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Preface

This book follows on from the book *Creative Mathematics* in this series which began with three essays (on research into, on writing about and on presenting mathematics) and then continued with a series of problems, each of which was divided into three parts. Part I provided an introduction to the problem followed by some elementary questions about the problem. Part II contained an answer to these questions, as well as a deeper discussion and a generalisation of the problem. This led to more advanced questions which were discussed in Part III.

This book is a natural development of this approach, the main purpose of which is to give the reader experience in working on (as far as the reader is concerned) unsolved problems. The problems in this book are, generally speaking, more difficult than those in *Creative Mathematics*, and we assume a greater level of maturity of the reader.

One of the main purposes of this book is to show that mathematical problems are often solved using mathematics that is not, at first sight, connected to the problem, and readers are encouraged (and even urged) to consider as wide a variety of mathematical ideas as possible when trying to solve a problem. The reader will no doubt have met problems in what might be called 'recreational mathematics' where problems are solved for amusement, without necessarily understanding or investigating the key mathematical ideas that lie behind the solution. Here, by contrast, we focus on the important underlying ideas rather than on the solution itself.

How to use this book

This book is written to help the reader learn how to do research in mathematics. Each chapter contains a project that has been chosen not because of its mathematical importance but because (in the view of the author) it provides a good illustration of how arguments develop, and how new questions arise once some progress is made. These projects have also been chosen because they do not require a deep mathematical background in order to understand the problem and start investigation. Nevertheless, the reader will probably have to learn some more mathematics in order to solve the problems. Some of the problems do not have easy answers, and some are not yet completely solved.

Each chapter focuses on one topic, and although some results and proofs are given in the discussion, many steps are omitted, and it is the responsibility of the reader to locate and fill these gaps. The general rule is that the reader should check every step and provide as much extra material as is necessary to ensure their complete understanding of each step. As we progress through a project, specific questions are asked, and the reader will need to interpret, or clarify, some of these before a solution is attempted. It hardly needs saying that the whole purpose of the book is that the reader should fully engage with these problems and fill in the (many) missing steps in the text itself. Although some theorems, and their proofs, are given, these do not have quite the same role as in most textbooks. The theorems given here serve the purpose of making further progress *in order that we can ask yet more questions*, for this is the real nature of research.