Index

A-Gr, category of graded left A-modules, 31
Aβ, homogeneous elements of A, 6
а-shift functor, 31
ann(m), annihilator ideal, 47
AΩ, a Ω-graded subring of A, 10
C(a), centre of A, 17, 23, 186
Cε, dimension, 85, 90
deg(a), degree of the homogeneous element a, 6
eij, matrix unit, 30, 54
Filt-A, category of filtered right A-modules, 49
γ-component of a graded ring, 6
G(gr), the cone of the group G, 149, 150
Gr-A, category of graded finitely generated right A-modules, 31
Gr-IBN, graded invariant basis number, 126
Gr-Gr, the category of graded A-bimodules, 34
Gr-Gr-Gr, 188
Gr-A, category of graded right A-modules, 31
GrA category of Γ-graded right A-modules, 1, 31
GrΩ-A, Ω a subgroup of Γ, 222
HH°(A), the Hochschild homology of A, 139
IBN, invariant basis number, 94, 126
J(A), Jacobson radical, 23
Jgr(A), graded Jacobson radical, 23
K0r(A), graded Grothendieck group, 126
K1r(A), the positive cone of K0r, 151
K1r(A), reduced graded Grothendieck group, 130
K0r(A), 177
K0, K1 of an exact category, 177
KΓ-functors, 214
L(F), Leavitt path algebra, 85
M̅0(A), 133
Mod-A, category of right A-modules, 4
mod-A, category of finitely generated right A-modules, 4
PGr-A, category of graded finitely generated projective A-modules, 41, 128, 177, 205
Pic(A), Picard group of A, 180
Picent(A), Picard group Picη(A), 187
Picη, graded Picard group, 181, 185
Pr-A, category of finitely generated projective A-modules, 4, 128, 214
QCoh-P, 5
QGr-A, 5
Γr category of Γ-graded rings, 10
R(X), free ring, 79
S∞, infinite symmetric group, 192
Sn, symmetric group on n objects, 169
S(M), symmetric algebra of the module M, 20
srη, graded stable rank, 97
η-field, 102, 141
η-proper ring, 102
η-ring, 100, 141
T(M), tensor algebra of the module M, 8, 14, 20
Vη, monoid of graded finitely generated projective modules, 123
V+, group completion, 124
∧ M, exterior algebra of the module M, 20, 182
Index

$\wedge^n M$, $n$th exterior algebra of the module $M$, 182
0-component ring of a graded ring, 7, 54, 128, 164, 167
acyclic graph, 85, 89
additive commutator subgroup, 140
additive functor, 105
adjacency matrix, 85, 161
adjoint functor, 36, 122
AF-algebras, 192
annihilator ideal, 47
Artinian ring, 20
associated graded functor, 114
associated graded ring, 71, 154, 189
Azumaya algebra, 187
base change functor, 215
basic set of graded simple modules, 170
block functor, 10, 37
centraliser, 21
centre of a ring, 21, 186
Chern map, 140
Clifford algebra, 21
closed path, 85
coarsening functor, 10, 37
cocycle map, 16
comet graph, 85, 90
complete set of coset representative, 38, 71, 152
component of a graded ring, 6
composition series, 169
cone of ordering, 149, 150
conjugate of subshifts, 172
contractive map, 204
corner isomorphism, 81
corner skew Laurent polynomial ring, 81, 102
crossed product ring, 14, 16, 62, 83, 88, 129
cycle in a graph, 85
Dade’s theorem, 73, 119, 128, 180
Dedekind finite ring, 83
dimension group, 150, 192
dimension module, 200
dimension of module, 66
directed graph, 84
directly finite, 83
dual module, 32
degree of a graph, 84
deedge shift, 172
elementary grading, 57
elementary shift equivalent, 172
enlargement of a homogeneous idempotent, 134
essential graph, 85
exact category, 177
exact functor, 214
exterior algebra, 20
exterior power, 182
filtered module, 49
filtered ring, 48
finite graph, 84
flat module, 26
forgetful functor, 10, 36
formal matrix ring, 8, 112
formal triangular matrix ring, 8
free ring, 79
Frobenius algebra, 8
full homogeneous idempotent, 114
full idempotent, 82
full set of matrix units, 59
full shift space, 172
fully graded ring, 79
generalised crossed product, 79
ghost edge, 87
global dimension, 216
good grading, 54, 57
gr-IBN, 94, 190
graded $\ast$-algebra, 101
graded $\ast$-field, 101
graded $\ast$-proper ring, 102
graded $\ast$-ring, 100, 141
graded $R$-algebra, 6
graded algebra, 6
graded Artin--Wedderburn Theorem, 72
graded Artinian ring, 20, 72, 170
graded Azumaya algebra, 187
graded bimodule, 34
graded central simple algebra, 188
graded composition factors, 169
graded composition series, 169
graded conjugate, 134
graded dimension group, 163
graded dimension module, 151
graded divisible module, 47
graded division algebra, 68, 188
graded division ring, 17, 64, 152
graded double ring, 142
graded equivalence, 114, 115
graded equivalent idempotents, 133
graded field, 17
graded flat module, 26
graded formal matrix ring, 35, 112
graded free module, 33
graded functor, 114
graded generator, 109
graded global dimension, 216
graded Grothendieck group, 123, 126
graded homomorphism of degree \(\delta\), 30
graded ideal, 20
graded injective module, 46
graded invariant basis number, 94
graded invertible bimodule, 185
graded invertible module, 181
graded Jacobson radical, 22, 27, 158
graded Jordan–Hölder theorem, 169
graded left ideal, 20
graded left module, 34
graded local ring, 25, 158
graded matricial algebra, 193
graded matrix ring, 49, 89, 105, 193
graded matrix units, 59
graded maximal ideal, 22
graded module, 33
graded module homomorphism, 28
graded module isomorphism, 28
graded Morita equivalent, 114, 185, 194, 203
graded Nakayama lemma, 158
graded Noetherian ring, 2, 20, 75, 222
graded Picard group, 181, 185
graded prime ideal, 22
graded prime ring, 22
graded primitive, 72
graded progenerator module, 111
graded projective dimension, 216
graded projective module, 41
graded proper, 102
graded regular ring, 216
graded right ideal, 20
graded right module, 28
graded right regular ring, 216
graded ring, 6
graded ring homomorphism, 6
graded ring isomorphism, 6
graded ring with involution, 100
graded semiprime ideal, 22, 27
graded semiprime ring, 22
graded semisimple ring, 72
graded simple ring, 23, 66, 71, 75, 188
graded stable rank, 97
graded stably isomorphic, 128
graded submodule, 29
graded subring, 20
graded trace ideal, 109
graded type of a ring, 95
graded ultramatricial algebra, 198
graded unimodular row, 97
graded unit regular, 89
graded von Neumann regular ring, 26, 48, 100
graded von Neumann unit regular, 89
grading concentrated in degree zero, 6
graph, 84
Grothendieck group, 123, 124
group completion, 124
group ring, 7, 17, 101
groupoid, 13
groupoid grading, 13, 113
Hattori–Stallings trace map, 140
henselian, 71, 189
Hermitian transpose, 101
Hochschild homology, 140
homogeneous element, 6
homogeneous ideal, 20
homogeneous idempotent, 58
homogeneous of degree \(\delta\), 6
Hopf algebra, 122
hyper-central group, 24
IBM, 94, 126
IBN in category, 95
idempotent matrix, 133
in-splitting of a graph, 174
induction functor, 73
invariant basis number, 94
invariant basis property, 94
invertible module, 78, 180
invertible graded ring, 100, 141
irreducible graph, 85
irreducible subshift, 172
Jacobson radical, 23
Jordan–Hölder theorem, 169
Krieger’s dimension group, 175
Leavitt path algebra, 86, 160, 176
length of a path, 85
length of an element, 191
local ring, 25
locally semisimple algebra, 192
loop edge, 84
LS-algebras, 192
matrix units, 30, 54, 57, 107
minimal support with respect to an ideal, 23
monomials, 87
Morita context, 104
Morita ring, 112
multiplicative commutator subgroup, 179
Noetherian ring, 20, 75, 94, 217
Index

order preserving homomorphism, 150, 194, 200
order-unit, 149, 150
out-splitting of a graph, 173
path algebra, 86, 207
permutation group, 169, 192
permutation matrix, 53
Picard group, 180, 191
positive cone of ordering, 149, 150, 194
positive-definite, 102
positively graded ring, 7, 206
pre-ordered abelian group, 149
pre-ordered module, 150
progenerator module, 111
projection, 102
quaternion algebra, 18
quotient category, 2, 194, 223
range of an edge, 85
reduced graded Grothendieck group, 131
reduced Grothendieck group, 130
relative graded Grothendieck group, 143
restriction functor, 73
row-finite graph, 84
Serre subcategory, 2, 215, 216, 222
shift equivalence, 173
shift functor, 31, 177
shift map, 172
shift of finite type, 172
shifted, 28, 108
simple ring, 23, 75, 191
sink vertex, 84
skew group ring, 17
skew Laurent polynomial ring, 17
smash product, 121
source of an edge, 85
source vertex, 84
stable rank, 96
stable unimodular row, 97
strong shift equivalence, 172
strONGLy graded Leavitt path algebra, 88, 94
strongly graded module, 75
strongly graded ring, 14, 72, 128, 189
subshift, 172
subshift of finite type associated with a graph, 172
support of a graded ring, 6
support of an element, 23
suspended module, 28
suspension functor, 31
symbol algebra, 19
symmetric algebra, 20
symmetric group, 169
tame valuation, 71
tensor algebra, 8
tensor product of graded modules, 35
tensor product of graded rings, 10
topological Markov chain, 172
topologically conjugate of subshifts, 172
totally ordered abelian group, 18, 70
totally ramified graded division ring, 69
trace map, 139
transfer function, 215
trivial extension, 9
trivial grading, 6, 40, 127
trivial path, 85
twisted group ring, 17
type of a ring, 94, 126
ultramarticial algebra, 88, 165, 192
unitisation ring, 145
unramified graded division ring, 69
valuation function, 70
Veronese module, 33, 101, 141
Veronese subring, 18, 101, 141
vertex of a graph, 84
von Neumann regular ring, 26, 77, 83, 89
weight of an edge, 93
Weyl algebra, 80, 168