ARTICLE 1

FUNDAMENTAL PRINCIPLES

THE aim of this, the introductory article, is to outline the underlying principles of phototopography, indicating the methods by which points in photographic perspective are reduced to their corresponding orthogonal plans.

Contrary to a common misapprehension, photogrammetry, the process of mapping from survey photographs, does not necessarily introduce actual perspective constructions, these normally being obviated by geodetic measurements; though, on the other hand, some knowledge of perspective is often very desirable, and, in consequence, such principles as have relation to iconometric plotting will be summarised in the latter portion of the present article.

I. FUNDAMENTAL PRINCIPLES OF PHOTOGRAMMETRY

Photographic Perspective. Now a photograph is a perspective view in which the "picture plane" is superseded by a vertical photographic plate, the "station point" by a camera station at the second nodal point of the lens, the "distance" by the focal length f of the lens, and the "centre of vision" by the *principal point,* which is the intersection of the optical axis of the lens and the photographic plate.

Also, to exhibit the elements of a perspective picture, a survey photograph must contain both a horizontal and a vertical line, these intersecting at the principal point.

Fig. 1 illustrates the conditions of perspective in regard to a photographic view. These involve a ground or datum plane, GG, a picture plane PP, a principal plane VV, an horizon plane HH, and a station or nodal point I, the first and last planes GG and HH being horizontal and parallel, while the second and third, PP and VV, are vertical and are mutually perpendicular. The traces, or intersections, of these planes are fundamental lines in perspective, and are often styled alike in perspective and photogrammetry; *gg*, the *ground line*, *hh*, the *horizon line*, and *vv*, the *vertical line*, the intersection of *hh* and *vv* being the principal point O. Commonly, however, the line *vv* is termed the *principal line* in phototopography, while, in plan, a horizontal line terminating or representing the picture is styled the *picture trace*, *pp* in future notation.

Now in perspective projection the picture would appear erect on the dotted picture plane P′P′, between the object A and the spectator I, whereas in photography it would be seen inverted

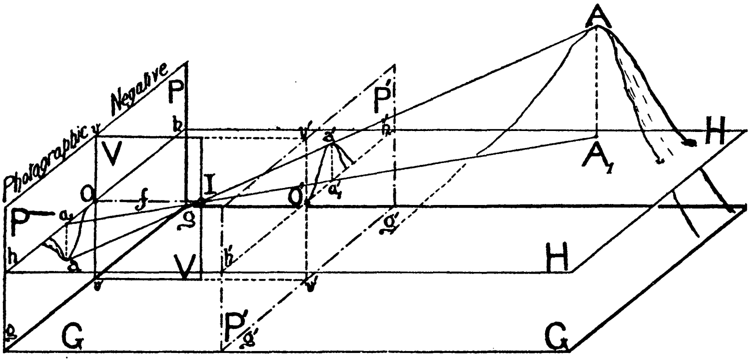

Fig. 1

on the ground glass plate, and would be projected thus on the negative PP. But this difference is immaterial. For since the photograph would be handled as an erect view, or *positive*, the plate may be regarded as having been exposed at a distance equal to the focal length *f* in front of the lens, thus occupying the dotted position P′P′ of Fig. 1. This, however, is merely equivalent to the interpretation of the eye, for though an inverted image is cast upon the retina, each ray is instantly and intuitively referred outwards, traversing the very path it, entering, took through the nodal point of the lens. *Hence*, in future, the photographic plate will be regarded as being at a "*distance*" *f* in front of the camera station I, thus avoiding reversed pictures and particularly the confusion of rays crossing the station point.

Geodiconometry. The iconometrical problem to be solved in photogrammetry is that of constructing a horizontal orthogonal projection, or plan, given a photographic perspective and the following elements:

(*a*) the focal length f,

(*b*) the horizon line hh, and

(*c*) the principal line vv,

the principal point O otherwise superseding one of these two lines.

Normally the focal length will be known, while the horizon and vertical lines will be delineated on the negatives by a specific function of the surveying camera. In fact these elements characterise a *phototopographic perspective*.

Even so, these data are insufficient for determining the horizontal distances and elevations of pictured terrene points, unless such points lie in a horizontal plane, the elevation of which is known with respect to the camera station. But if *two* photographs from two suitably situated and surveyed stations be obtained, each view containing a selected terrene point, then that point will be completely determined, the location in the horizontal plane being analogous to that of the well-known *method of intersections* in plane tabling. This introduction of geodetic data is characteristic of photographic surveying, though resort to single-view reduction is sometimes made, mainly in connection with details in the horizontal plane.

The photographic method lends itself both to *graphical* and *arithmetical* solutions, the former being more closely akin to plane tabling, and the latter to ordinary trigonometrical plotting. But since the outcome of a survey is a map, a graphical production, the bulk of the following pages will be devoted to graphical methods, whereas only brief reference will be made to arithmetical methods, though such, as a rule, lead to greater accuracy, but on the other hand do not always warrant the consequent expense, nor readily admit the application of various artifices in perspective. Incidentally it may be mentioned that the surveys of Laussedat and Deville were graphical and those of Jordan and Paganini arithmetical, these being representative examples of the photographic method.

The geodetical elements of a photographic survey may be said to be introduced through the triangulation system which forms the basis of the survey. Such systems are established and surveyed in the manner of those of ordinary topographical surveys, though in general they are adapted to suit the peculiar demands of photo-intersections.

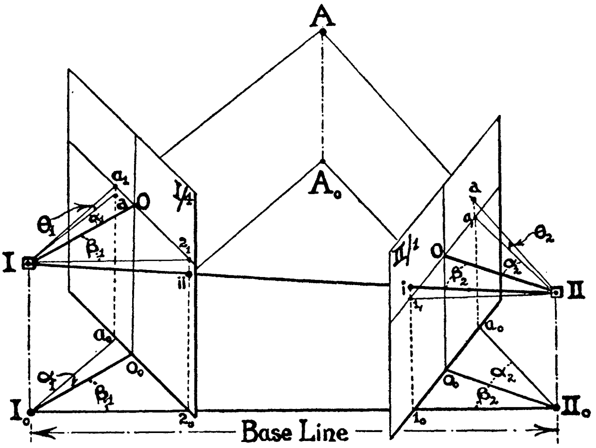

Fig. 2

(*a*) Let it therefore be assumed that the camera takes the place of a plane table with which points are being plotted by "intersections" from the extremities of the bases or sides as computed and plotted in the triangulation system.

Consider the process of photo-intersections as illustrated in Fig. 2, which may be regarded as both exaggerating the conditions in space and reproducing the conditions of plotting.

Here two photographs are shown in "position" at a distance equal to the focal length *f* of the camera in front of two known stations I and II, which are projected on to the ground or datum plane as I_0 and II_0 respectively, the distance $I_0 II_0$ being known. Each plate shows the horizon and principal lines together with the picture **a** of a terrene point A, while the stations I and II appear as *i* and *ii* on opposite plates. The pictured points and the principal point are projected both to the horizon line and the ground line, while the remaining data refer more particularly to the arithmetical method, an outline of which follows the description of the graphical method.

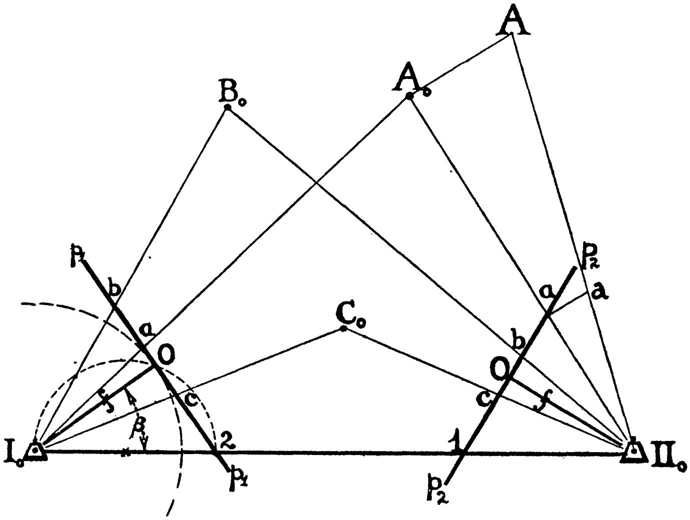

Fig. 3

General Graphical Method. Now if the stations, or nodal points, I and II, be plotted in plan, and the bases of the plates be represented by lines, or *picture traces*, p_1p_1 and p_2p_2, in "position" at a perpendicular distance f from I_0 and II_0, then will rays through the projection *a* of each photograph intersect, giving the plan A_0 of the point A in the terrene (Figs. 2 and 3). Also, if any other points, **b** for B, **c** for C, etc., appear on each plate, the plans of these points may be plotted in like manner, as indicated by the rays from I_0 and II_0 through the projections *b* and *c*. Further, the horizontal distance II_0A_0 being known, it follows from Fig. 2 that the difference in elevation H_1 between the point A and the camera station II is such that

$$H_1 : \mathbf{a}a_1 :: II_0A_0 : II_0a_0,$$

where $\mathbf{a}a_1$ and Oa_1 can be scaled from the photograph, and II_0a_0 appears on the plotting sheet, being $\sqrt{f^2 + Oa_1^2}$. The corresponding graphical construction is shown on the right of Fig. 3, $\mathbf{a}a$ being equal to the ordinate $\mathbf{a}a_1$, as scaled from the photograph.

(*b*) The first step at any plane table station consists in orienting the board, placing it so that plotted lines are parallel to, or coincident with, the corresponding lines in the field, whereas in phototopography the analogous step is effected in the office, and the trace of the photograph, or picture trace, is oriented on the paper, so that it occupies a position in plan corresponding to that which the plate occupied in space during exposure in the field.

Orientation. When the stations of the survey have been plotted, the first step consists in orienting a number of picture traces, p_1p_1, p_2p_2, etc., and the method to be used in effecting this will normally depend upon the phototopographic instrument employed or the data pictured or otherwise known. Thus, if a camera without means of measuring horizontal angles be employed, it is necessary that in addition to the principal point O a station be both pictured and plotted, as indicated by the circular arcs on the left of Fig. 3, the diameter of the semicircle $I_0 2$ being equal to $\sqrt{f^2 + (O2)^2}$, which readily admits graphical construction. If, however, as with the phototheodolite, the angle β_1 between the line I_0II_0 and the principal ray I_0O be measured, the pictured point *ii* becomes redundant, being superseded by a geodetical element β_1.

But orientation is a part of the routine of photogrammetry and, in consequence, will be fully discussed in Article 4.

General Arithmetical Method. The following relations may be shown to exist with regard to two vertical photographic plates, such as those oriented in front of stations I and II of

Fig. 2, where I_0 and II_0 are the projections of these stations on the ground plane, and A_0 is the like projection of a terrene point A, pictured as **a** in each photograph, I and II being pictured as *i* and *ii* on opposite plates. The projections of the pictured points are inserted in Fig. 2 on both the horizon and the ground line, and, being common, are indicated alike on each photograph, a_1 and a_0 denoting the respective projections of **a**, 1_1 and 1_0 those of *i*, and 2_1 and 2_0 those of *ii*. The focal length f, or distance line $I_0 o_0$ or $II_0 o_0$ will be normally constant and known.

Let β_1 and β_2 represent the horizontal angles observed at I and II as the angles between the principal line and the opposite camera station, II and I accordingly. Then the data additional to that of the graphical method will consist of the following angles and distances, all of which are determined from already known values or measurements;

α_1 and α_2, the horizontal angles between the principal line and the point A as measured from stations I and II respectively;

θ_1 and θ_2, the vertical angles of the point A with respect to the horizon plane at stations I and II respectively;

d_1 and d_2, the lines of direction Ia_1 and IIa_1, these being also equal to $I_0 a_0$ and $II_0 a_0$ respectively.

Now let the rectangular co-ordinates of the pictured points be referred to the principal and horizon lines as axes, with the principal point O as origin, the abscissae a_1O of **a** being x_1 and x_2 for the respective pictures, and the ordinates a_1**a** similarly y_1 and y_2.

Thus the known data now consist of the distance $I_0 II_0$ or base X, the focal length f, the abscissae and ordinates x_1, x_2, y_1, y_2, and the observed horizontal angles β_1 and β_2.

(*a*) To find the horizontal angles α_1 and α_2, which respectively determine the horizontal directions $I_0 a_0$ and $II_0 a_0$ of the terrene point A. (Fig. 2)

$$\tan \alpha_1 = \frac{x_1}{f} \text{ and } \tan \alpha_2 = \frac{x_2}{f}. \qquad ......(i), (ii).$$

(*b*) To determine the horizontal distances D_1 and D_2 of the terrene point A.

In the triangle $I_0 A_0 II_0$, the side $X = I_0 II_0$ is known, as also are the angles α_1, β_1, α_2, β_2. Hence if D_1 and D_2 denote the respective sides I_0A_0 and II_0A_0,

$$\frac{D_1}{\sin(\alpha_2+\beta_2)} = \frac{X}{\sin\{180° - (\alpha_1+\beta_1+\alpha_2+\beta_2)\}};$$

whence $$D_1 = X \cdot \frac{\sin(\alpha_2+\beta_2)}{\sin(\alpha_1+\beta_1+\alpha_2+\beta_2)}. \quad \text{......(1)}$$

Similarly $$D_2 = X \cdot \frac{\sin(\alpha_1+\beta_1)}{\sin(\alpha_1+\beta_1+\alpha_2+\beta_2)}. \quad \text{......(2)}$$

(*c*) To determine θ_1 and θ_2, the vertical angles between the horizon planes at I and II and the terrene point A.

$$\tan\theta_1 = \frac{y_1}{d_1} \quad \text{and} \quad \tan\theta_2 = \frac{y_2}{d_2},$$

d_1 and d_2 being respectively the lines of direction Ia_1 and IIa_1 in Fig. 2.

But since $d_1 = \sqrt{f^2+x_1^2}$ and $d_2 = \sqrt{f^2+x_2^2}$,

$$\tan\theta_1 = \frac{y_1}{\sqrt{f^2+x_1^2}} \quad \text{and} \quad \tan\theta_2 = \frac{y_2}{\sqrt{f^2+x_2^2}} \quad (iii), (iv).$$

(*d*) To determine the differences in elevation H_1 and H_2 between the terrene point A and stations I and II respectively.

$$\tan.\theta_1 = \frac{H_1}{D_1}, \quad \text{and} \quad H_1 = D_1\tan\theta_1, \quad \text{......(3)}$$

$$\tan.\theta_2 = \frac{H_2}{D_2}, \quad \text{and} \quad H_2 = D_2\tan\theta_2. \quad \text{......(4)}$$

II. FUNDAMENTAL PRINCIPLES OF PERSPECTIVE

Radial Projection. A perspective, or radial projection of an object, may be defined as the view obtained on an interposed vertical plane by rays converging from the object to a point known as the station point, or eye-point, the latter term suggesting the eye in monocular vision.

Fig. 4 is a pictorial view exhibiting a right-angled triangular prism ABCDEF with reference to the perspective planes and a station point I, these being indicated by the letters adopted conventionally in Fig. 1, viz. GG the ground plane, HH the horizon plane, PP the picture plane, and VV the vertical or principal plane, while accordingly *gg* is the ground line, *hh* the horizon line, and *vv* the vertical line.

Rays are shown drawn from the corners of the prism to the point of view I, and these, penetrating the picture plane PP, determine upon that plane the perspective picture **a**, **b**, **c**, **d**, **e**, **f**, which is indicated by fine lines. Also, to illustrate various steps in the following construction, two points are projected

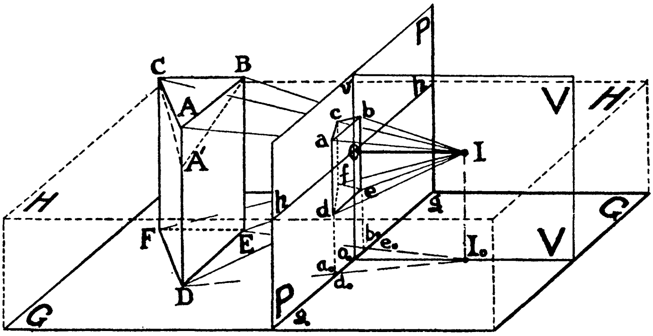

Fig. 4

on to the ground line *gg*, as a_0 and b_0, the corresponding plans of the rays I_0a_0 and I_0b_0, being drawn from I_0, the plan of the station I.

Fig. 5 following shows the plan A/D, B/E, C/F of the right triangular prism of Fig. 4 with respect to the ground trace *pp* of the picture plane PP and the plan I_0 of the station I. It will be seen that the prism stands on the ground plane, and thus has its long edges vertical, while one side of each face is parallel to the picture plane, and another perpendicular thereto.

These simple conditions will introduce practically all the essential rules of perspective.

So far the data is wholly orthogonal, the height of the prism being stated or indexed upon the given plan. The only other dimension requisite to drawing the perspective **abcdef** of the prism is the height of the eye, which in the present connection is commonly 5 feet to scale above the ground plane. But since the picture plane will have to be rabatted about its base *gg* into the horizontal plane—the paper—in order to exhibit the required view, it will be advisable to move the picture plane from the position indicated by its ground trace *pp* to a convenient parallel position, and there rabat the plane on to the ground, the trace

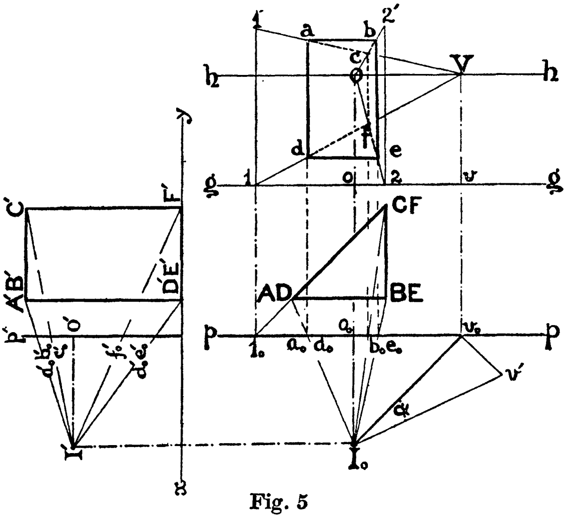

Fig. 5

pp becoming the base of picture *gg*. Now, if rays be drawn from the station plan I_0 to the corners of the prism, these rays will intersect the ground trace *pp* in three points a_0/d_0, b_0/e_0, and c_0/f_0, which are the *plans* of the required intersections of the rays with the picture plane, the plan of each point here corresponding to two points in the same vertical line, A and D, B and E, and C and F. Hence it remains to determine the corresponding elevations a_0'/d_0', b_0'/e_0', c_0'/f_0', in order to draw the perspective **abcdef** on the rabatted picture plane.

(1) The obvious method of finding the elevations a_0'–, b_0'–, c_0'– would be by means of orthogonal projection, as indicated