

Index

- abelian group, 1, 10, 14
- affine combination, 174
- Allais paradox, 121, 133
- almost complementary, 251
- almost completely labeled, 231, 251, 259
- angle, 192, 194
- antisymmetric, 8
- artificial equilibrium, 230, 231, 235, 250, 251
- associative, 10
- Aumann, Robert, xii, 74, 334
- background material, xi
 - converging subsequences, 188
 - directed graphs and trees, 80
 - Euclidean norm and distance, 187
 - expected value, 81
 - line segments, 130
 - matrix multiplication, 141, 143
 - partial orders, 8
 - undirected graphs, 170
 - visualizing matrix multiplication, 143
- backward induction, 83, 314, 319
 - more special than SPE, 284, 293
- bargaining axioms, 302, 303
- bargaining solution, 303, 329
- barycenter, 176–178, 190
- barycentric coordinates, 176
- basic solution, 252
- basic variables, 251
- Battle of the Sexes, 60
- behavior strategy, **280**, 285
- best response, 46, 57, **58**
- best-response condition, 135, **145**, **148**
- best-response diagram, 227
- bimatrix game, **142**
- binary system, 1, 5
- bluff, 269
- Bolzano–Weierstrass theorem, **188**
- Braess paradox, 43, 44, 49
- Brouwer’s fixed-point theorem, **151**, 153, 189, **195**, 203, 258
- castle, 234
- Chicken, 60, 74, 334
- Chomp, 31, 33, 35
- cold game, 30
- column shift, 239, 240
- commitment game, **93**
- common knowledge, 55, 132
- commutative, 10
- compact set, **149**, 151, 171, 190, 212, 248, 302
- complementary, 251
- complementary pivoting, 252, 255, 260
- completely labeled, 172, 173, 181, 227–229, 247, 249, 251
- concave function, **130**
- concave utility function, 131, 312
- conditional probability, 269, 337
- Condorcet Paradox, 110, 133
- congestion game, **45**
- congestion network, 40, 43, **44**
- continuity axiom, 108
- Continuous Ultimatum game, 316
- contractive map, 195
- convex combination, 150, 165, 174
- convex hull, **174**, 301
- convex set, **149**, **174**

- cooperative game theory, 299, 330
 coordination game, 61, 236
 copycat principle, 13
 correlated equilibrium, **338, 339**
 correlated moves, 282, 290
 correlated strategies, 336
 correlation device, 337, 338
 cost, 41, 206
 Cournot duopoly, 66, 74
 Cram, 25, 32, 34, 36

 decision theory, 110
 Dedekind cut, 31
 degenerate game, 162, **164**, 233, 238
 degree of a map (Brouwer), 203
 degree of graph node, **170**
 descendant, **80**, 91
 deviation, 41, 46
 dictionary, 251
 difference trick, **156**, 209
 digraph, 45
 digraph game, 17, 31, 35
 dimension, 174
 dominance solvable, 53, **65**, 74, 342
 dominated
 strategy, 342
 dominated option, 27
 Domineering, 24
 domino, 24, 32, 34, 36
 downward closed set, 313
 duality of linear programming, 218
 duplicate label, 231

 edge, 40, 170, 176, 181, 230
 end node of a path, 170, 232
 ending condition, 2, 7, 10
 endpoint of an edge, 170
 entering variable, 252
 equilibria (plural of equilibrium), 59
 equilibrium, 53, **58**, 73
 correlated, 338, 339
 in a congestion game, 41, 46, 48
 mixed, 148, 152
 Nash, 59, 338
 pure, 53
 refinement, 62, 74
 selection, 62
 equilibrium path, 95
 equivalence
 of impartial games, 11
 of partizan games, 25, 31
 relation, 11
 strategic, 239, 259
 strong strategic, 241, 259
 Euclidean distance, **187**
 Euclidean norm, **187**
 expected payoff, 85, 148
 expected value, 81
 expected-utility function, 103, 106, 112,
 117, 124, 208
 exponential size of game tree, 90
 extreme point, 176, 204

 face, 176, 245
 facet, 176
 Farkas's Lemma, 220
 first-mover advantage, 96
 flow, 41
 atomic (non-splittable), 39, 48
 splittable, 40, 48, 50
 follower, 96
 Freudenthal triangulation, **197**
 full-dimensional, 197

 Gambit software, xii
 game position, 7
 game sum, 10
 Game Theory Explorer (GTE), xii
 game tree, 3, **79**, 270
 generic game, 84
 goalpost method, 159, 208, 226
 graph, **170**
 Grundy value, 16, 31

 Hackenbush
 Black-White, 30, 37
 Gray, 36
 halfline, 192
 halfspace, 193, 245
 heap, *see* Nim heap
 history of moves, 3, 81

- Homo Ludens (Huizinga), 330
 hot game, 30
 hotel with seaview, 188, 204
 hyperplane, 193
- identity matrix, 344
 impartial game, 2
 imperfect recall, 275
 incentive constraints, 338
 independence axiom, 104, 108, **121**
 index of an equilibrium, 259
 indifference, 106, **112**, 139
 information set, 265, 266
 Inspection game, 137
 insurance
 accidental breakdown, 109
 car rental, 109, 110
 intransitive preferences, 110, 114, 133
 invariant under utility scaling, 303
 irrelevant alternatives, 303
 iterated elimination of dominated
 strategies, 64, 65, 69, 71, 222
 iterated-offers bargaining game, 317–330
- Kayles, 17, 23, 31
 Kuhn's theorem, 284
- label, 172, 227
 Last Year in Marienbad, 4
 leader, 96
 leadership game, 96
 leaf, **80**
 learning, 349
 leaving variable, 252
 Left player, 24
 Lemke's algorithm, 260
 Lemke–Howson
 algorithm, 232
 graph, 233, 235, 259
 path, 231, 232, 244
 lexico-minimum ratio test, 258
 lexico-positive, 257
 lexicographic order, 105, **114**, 123, 124
 LH, *see* Lemke–Howson
 line segment, 130
 linear complementarity problem, 260
 linear programming, 218, 221, 255, 260,
 296
 linear programming duality, 206, 218
 losing position, 3, 25
 lottery, **106**, **111**
 simple, 104, **117**
 lower envelope, 209
 \mathcal{L} -position, 25
- majority vote, 110
 Marienbad, 4
 Markov chain, **344**
 Maschler, Michael, 334
 Matching Pennies, **72**
 matrix game, **207**
 matrix multiplication, 141, 143
 max-min strategy, 138, 210, **211**
 maximizer, **207**
 mex rule, 15, **16**, 31
 min-max strategy, **211**, 212
 minimal element, 8
 minimax theorem, 213, **214**, 216, 344, 347
 minimizer, **207**
 minimum ratio test, 255
 misère Nim, 4, 32
 misère play, 2, 14
 missing label, 231
 mixed equilibrium, **148**, 152, 228
 mixed extension, 139, **148**, 212
 mixed strategy, **143**
 mixed-strategy profile, 148
 mixed-strategy set, 144
- negative game, 11, 13, 25
 neighbor, **170**
 Nim, **2**, 32
 Nim heap, 2, 15
 notation n^* , 15
 Nim sum, 5, 20
 Nim value, **16**, 27
 nimer, 27
 node, 170, 230
 nonbasic variables, 252
 nondegenerate, 254
 normal form, 74

- normal play, 2
 normal vector, 193, 245
 normative, 112, 114, 120, 133
 Northcott's game, 31, 34
 \mathcal{N} -position, 25
 number (partizan game), 27, 28

 optimal strategy, 215
 option, 3, 7, 10, 26
 order
 partial, 8
 total, 8
 origin of utility scale, 104, 107, 116
 orthogonal, 191
 outcome class, 3, 25
 outcome set for decisions, 111

 parallel edges, 40, 41, 49
 Pareto-optimal, 303
 parity argument, 170, 184, 203, 232, 234
 partial order, 8, 32, 142
 partial strategy profile, 339
 partizan game, 2, 24, 37, 38
 path, 45, 80, 170
 payoff equivalent, 62
 perfect information, 2
 perfect recall, 275–279, 285
 permutation, 198
 Pigou network, 40, 42, 44, 49
 pivoting, 252
 play of a game, 77
 Poker, 268, 297
 Poker Nim, 15, 16, 17
 polyhedron, 245, 341
 polytope, 175, 245
 potential function, 39, 46, 48, 49
 potential game, 50, 74
 PPAD, 259
 \mathcal{P} -position, 25
 preference
 relation, 111
 strict, 106, 112
 Price of Anarchy, 49
 prime number decomposition theorem, 9
 Prisoner's Dilemma, 55, 73, 300

 product graph, 231
 projection, 171, 191

 Quality game, 62
 quantity competition, 66
 Queen-move game, 22, 34

 random variable, 81
 rationality, 66, 132
 sequential, 91
 ray, 192
 ray termination, 255
 realization equivalence, 284
 recursive, 4, 10
 reduced strategic form, 87, 275
 reduced strategy, 78, 87, 274
 reflexive, 8, 11
 regret, 109, 122, 349
 rental car insurance, 109
 Right player, 24
 Rock-Paper-Scissors, 72, 74
 Rook-move game, 17
 \mathcal{R} -position, 25

 scalar product, 142, 144
 scale-equivalent, 115, 116, 118
 Schelling, Thomas, 74
 seaview hotel, 188, 204
 security strategy, 212
 Seoul (Korea), 44, 49
 separating hyperplane, 193, 220
 sequence form, 295
 set inclusion, 8
 sg-value, 16
 simple lottery, 104, 117
 simplex, 175, 339
 simplex algorithm, 255, 260
 simplicial subdivision, 176
 slack variables, 251
 Social Choice, 110, 133
 social optimum, 41, 42, 49
 SPE, *see* subgame-perfect equilibrium
 Sperner condition, 180, 184
 Sperner labeling, 180
 Sprague–Grundy value, 16, 31
 Stackelberg game, 96, 98

- Stag Hunt, 73
 staggered payoffs, 56, 74
 stationary distribution, **344**
 stationary strategy, 325
 strategic equivalence, 239
 strategic form, 74, 273
 strict order, 8, 32
 strict preference, 106, **112**
 strictly concave, 131
 strictly dominated strategy, 62
 strong strategic equivalence, 241
 sub-facet, **176**, 178, 182
 sub-segment, 173
 sub-simplex, 176
 subgame, 91, **291**
 subgame-perfect equilibrium, 91, 292
 subjective probability, 111
 subtree, 91, 291
 symmetric
 bimatrix game, **142**
 equilibrium, 61, 73
 game, 55, 60, 66, **72**
 symmetry axiom, 303
- temperature scale, 104, 107, 116
 terminal node, 80
 threat point, 302
 tight inequality, 245
 top-down induction, **8**
 total order, 8
 total relation, 112
 tradeoff, 114
- transformation
 positive-affine, **115**
 projective, 249
 strictly increasing, 115
 transitive, 8, 11
 transposed matrix, **141**
 triangulation, **176**
 Trust Dilemma, 61
- Ultimatum game, 314
 unit cube, **196**
 unit of utility scale, 104, 107, 116
 unit simplex, 151, 169–171, 175, 190
 unit vector, **150**, 169, **171**
 upper envelope, 158, 225, 241, 246
 utility function, 103–133
 cardinal, 104, 107, **115**
 ordinal, 106, 115
 utility scale, 104, 116, 137, 240
- vertex, 176, 204, 245
- walk, **80**
 Wardrop equilibrium, 50
 weakly dominated strategy, **62**, 63, 64, 75, 219
 elimination may lose equilibria, 63, 220
 Wilson, Robert, 259
 winning move, 3, 4
 winning position, 3, 25
- zero-sum game, 207