Applied Stochastic Differential Equations

Stochastic differential equations are differential equations whose solutions are stochastic processes. They exhibit appealing mathematical properties that are useful in modeling uncertainties and noisy phenomena in many disciplines. This book is motivated by applications of stochastic differential equations in target tracking and medical technology and, in particular, their use in methodologies such as filtering, smoothing, parameter estimation, and machine learning. It builds an intuitive hands-on understanding of what stochastic differential equations are all about, but also covers the essentials of Itô calculus, the central theorems in the field, and such approximation schemes as stochastic Runge–Kutta. Greater emphasis is given to solution methods than to analysis of theoretical properties of the equations. The book's practical approach assumes only prior understanding of ordinary differential equations. The numerous worked examples and end-of-chapter exercises include application-driven derivations and computational assignments. MATLAB/Octave source code is available for download, promoting hands-on work with the methods.

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Contents

Preface

This book is an outgrowth of a set of lecture notes that has been extended with material from the doctoral theses of both authors and with a large amount of completely new material. The main motivation for the book is the application of stochastic differential equations (SDEs) in domains such as target tracking and medical technology and, in particular, their use in methodologies such as filtering, smoothing, parameter estimation, and machine learning. We have also included a wide range of examples of applications of SDEs arising in physics and electrical engineering.

Because we are motivated by applications, much more emphasis is put on solution methods than on analysis of the theoretical properties of equations. From the pedagogical point of view, one goal of this book is to provide an intuitive hands-on understanding of what SDEs are all about, and if the reader wishes to learn the formal theory later, she can read, for example, the brilliant books of Øksendal (2003) and Karatzas and Shreve (1991).

Another pedagogical aim is to overcome a slight disadvantage in many SDE books (e.g., the aforementioned ones), which is that they lean heavily on measure theory, rigorous probability theory, and the theory of martingales. There is nothing wrong in these theories – they are very powerful theories and everyone should indeed master them. However, when these theories are explicitly used in explaining SDEs, they bring a flurry of technical details that tend to obscure the basic ideas and intuition for the first-time reader. In this book, without shame, we trade rigor for readability by treating SDEs completely without measure theory.

The book's low learning curve only assumes prior knowledge of ordinary differential equations and basic concepts of statistics, together with understanding of linear algebra, vector calculus, and Bayesian inference. The book is mainly intended for advanced undergraduate and graduate students in applied mathematics, signal processing, control engineering, statistics, and computer science. However, the book is suitable also for researchers and practitioners who need a concise introduction to the topic at a level that enables them to implement or use the methods.

The worked examples and numerical simulation studies in each chapter illustrate how the theory works in practice and can be implemented for solving the problems. End-of-chapter exercises include application-driven derivations and computational assignments. The MATLAB® source code for reproducing the example results is available for download through the book's web page, promoting hands-on work with the methods.

We have attempted to write the book to be freestanding in the sense that it can be read without consulting other material on the way. We have also attempted to give pointers to work that either can be considered as the original source of an idea or just contains more details on the topic at hand. However, this book is not a survey, but a textbook, and therefore we have preferred citations that serve a pedagogical purpose, which might not always explicitly give credit to all or even the correct inventors of the technical ideas. Therefore, we need to apologize to any authors who have not been cited although their work is clearly related to the topics that we cover. We hope you understand.

The authors would like to thank Aalto University for providing the chance to write this book. We also would like to thank Robert Piché, Petteri Piiroinen, Roland Hostettler, Filip Tronarp, Santiago Cortés, Johan Westö, Joonas Govenius, Ángel García-Fernández, Toni Karvonen, Juha Sarmavuori, and Zheng Zhao for providing valuable comments on early versions of the book.

Simo and Arno