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Cambridge International
AS and A Level Mathematics:
Pure Mathematics 1

Coursebook

Revised Edition



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Introduction

Cambridge International AS and A Level Mathematics has been written especially for the Cambridge International Examinations syllabus 9709. There is one book corresponding to each syllabus unit, except that units P2 and P3 are contained in a single book. This book covers the first Pure Mathematics unit, P1.

The syllabus content is arranged by chapters which are ordered so as to provide a viable teaching course. The early chapters develop the foundations of the syllabus; students may already be familiar with some of these topics. Later chapters, however, are largely independent of each other, and teachers may wish to vary the order in which they are used.

Some chapters, particularly Chapters 2, 3 and the first four sections of Chapter 8, contain material which is not in the examination syllabus for P1, and which therefore cannot be the direct focus of examination questions. Some of this is necessary background material, such as indices and surds; some is useful knowledge, such as graphs of powers of x , the use and meaning of modulus, and work on sequences.

A few sections include important results which are difficult to prove or outside the syllabus. These sections are marked with a coloured bar, and there is usually a sentence early on explaining precisely what it is that the student needs to know.

Occasionally within the text paragraphs appear in *this type style*. These paragraphs are usually outside the main stream of the mathematical argument, but may help to give insight, or suggest extra work or different approaches.

Graphic calculators are not permitted in the examination, but they are useful aids in learning mathematics. In the book the authors have noted where access to a graphic calculator would be especially helpful but have not assumed that they are available to all students.

Numerical work is presented in a form intended to discourage premature approximation. In ongoing calculations inexact numbers appear in decimal form like 3.456..., signifying that the number is held in a calculator to more places than are given. Numbers are not rounded at this stage; the full display could be, for example, 3.456123 or 3.456789. Final answers are then stated with some indication that they are approximate, for example '1.23 correct to 3 significant figures'.

There are plenty of exercises, and each chapter ends with a Miscellaneous exercise which includes some questions of examination standard. Three Revision exercises consolidate work in preceding chapters. The book concludes with two Practice exam-style papers.

In some exercises a few of the later questions may go beyond the likely requirements of the P1 examination, either in difficulty or in length, or both. Again, these questions are marked with coloured bar, which indicates that they require knowledge of results outside the syllabus.

Cambridge University Press would like to thank Cambridge International Examinations for permission to use past examination questions set internationally.

The authors thank Cambridge International Examinations and Cambridge University Press, in particular Diana Gillooly, for their help in producing this book. However, the responsibility for the text, and for any errors, remains with the authors.

Hugh Neill and Douglas Quadling, 2002

Introduction

Introduction to the revised edition

This revised edition has been prepared to bring this textbook in line with the current version of the Cambridge International Examinations specification. As much as possible of the original edition has been left unchanged to assist teachers familiar with the original edition; this includes section numbers, question numbers and so on. The principal additions to this edition are examples and exercises on sketching of graphs (Sections 4.2, 4.3, 7.3 and Exercises 4A and 7B) and a new section on using discriminants to identify tangents to quadratic curves (Section 4.8 and new questions in Exercise 4C). Chapter 11 has also been significantly edited, as the previous edition built much of the understanding of functions on students' experience with the calculators that were common at the time of writing. The current generation of calculators use a more natural form of input which does not translate easily to understanding functions, so most references to calculators have been removed and an alternative approach has been used in the text.

The other major change in this edition is the replacement of all of the older OCR examination questions in the Miscellaneous exercises with more recent Cambridge International Examinations questions. This will be of benefit to students preparing for the current style of examination questions. In order to maintain the numbering of the other questions, the newer questions have been slotted in to the exercises. While this has inevitably meant some loss of order within the miscellaneous exercises, this was felt to be more than compensated by the preservation of the original numbering. All questions and answers taken from Cambridge International Examinations past papers have been clearly referenced. All other questions and answers have been written by the authors of this book.

The editor of this edition thanks Cambridge International Examinations and Cambridge University Press, in particular Cathryn Freear and Andrew Briggs, for their great help in preparing this revised edition.

Julian Gilbey

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