models proposed, 'the most revolutionary development in the theory of speculative prices since Bachelier's initial work' [38], is Mandelbrot's hypothesis that price changes follow a Lévy stable distribution [102]. Lévy stable processes are stochastic processes obeying a generalized central limit theorem. By obeying a generalized form of the central limit theorem, they have a number of interesting properties. They are stable (as are the more common Gaussian processes) – i.e., the sum of two independent stochastic processes x_1 and x_2 characterized by the same Lévy distribution of index α is itself a stochastic process characterized by a Lévy distribution of the same index. The shape of the distribution is maintained (is stable) by summing up independent identically distributed Lévy stable random variables.

As we shall see, Lévy stable processes define a basin of attraction in the functional space of probability density functions. The sum of independent identically distributed stochastic processes $S_n \equiv \sum_{i=1}^{n} x_i$ characterized by a probability density function with power-law tails,

$$P(x) \sim x^{-(1+\alpha)}, \tag{1.2}$$

will converge, in probability, to a Lévy stable stochastic process of index α when n tends to infinity [66].

This property tells us that the distribution of a Lévy stable process is a power-law distribution for large values of the stochastic variable x. The fact that power-law distributions may lack a typical scale is reflected in Lévy stable processes by the property that the variance of Lévy stable processes is infinite for $\alpha < 2$. Stochastic processes with infinite variance, although well defined mathematically, are extremely difficult to use and, moreover, raise fundamental questions when applied to real systems. For example, in physical systems the second moment is often related to the system temperature, so infinite variances imply an infinite (or undefined) temperature. In financial systems, an infinite variance would complicate the important task of risk estimation.

1.3 The chaos approach

A widely accepted belief in financial theory is that time series of asset prices are unpredictable. This belief is the cornerstone of the description of price dynamics as stochastic processes. Since the 1980s it has been recognized in the physical sciences that unpredictable time series and stochastic processes are not synonymous. Specifically, chaos theory has shown that unpredictable time series can arise from deterministic nonlinear systems. The results obtained in the study of physical and biological systems triggered an interest in economic systems, and theoretical and empirical studies have investigated whether the time evolution of asset prices in financial markets might indeed be due to underlying nonlinear deterministic dynamics of a (limited) number of variables.

One of the goals of researchers studying financial markets with the tools of nonlinear dynamics has been to reconstruct the (hypothetical) strange attractor present in the chaotic time evolution and to measure its dimension d. The reconstruction of the underlying attractor and its dimension d is not an easy task. The more reliable estimation of d is the inequality $d > 6$. For chaotic systems with $d > 3$, it is rather difficult to distinguish between a chaotic time evolution and a random process, especially if the underlying deterministic dynamics are unknown. Hence, from an empirical point of view, it is quite unlikely that it will be possible to discriminate between the random and the chaotic hypotheses.

Although it cannot be ruled out that financial markets follow chaotic dynamics, we choose to work within a paradigm that asserts price dynamics are stochastic processes. Our choice is motivated by the observation that the time evolution of an asset price depends on all the information affecting (or believed to be affecting) the investigated asset and it seems unlikely to us that all this information can be essentially described by a small number of nonlinear deterministic equations.

1.4 The present focus

Financial markets exhibit several of the properties that characterize complex systems. They are open systems in which many subunits interact nonlinearly in the presence of feedback. In financial markets, the governing rules are rather stable and the time evolution of the system is continuously monitored. It is now possible to develop models and to test their accuracy and predictive power using available data, since large databases exist even for high-frequency data.

One of the more active areas in finance is the pricing of derivative instruments. In the simplest case, an asset is described by a stochastic process and a derivative security (or contingent claim) is evaluated on the basis of the type of security and the value and statistical properties of the underlying asset. This problem presents at least two different aspects: (i) 'fundamental' aspects, which are related to the nature of the random process of the asset, and (ii) 'applied' or 'technical' aspects, which are related to the solution of the option-pricing problem under the assumption that the underlying asset performs the proposed random process.

Recently, a growing number of physicists have attempted to analyze and model financial markets and, more generally, economic systems. The interest of this community in financial and economic systems has roots that date back to 1936, when Majorana wrote a pioneering paper on the essential analogy between statistical laws in physics and in the social sciences [101]. This unorthodox point of view was considered of marginal interest until recently. Indeed, prior to the 1990s, very few professional physicists did any research associated with social or economic systems. The exceptions included Kadanoff [76], Montroll [125], and a group of physical scientists at the Santa Fe Institute [5].

Since 1990, the physics research activity in this field has become less episodic and a research community has begun to emerge. New interdisciplinary journals have been published, conferences have been organized, and a set of potentially tractable scientific problems has been provisionally identified. The research activity of this group of physicists is complementary to the most traditional approaches of finance and mathematical finance. One characteristic difference is the emphasis that physicists put on the empirical analysis of economic data. Another is the background of theory and method in the field of statistical physics developed over the past 30 years that physicists bring to the subject. The concepts of scaling, universality, disordered frustrated systems, and self-organized systems might be helpful in the analysis and modeling of financial and economic systems. One argument that is sometimes raised at this point is that an empirical analysis performed on financial or economic data is not equivalent to the usual experimental investigation that takes place in physical sciences. In other words, it is impossible to perform large-scale experiments in economics and finance that could falsify any given theory.

We note that this limitation is not specific to economic and financial systems, but also affects such well developed areas of physics as astrophysics, atmospheric physics, and geophysics. Hence, in analogy to activity in these more established areas, we find that we are able to test and falsify any theories associated with the currently available sets of financial and economic data provided in the form of recorded files of financial and economic activity.

Among the important areas of physics research dealing with financial and economic systems, one concerns the complete statistical characterization of the stochastic process of price changes of a financial asset. Several studies have been performed that focus on different aspects of the analyzed stochastic process, e.g., the shape of the distribution of price changes [22, 64, 67, 105, 111, 135], the temporal memory [35, 93, 95, 112], and the higher-order statistical properties [6, 31, 126]. This is still an active area, and attempts are ongoing to develop the most satisfactory stochastic model describing all the features encountered in empirical analyses. One important accomplishment in this area is an almost complete consensus concerning the finiteness of the second moment of price changes. This has been a longstanding problem in finance, and its resolution has come about because of the renewed interest in the empirical study of financial systems.

A second area concerns the development of a theoretical model that is able to encompass all the essential features of real financial markets. Several models have been proposed [10, 11, 23, 25, 29, 90, 91, 104, 117, 142, 146, 149–152], and some of the main properties of the stochastic dynamics of stock price are reproduced by these models as, for example, the leptokurtic 'fat-tailed' non-Gaussian shape of the distribution of price differences. Parallel attempts in the modeling of financial markets have been developed by economists [98–100].

Other areas that are undergoing intense investigations deal with the rational pricing of a derivative product when some of the canonical assumptions of the Black & Scholes model are relaxed [7, 21, 22] and with aspects of portfolio selection and its dynamical optimization [14, 62, 63, 116, 145]. A further area of research considers analogies and differences between price dynamics in a financial market and such physical processes as turbulence [64, 112, 113] and ecological systems [55, 135].

One common theme encountered in these research areas is the time correlation of a financial series. The detection of the presence of a higher-order correlation in price changes has motivated a reconsideration of some beliefs of what is termed 'technical analysis' [155].

In addition to the studies that analyze and model financial systems, there are studies of the income distribution of firms and studies of the statistical properties of their growth rates [2, 3, 148, 153]. The statistical properties of the economic performances of complex organizations such as universities or entire countries have also been investigated [89].

This brief presentation of some of the current efforts in this emerging discipline has only illustrative purposes and cannot be exhaustive. For a more complete overview, consider, for example, the proceedings of conferences dedicated to these topics [78, 88, 109].