

**Steve Dobbs, Jane Miller and Julian Gilbey**

Cambridge International  
AS and A Level Mathematics:  
**Statistics 2**

Coursebook

Revised Edition

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# Introduction

*Cambridge International Examinations Cambridge Advanced Level Mathematics* has been written especially for the 2002 Cambridge International Examinations syllabus. There is one book corresponding to each syllabus unit, except that units P2 and P3 are contained in a single book. This book is the second Probability and Statistics unit, S2.

The syllabus content is arranged by chapters which are ordered so as to provide a viable teaching course. A few sections include important results that are difficult to prove or outside the syllabus. These sections are marked with a vertical coloured bar in the section heading, and there is usually a sentence early on explaining precisely what it is that the student needs to know.

Some paragraphs within the text appear in *this type style*. These paragraphs are usually outside the main stream of the mathematical argument, but may help to give insight, or suggest extra work or different approaches.

Graphic calculators are not permitted in the examination, but they can be useful aids in learning mathematics. In the book the authors have noted where access to graphic calculators would be especially helpful but have not assumed that they are available to all students.

The authors have assumed that students have access to calculators with built-in statistical functions.

Numerical work is presented in a form intended to discourage premature approximation. In ongoing calculations inexact numbers appear in decimal form like 3.456..., signifying that the number is held in a calculator to more places than are given. Numbers are not rounded at this stage; the full display could be either 3.456 123 or 3.456 789. Final answers are then stated with some indication that they are approximate, for example '1.23 correct to 3 significant figures'.

Most chapters contain Practical activities. These can be used either as an introduction to a topic, or, later on, to reinforce the theory. Two Practical activities, in Sections 4.5 and 5.4, require access to a computer.

There are also plenty of exercises, and each chapter ends with a Miscellaneous exercise which includes some questions of examination standard. There is a Revision exercise, and two Practice exam-style papers. In some exercises a few of the later questions may go beyond the likely requirements of the examination, either in difficulty or in length or both. Some questions are marked with a vertical coloured bar, which indicates that they require knowledge of results outside the syllabus.

Cambridge University Press would like to thank Cambridge International Examinations for permission to use past paper examination questions.

The authors thank Cambridge International Examinations and Cambridge University Press for their help in producing this book. However, the responsibility for the text, and for any errors, remains with the authors.

Steve Dobbs and Jane Miller, 2002

## Introduction to the revised edition

This revised edition has been prepared to bring this textbook in line with the current version of the Cambridge International Examinations specification. Most of the original edition has been left unchanged to assist teachers familiar with the original edition; this includes section numbers, question numbers and so on. A new starred section has been added at the end of Chapter 6 (Hypothesis testing) with a brief discussion of two different schools of thought on statistical inference.

The other major change in this edition is the replacement of all of the older OCR examination questions in the exercises by more recent Cambridge International Examinations questions. This will be of benefit to students preparing for the current style of examination questions. In order to maintain the numbering of the other questions, the newer questions have been slotted in to the exercises. While this has inevitably meant some loss of order within the miscellaneous exercises, this was felt to be more than compensated by the preservation of the original numbering. In a couple of cases, an insufficient number of past paper questions were available to replace the existing questions, and so the exercises have been slightly shortened. In Chapter 8, there are many recent examination questions and so the miscellaneous exercise has been extended to include illustrative examples. Further past papers can, of course, be requested from Cambridge International Examinations. In addition to this, there were also six examples within the text based on such questions; these have been replaced by new examples. (Specifically, they are Examples 4.2.1, 5.6.1, 6.4.1, 6.5.2, 6.6.1 and 8.2.3.)

All questions and answers taken from Cambridge International Examinations past papers have been clearly referenced. All other questions and answers have been written by the authors of this book.

The editor of this edition thanks Cambridge International Examinations and Cambridge University Press, in particular Cathryn Freear and Andrew Briggs, for their great help in preparing this revised edition.

Julian Gilbey

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