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LINEARIZED THEORY OF STEADY HIGH-SPEED FLOW

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TO MY FRIENDS

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PREFACE

The difficulty of solving the non-linear equations of motion of a compressible fluid has led to the extensive use of linear approximations to these equations in applications to aeronautics. The solutions of these linearized equations are subject to somewhat severe limitations, and a knowledge of the nature of the physical and mathematical approximations made in their derivation is helpful in assessing their applicability to particular problems. In the first part of this monograph, I have tried to set down a logical development of the theory for steady flows, giving due attention to the assumptions on which the theory is based. Perhaps an ideal course would have been to give an account of such more accurate theory as exists, and then to compare the linear approximations with it at every stage, but this would have greatly increased the size of the volume. Instead, I have quoted briefly the results of more exact theory wherever it has seemed necessary, and assumed the rest to be known or obtainable from other more complete works on the subject. Thus a knowledge of at least the elements of the theory and practice of compressible fluid flow is required from the reader. The second part deals with applications to special problems, and is mostly concerned with supersonic flows. The third part contains an account of slender-body theory, which in some ways is distinct from ordinary linearized theory and requires a separate derivation, since it is generally more accurate on and near the body and less accurate at great distances.

In developing the general theory, I have made extensive use of vectorial notation, which not only leads to a concise exposition, but, even more importantly in my opinion, helps to reveal the physical meaning of the equations. I am well aware that this will be a slight hindrance to some readers, but the notation is becoming more and more widely used, and I feel sure that the comparatively small amount of time required to learn it will be amply repaid in many other contexts. In this notation the linearized equations of

motion are first-order differential equations, each of which has a simple physical interpretation, and which are shown to be integrable directly without introducing auxiliary potential functions even if they exist. The possibility thus opened up, namely to work entirely in terms of the particle velocity, appeals to me as giving a more physically satisfying approach to the subject than to work in terms of potentials. It turns out that this approach has many advantages over the more conventional treatment, and actually simplifies the analysis in many cases. Nevertheless, in order not to depart too far from standard treatments, I have exploited this possibility only in some parts of the development of the general theory, and in the chapter on conical fields. Similar considerations apply to the introduction of sources and sinks, which can have no physical reality and moreover in supersonic flow have a non-existent (infinite) mass flux unless some artifice such as that of taking the finite part of a divergent integral is employed. But the concept of sources and sinks can be a useful aid to thought, and there seems to be no point in omitting mention of these and other singularities.

As readers will rapidly become aware, the monograph shows a strong personal bias in the selection of material. This is mainly due to the fact that the monograph is a slightly revised and shortened version of an essay submitted in competition for the Adams Prize for 1949–50 in the University of Cambridge. My original intention was to expand the essay into a full-length treatise, but on consideration of the rapid rate at which the subject is developing, I have come to the conclusion that a greatly extended permanent account is not desirable at the present time. This has meant that some interesting approaches to specific problems are only mentioned briefly or are not mentioned at all, and I can only hope that this monograph with all its limitations will stimulate readers to refer to the original papers listed in the Bibliography.

I am very pleased to be able to take this opportunity of acknowledging my indebtedness to Professor S. Goldstein, Professor

M. J. Lighthill and my many other friends in the Department of Mathematics at Manchester University for the many very happy years that I spent working with them, and for all the assistance that they gave me so generously, without which it would not have been possible for me to write this monograph. My thanks are also due to Mrs D. Fahy for her careful typing of my manuscript, and to my wife for her assistance, particularly during proof-reading.

G. N. W.

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