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Douglas Quadling , Julian Gilbey
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Chapter 1
The motion of projectiles

In this chapter the model of free motion under gravity is extended to objects projected at an angle. When you have completed it, you should

- understand displacement, velocity and acceleration as vector quantities
- be able to interpret the motion as a combination of the effects of the initial velocity and of gravity
- know that this implies the independence of horizontal and vertical motion
- be able to use equations of horizontal and vertical motion in calculations about the trajectory of a projectile
- know and be able to obtain general formulae for the greatest height, time of flight, range on horizontal ground and the equation of the trajectory
- be able to use your knowledge of trigonometry in solving problems.

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Any object moving through the air will experience air resistance, and this is usually significant for objects moving at high speeds through large distances. The answers obtained in this chapter, which assume that air resistance is small and can be neglected, are therefore only approximate.

1.1 Velocity as a vector

When an object is thrown vertically upwards with initial velocity u, its displacement s after time t is given by the equation

$$s = ut - \tfrac{1}{2}gt^2,$$

where g is the acceleration due to gravity.

One way to interpret this equation is to look at the two terms on the right separately. The first term, ut, would be the displacement if the object moved with constant velocity u, that is if there were no gravity. To this is added a term $\tfrac{1}{2}(-g)t^2$, which would be the displacement of the object in time t if it were released from rest under gravity.

You can look at the equation

$$v = u - gt$$

in a similar way. Without gravity, the velocity would continue to have the constant value u indefinitely. To this is added a term $(-g)t$, which is the velocity that the object would acquire in time t if it were released from rest.

Now suppose that the object is thrown at an angle, so that it follows a curved path through the air. To describe this you can use the vector notation which you have already used (in M1 Chapter 10) for force. The symbol **u** written in bold stands for the velocity with which the object is thrown, that is a speed of magnitude u in a given direction. If there were no gravity, then in time t the object would have a displacement of magnitude ut in that direction. It is natural to denote this by $\mathbf{u}t$, which is a vector displacement. To this is added a vertical displacement of magnitude $\tfrac{1}{2}\mathbf{g}t^2$ vertically downwards. In vector notation this can be written as $\tfrac{1}{2}gt^2$, where the symbol **g** stands for an acceleration of magnitude g in a direction vertically downwards.

To make an equation for this, let **r** denote the displacement of the object from its initial position at time $t = 0$. Then, assuming that air resistance can be neglected,

$$\mathbf{r} = \mathbf{u}t + \tfrac{1}{2}\mathbf{g}t^2.$$

In this equation the symbol + stands for vector addition, which is carried out by the triangle rule, the same rule that you use to add forces.

EXAMPLE 1.1.1

A ball is thrown in the air with speed $12\,\text{m}\,\text{s}^{-1}$ at an angle of $70°$ to the horizontal. Draw a diagram to show where it is 1.5 seconds later.

If there were no gravity, in 1.5 seconds the ball would have a displacement of magnitude 12×1.5, that is 18 m, at $70°$ to the horizontal. This is represented by the arrow $\overrightarrow{OA}$ in Fig. 1.1, on a scale of 1 cm to 5 m. To this must be added a

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displacement of magnitude $\frac{1}{2} \times 10 \times 1.5^2$ m, that is 11.25 m, vertically downwards, represented by the arrow $\overrightarrow{AB}$. The sum of these is the displacement $\overrightarrow{OB}$. So after 1.5 seconds the ball is at B. You could if you wish calculate the coordinates of B, or the distance OB, but in this example these are not asked for.

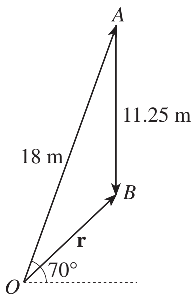

Fig. 1.1

EXAMPLE 1.1.2

A stone is thrown from the edge of a cliff with speed 18 m s^{-1}. Draw diagrams to show the path of the stone in the next 4 seconds if it is thrown

a horizontally,

b at 30° to the horizontal.

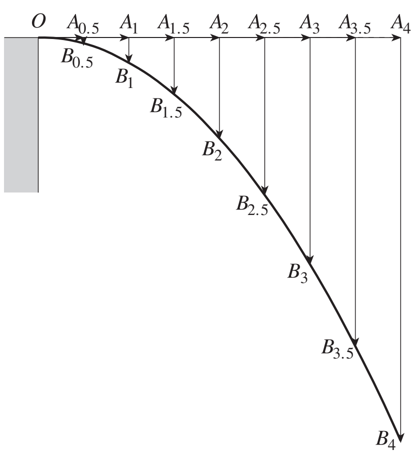

Fig. 1.2

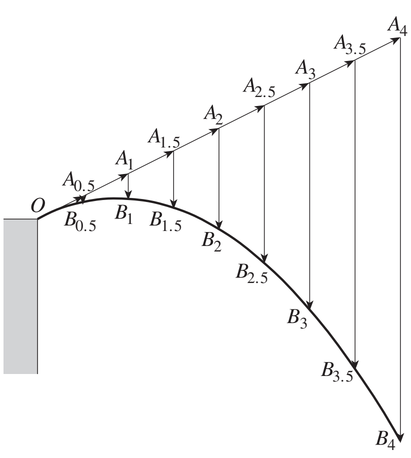

Fig. 1.3

These diagrams were produced by superimposing several diagrams like Fig. 1.1. In Figs. 1.2 and 1.3 (for parts **a** and **b** respectively) this has been done at intervals of 0.5 s, that is for $t = 0.5, 1, 1.5, \ldots, 4$. The displacements $\mathbf{u}t$ in these times have magnitudes 9 m, 18 m, … , 72 m. The vertical displacements have magnitudes 1.25 m, 5 m, 11.25 m, …, 80 m. The points corresponding to A and B at time t are denoted by A_t and B_t.

You can now show the paths by drawing smooth curves through the points O, $B_{0.5}$, B_1, …, B_4 for the two initial velocities.

The word **projectile** is often used to describe an object thrown in this way. The path of a projectile is called its **trajectory**.

A vector triangle can also be used to find the velocity of a projectile at a given time. If there were no gravity the velocity would have the constant value $\mathbf{u}$ indefinitely. The effect of gravity is to add to this a velocity of magnitude gt vertically downwards, which can be written as the vector $\mathbf{g}t$. This gives the equation

$$\mathbf{v} = \mathbf{u} + \mathbf{g}t,$$

assuming that air resistance can be neglected.

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EXAMPLE 1.1.3

For the ball in Example 1.1.1, find the velocity after 1.5 seconds.

The vector **u** has magnitude $12\,\text{m}\,\text{s}^{-1}$ at 70° to the horizontal. The vector $\mathbf{g}t$ has magnitude $10 \times 1.5\,\text{m}\,\text{s}^{-1}$, that is $15\,\text{m}\,\text{s}^{-1}$, directed vertically downwards.

To draw a vector triangle you need to choose a scale in which velocities are represented by displacements. Fig. 1.4 is drawn on a scale of 1 cm to $5\,\text{m}\,\text{s}^{-1}$. You can verify by measurement that the magnitude of **v** is about $5.5\,\text{m}\,\text{s}^{-1}$, and it is directed at about 42° below the horizontal.

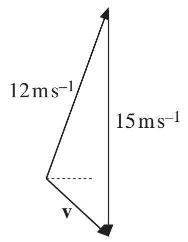

Fig. 1.4

Fig. 1.5 combines the results of Examples 1.1.1 and 1.1.3, showing both the position of the ball after 1.5 seconds and the direction in which it is moving.

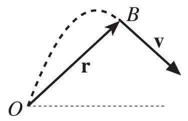

Fig. 1.5

As a reminder of what was said at the very start of this chapter, any object moving through the air will experience air resistance. How significant this is will depend upon a number of factors, including the nature of the object. For example, a feather is far more affected by air resistance than a solid metal ball. Also, experiments show that air resistance increases with speed. Air resistance is usually significant for objects moving at high speeds through large distances. In this chapter, it is assumed that air resistance is small enough that it can be ignored, and so all of the answers are only approximate.

Some people think that another limitation of this model for projectiles is the assumption that the acceleration is constant throughout the motion. They are aware that gravity reduces with height. However, while this last point is true, the acceleration change is miniscule: it only reduces by about 0.003% for every 100 metres above the ground. This is far less significant than any measurement error or air resistance effects for the type of projectiles we are considering, and so cannot be considered a significant limitation. The change in gravity does need to be taken into account when designing spacecraft launchers, but that is beyond the scope of this course.

Exercise 1A

1 A stone is thrown horizontally with speed $15\,\text{m}\,\text{s}^{-1}$ from the top of a cliff 30 metres high. Construct a diagram showing the positions of the particle at 0.5 second intervals. Estimate the distance of the stone from the thrower when it is level with the foot of the cliff, and the time that it takes to fall.

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2 A pipe discharges water from the roof of a building, at a height of 60 metres above the ground. Initially the water moves with speed $1\,\text{m s}^{-1}$, horizontally at right angles to the wall. Construct a diagram using intervals of 0.5 seconds to find the distance from the wall at which the water strikes the ground.

3 A particle is projected with speed $10\,\text{m s}^{-1}$ at an angle of elevation of 40°. Construct a diagram showing the position of the particle at intervals of 0.25 seconds for the first 1.5 seconds of its motion. Hence estimate the period of time for which the particle is higher than the point of projection.

4 A ball is thrown with speed $14\,\text{m s}^{-1}$ at 35° above the horizontal. Draw diagrams to find the position and velocity of the ball 3 seconds later.

5 A particle is projected with speed $9\,\text{m s}^{-1}$ at 40° to the horizontal. Calculate the time the particle takes to reach its maximum height, and find its speed at that instant.

6 A cannon fires a shot at 38° above the horizontal. The initial speed of the cannonball is $70\,\text{m s}^{-1}$. Calculate the distance between the cannon and the point where the cannonball lands, given that the two positions are at the same horizontal level.

7 A particle projected at 40° to the horizontal reaches its greatest height after 3 seconds. Calculate the speed of projection.

8 A ball thrown with speed $18\,\text{m s}^{-1}$ is again at its initial height 2.7 seconds after projection. Calculate the angle between the horizontal and the initial direction of motion of the ball.

9 A particle reaches its greatest height 2 seconds after projection, when it is travelling with speed $7\,\text{m s}^{-1}$. Calculate the initial velocity of the particle. When it is again at the same level as the point of projection, how far has it travelled horizontally?

10 Two particles A and B are simultaneously projected from the same point on a horizontal plane. The initial velocity of A is $15\,\text{m s}^{-1}$ at 25° to the horizontal, and the initial velocity of B is $15\,\text{m s}^{-1}$ at 65° to the horizontal.

- **a** Construct a diagram showing the paths of both particles until they strike the horizontal plane.
- **b** From your diagram estimate the time that each particle is in the air.
- **c** Calculate these times, correct to 3 significant figures.

1.2 Coordinate methods

For the purposes of calculation it often helps to use coordinates, with column vectors representing displacements, velocities and accelerations, just as was done for forces in M1 Chapter 10. It is usual to take the x-axis horizontal and the y-axis vertical.

For instance, in Example 1.1.2(a), the initial velocity $\mathbf{u}$ of the stone was $18\,\text{m s}^{-1}$ horizontally, which could be represented by the column vector $\begin{pmatrix} 18 \\ 0 \end{pmatrix}$. Since the units are metres and seconds, $\mathbf{g}$ is $10\,\text{m s}^{-2}$ vertically downwards, represented by $\begin{pmatrix} 0 \\ -10 \end{pmatrix}$. Denoting the displacement $\mathbf{r}$ by $\begin{pmatrix} x \\ y \end{pmatrix}$, the equation becomes $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{g}t^2$ becomes

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 0 \end{pmatrix} t + \tfrac{1}{2}\begin{pmatrix} 0 \\ -10 \end{pmatrix} t^2, \text{ or more simply } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18t \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -5t^2 \end{pmatrix} = \begin{pmatrix} 18t \\ -5t^2 \end{pmatrix}.$$

You can then read off along each line to get the pair of equations

$$x = 18t \quad \text{and} \quad y = -5t^2.$$

From these you can calculate the coordinates of the stone after any time t.

You can make t the subject of the first equation as $t = \frac{1}{18}x$ and then substitute this in the second equation to get $y = -5\left(\frac{1}{18}x\right)^2$, or (approximately) $y = -0.015x^2$. This is the equation of the trajectory. You will recognise this as a parabola with its vertex at O.

You can do the same thing with the velocity equation $\mathbf{v} = \mathbf{u} + \mathbf{g}t$, which becomes

$$\mathbf{v} = \begin{pmatrix} 18 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -10 \end{pmatrix} t = \begin{pmatrix} 18 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -10t \end{pmatrix} = \begin{pmatrix} 18 \\ -10t \end{pmatrix}.$$

This shows that the velocity has components 18 and $-10t$ in the x- and y-directions respectively.

Notice that 18 is the derivative of $18t$ with respect to t, and $-10t$ is the derivative of $-5t^2$. This is a special case of a general rule.

If the displacement of a projectile is $\begin{pmatrix} x \\ y \end{pmatrix}$, its velocity is $\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix}$.

This is a generalisation of the result given in M1 Section 11.2 for motion in a straight line.

Here is a good place to use the shorthand notation (dot notation) introduced in M1 Section 11.5, using $\dot{x}$ to stand for $\frac{dx}{dt}$ and $\dot{y}$ for $\frac{dy}{dt}$. You can then write the velocity vector as $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$.

Now consider the general case, when the projectile starts with an initial speed u at an angle θ to the horizontal. Its initial velocity $\mathbf{u}$ can be described either in terms of u and θ, or in terms of its horizontal and vertical components p and q. These are connected by $p = u\cos\theta$ and $q = u\sin\theta$. The notation is illustrated in Figs. 1.6 and 1.7.

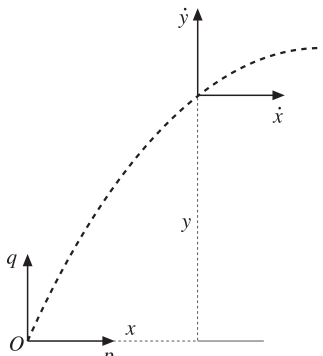

Fig. 1.6

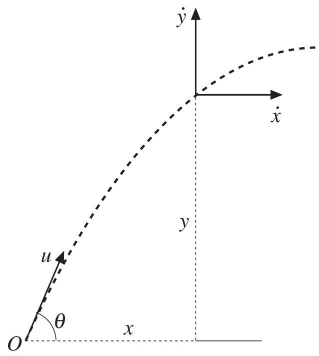

Fig. 1.7

The acceleration **g** is represented by $\begin{pmatrix} 0 \\ -g \end{pmatrix}$, so the equation

$$\mathbf{r} = \mathbf{u}t + \tfrac{1}{2}\mathbf{g}t^2$$

becomes

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} pt \\ qt \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{2}gt^2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u\cos\theta\, t \\ u\sin\theta\, t \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{2}gt^2 \end{pmatrix}.$$

By reading along each line in turn, the separate equations for the coordinates are

$$x = pt \qquad \text{or} \qquad x = u\cos\theta\, t,$$
$$\text{and} \quad y = qt - \tfrac{1}{2}gt^2 \qquad \text{or} \qquad y = u\sin\theta\, t - \tfrac{1}{2}gt^2.$$

In a similar way, $\mathbf{v} = \mathbf{u} + \mathbf{g}t$ becomes

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 \\ -gt \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix} + \begin{pmatrix} 0 \\ -gt \end{pmatrix}.$$

So

$$\dot{x} = p \qquad \text{or} \qquad \dot{x} = u\cos\theta,$$
$$\text{and} \quad \dot{y} = q - gt \qquad \text{or} \qquad \dot{y} = u\sin\theta - gt.$$

Since g, p, q, u and θ are all constant, you can see again that $\dot{x}$ and $\dot{y}$ are the derivatives of x and y with respect to t.

Now the equations $x = pt$ and $\dot{x} = p$ are just the same as those you would use for a particle moving in a straight line with constant velocity p. And the equations $y = qt - \frac{1}{2}gt^2$ and $\dot{y} = q - gt$ are the same as those for a particle moving in a vertical line with initial velocity q and acceleration $-g$. This establishes the **independence of horizontal and vertical motion**.

If a projectile is launched from O with an initial velocity having horizontal and vertical components p and q, under the action of the force of gravity alone and neglecting air resistance, and if its coordinates at a later time are (x,y), then

- **the value of x is the same as for a particle moving in a horizontal line with constant velocity p;**
- **the value of y is the same as for a particle moving in a vertical line with initial velocity q and acceleration $-g$.**

EXAMPLE 1.2.1

A golf ball is driven with a speed of $45\,\mathrm{m\,s^{-1}}$ at $37°$ to the horizontal across horizontal ground. How high above the ground does it rise, and how far away from the starting point does it first land?

To a good enough approximation $\cos 37° = 0.8$ and $\sin 37° = 0.6$, so the horizontal and vertical components of the initial velocity are $p = 45 \times 0.8\,\mathrm{m\,s^{-1}} = 36\,\mathrm{m\,s^{-1}}$ and $q = 45 \times 0.6\,\mathrm{m\,s^{-1}} = 27\,\mathrm{m\,s^{-1}}$. The approximate value of g is $10\,\mathrm{m^{-2}}$.

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To find the height you only need to consider the y-coordinate. To adapt the equation $v^2 = u^2 + 2as$ with the notation of Fig. 1.6, you have to insert the numerical values u (that is q) = 27 and $a = -10$, and replace s by y and v by $\dot{y}$. This gives

$$\dot{y}^2 = 27^2 - 2 \times 10 \times y = 729 - 20y$$

When the ball is at its greatest height, $\dot{y} = 0$, so $729 - 20y = 0$. This gives $y = \frac{729}{20} = 36.45$.

To find how far away the ball lands you need to use both coordinates, and the link between these is the time t. So use the y-equation to find how long the ball is in the air, and then use the x-equation to find how far it goes horizontally in that time.

Adapting the equation $s = ut + \frac{1}{2}at^2$ for the vertical motion,

$$y = 27t - 5t^2.$$

When the ball hits the ground $y = 0$, so that $t = \frac{27}{5} = 5.4$. A particle moving horizontally with constant speed $36\,\text{m s}^{-1}$ would go 36×5.4 m, that is 194.4 m, in this time.

So, according to the gravity model, the ball would rise to a height of about 36 metres, and first land about 194 metres from the starting point.

In practice, these answers would need to be modified to take account of air resistance and the aerodynamic lift on the ball.

EXAMPLE 1.2.2

In a game of tennis a player serves the ball horizontally from a height of 2 metres. It has to satisfy two conditions.

i It must pass over the net, which is 0.9 metres high at a distance of 12 metres from the server.

ii It must hit the ground less than 18 metres from the server.

At what speeds can it be hit?

It is simplest to take the origin at ground level, rather than at the point from which the ball is served, so add 2 to the y-coordinate given by the general formula. If the initial speed of the ball is $p\,\text{m s}^{-1}$,

$$x = pt \quad \text{and} \quad y = 2 - 5t^2.$$

Both conditions involve both the x- and y-coordinates, and the time t is used as the link.

i The ball passes over the net when $12 = pt$, that is $t = \frac{12}{p}$. The value of y is then $2 - 5\left(\frac{12}{p}\right)^2 = 2 - \frac{720}{p^2}$, and this must be more than 0.9. So

$$2 - \frac{720}{p^2} > 0.9.$$

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This gives $\frac{720}{p^2} < 1.1$, which is $p > \sqrt{\frac{720}{1.1}} \approx 25.6$.

ii The ball lands when $y = 0$, that is when $2 - 5t^2 = 0$, or $t\sqrt{\frac{2}{5}}$. It has then gone a horizontal distance of $p\sqrt{\frac{2}{5}}$ metres, and to satisfy the second condition you need $p\sqrt{\frac{2}{5}} < 18$. This gives $p < 18\sqrt{\frac{5}{2}} \approx 28.5$.

So the ball can be hit with any speed between about 25.6 m s^{-1} and 28.5 m s^{-1}.

EXAMPLE 1.2.3

A cricketer scores a six by hitting the ball at an angle of 30° to the horizontal. The ball passes over the boundary 90 metres away at a height of 5 metres above the ground. Neglecting air resistance, find the speed with which the ball was hit.

If the initial speed was u m s^{-1}, the equations of horizontal and vertical motion are

$$x = u\cos 30°t \quad \text{and} \quad y = u\sin 30°t - 5t^2.$$

You know that, when the ball passes over the boundary, $x = 90$ and $y = 5$. Using the values $\cos 30° = \frac{1}{2}\sqrt{3}$ and $\sin 30° = \frac{1}{2}$,

$$90 = u \times \tfrac{1}{2}\sqrt{3} \times t = \tfrac{1}{2}\sqrt{3}ut \qquad \text{and} \qquad 5 = u \times \tfrac{1}{2} \times t - 5t^2 = \tfrac{1}{2}ut - 5t^2$$

for the same value of t.

From the first equation, $ut = \frac{180}{\sqrt{3}} = 60\sqrt{3}$. Substituting this in the second equation gives $5 = 30\sqrt{3} - 5t^2$, which gives $t = \sqrt{6\sqrt{3} - 1} = 3.06\ldots$.

It follows that $u = \frac{60\sqrt{3}}{t} = \frac{60\sqrt{3}}{3.06\ldots} \approx 33.9$.

The initial speed of the ball was about 34 m s^{-1}.

EXAMPLE 1.2.4

A boy uses a catapult to send a small ball through his friend's open window. The window is 8 metres up a wall 12 metres away from the boy. The ball enters the window descending at an angle of 45° to the horizontal. Find the initial velocity of the ball.

One of the modelling assumptions we are making here is that the window is just a point. In the real world, the window has a significant height, so in a more sophisticated model, we could take this into account to obtain a range of possible velocities.

Denote the horizontal and vertical components of the initial velocity by p m s^{-1} and q m s^{-1}. If the ball enters the window after t seconds,

$$12 = pt \quad \text{and} \quad 8 = qt - 5t^2.$$

Also, as the ball enters the window, its velocity has components $\dot{x} = p$ and $\dot{y} = q - 10t$. Since this is at an angle of 45° below the horizontal, $\dot{y} = -\dot{x}$, so $q - 10t = -p$, or

$$p + q = 10t.$$

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You now have three equations involving p, q and t. From the first two equations, $p = \frac{12}{t}$ and $q = \frac{8+5t^2}{t}$. Substituting these expressions in the third equation gives $\frac{12}{t} + \frac{8+5t^2}{t} = 10t$, that is

$$12 + (8 + 5t^2) = 10t^2, \quad \text{which simplifies to} \quad 5t^2 = 20.$$

So $t = 2$, from which you get $p = \frac{12}{2} = 6$ and $q = \frac{8+5\times 2^2}{2} = 14$.

Fig. 1.8 shows how these components are combined by the triangle rule to give the initial velocity of the ball. This has magnitude $\sqrt{6^2+14^2}\,\text{m s}^{-1} \approx 15.2\,\text{m s}^{-1}$ at an angle $\tan^{-1}\frac{14}{6} \approx 66.8°$ to the horizontal.

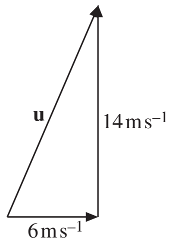

Fig. 1.8

The ball is projected at just over $15\,\text{m s}^{-1}$ at $67°$ to the horizontal.

Exercise 1B

Assume that all motion takes place above a horizontal plane.

1 A particle is projected horizontally with speed $13\,\text{m s}^{-1}$, from a point high above a horizontal plane. Find the horizontal and vertical components of the velocity of the particle after 2 seconds.

2 The time of flight of an arrow fired with initial speed $30\,\text{m s}^{-1}$ horizontally from the top of a tower was 2.4 seconds. Calculate the horizontal distance from the tower to the arrow's landing point. Calculate also the height of the tower.

3 Show that the arrow in Question 2 enters the ground with a speed of about $38\,\text{m s}^{-1}$ at an angle of about 39° to the horizontal.

4 A stone is thrown from the point O on top of a cliff with velocity $\begin{pmatrix}15\\0\end{pmatrix}\text{m s}^{-1}$. Find the position vector of the stone after 2 seconds.

5 A particle is projected with speed $35\,\text{m s}^{-1}$ at an angle of 40° above the horizontal. Calculate the horizontal and vertical components of the displacement of the particle after 3 seconds. Calculate also the horizontal and vertical components of the velocity of the particle at this instant.

6 A famine relief aircraft, flying over horizontal ground at a height of 245 metres, drops a sack of food.

- **a** Calculate the time that the sack takes to fall.
- **b** Calculate the vertical component of the velocity with which the sack hits the ground.

