AN ELEMENTARY TREATISE ON ACTUARIAL MATHEMATICS

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by

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INTRODUCTION

WHILE to a mathematician the actual knowledge of pure mathematics required for a student to pass the Examinations of the Institute of Actuaries may not be large, to the average candidate who is breaking fresh ground the amount appears to be heavy, and when that knowledge has to be gained by reading parts only of standard works on mathematics his difficulties are multiplied.

It was for this reason that the Council invited Alfred Henry to produce his book on *Calculus and Probability*. That book is now out of print and Henry is no longer with us. Had he been alive he would have been the first to acknowledge that his book did not cover all the ground that is now required.

The Council have thought it to be very desirable that a new Text Book of a more comprehensive design should be published for the benefit of our students, and in looking round for a man of sufficient experience to undertake such difficult and laborious work, they were happy in giving a unanimous invitation to Mr Freeman.

This book is the result and it will, I am confident, be found to be a comprehensive work, at any rate so far as the student is concerned, on Finite Differences, Summation, Differential and Integral Calculus and Probability.

Mr Freeman has, I am glad to see, commenced with a chapter on Elementary Trigonometry, a knowledge of which, as he states himself, is essential to a proper understanding of the Differential and Integral Calculus.

The whole work should prove to be a Text Book of great value to actuarial students.

H. M. T.

AUTHOR'S PREFACE

It has been my experience in dealing with the subjects of the syllabus for Part I of the Examinations of the Institute of Actuaries that it is essential to interest the student in the subject and then, when his interest has been aroused, to supply him with numerous examples illustrative of the principles that have been expounded. This is of particular importance when treating of Finite Differences and Probability, which are almost invariably new to the student. In pursuance of the first of these aims, I have had no compunction when occasion has arisen in wandering from the strict confines of the examination syllabus. For example, a little information on modern symbolic notation, or a paragraph on osculatory interpolation, while not essential to the immediate needs of the Part I student, may however prove of interest to him and, it is hoped, may stimulate him to further researches. Again, in accordance with my belief for the necessity for examples, I have included in the text many varied types of question and have set numerous examples for solution. The student should find considerable scope for his industry and ingenuity in working the examples set after each chapter and at the end of the book.

I have been fortunate in the assistance rendered me by my friends. In the first place, my sincerest thanks are due to Mr D. C. Fraser and Mr G. Green, who read through the book in manuscript and who were always willing to confer with me at any time and on any subject connected with the book. Then again, my colleagues on the panel of Tutors, Mr C. D. Rich and Mr C. W. Sanger, have rendered me much help at various stages of the work, for which I am very grateful. Finally I am deeply indebted to Mr S. H. Alison and Mr G. J. Lidstone for their expert advice. Mr Alison's suggestions on the chapters dealing with Limits and the elements of the Calculus were particularly valuable. Mr Lidstone not only read the whole of the proofs, but put at my disposal his unique knowledge of actuarial mathematics. It would not be too much to say that certain of the chapters on Finite Differences would have presented a comparatively meagre appearance had it not been for the inspiration afforded by the published writings and the helpful suggestions of Mr Fraser and Mr Lidstone.

H. F.

December, 1930

NOTE ON SYMBOLS

The following symbols and abbreviations have been used in the text:

$n!$ "factorial n." $n! = n(n-1)(n-2)\ldots 3.2.1$, where n is a positive integer.

n_r $\dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots 3.2.1}$, where n may be positive or negative, integral or fractional, and r is a positive integer.

$\equiv$ "is equivalent to." Thus $E \equiv 1 + \Delta$ means that the operation E is equivalent to the operations $1 + \Delta$.

$\rightarrow$ "tends to the value." For example, $\underset{n\to\infty}{\text{Lt}}\left(1+\frac{1}{n}\right)^n$ is an abbreviated form of "the limit of $\left(1+\frac{1}{n}\right)^n$ as n tends to infinity."

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J.S.S. *Journal of the Institute of Actuaries Students' Society.*