

STATISTICS





STATISTICS

SECOND EDITION OF 'A SECOND COURSE IN STATISTICS'

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CAMBRIDGE
AT THE UNIVERSITY PRESS
1971



CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org Information on this title: www.cambridge.org/9781316606940

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The edition © Cambridge University Press 1969

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First edition 1961 Reprinted 1965, 1966 Second edition [metric] 1969 Reprinted 1971 First paperback edition 2016

A catalogue record for this publication is available from the British Library

ISBN 978-1-316-60694-0 Paperback

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PREFACE

The object of this volume is to establish firmly the bridge between the elementary treatment given in *A First Course in Statistics* and the rigorous treatment given in more advanced University courses.

The keyword of A First Course in Statistics is observation. The uninitiated student is unable to extract ideas from Statistics until he has learnt the common methods of classifying and representing data. He has to learn what meaning can be attached to the terms commonly used in Statistical Analysis. In a first course the idea of significance should be avoided and all differences at this stage should be absolutely blatant.

In this course, however, the important idea is *probability*. At this stage the student must be able to decide how much *confidence* he can place in his results; whether the small differences he observes are *significant* or not.

The opening chapter on location and dispersion makes it possible to begin this course without having previously read the *First Course*. In other words, this volume is complete in itself. The normal distribution is introduced in chapter 2 because experience shows that, immediately after they have become acquainted with *frequency distributions*, students feel an urgent need to know something about the *mathematical model* which fits so many of them so well. The brief treatment of probability in chapter 3, which includes Bayes' theorem, is sufficient to lead naturally to probability distributions in general and to the binomial and Poisson distributions in particular.

Chapters 7, 8 and 9 deal with the use of χ^2 and t-tables for testing significance and a short account of Quality Control follows in chapter 10. A treatment of regression by the method of least squares and of correlation coefficients (including dichotomy, Spearman's ρ and Kendall's τ) is given in chapters 10–13 and the final chapter covers the use of F-tables for the analysis of variance.

The glossary at the end of the book is of interest in that it not only summarises the terms and formulae introduced in the earlier part of the text but it also extends them. Thus, by reference to the glossary, a forward look can be taken to ideas left open for exploration in a more advanced treatise.

1 July 1969 R.L.

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