Online and Matching-Based Market Design

The rich, multi-faceted, and multi-disciplinary field of matching-based market design is active and important owing to its highly successful applications, with economic and sociological impact. Its home is economics but with intimate connections to algorithm design and operations research. With chapters contributed by over 50 top researchers from all three disciplines, this volume is unique in its breadth and depth while still giving a cohesive and unified picture of the field, suitable for the uninitiated as well as the expert. It explains the dominant ideas from computer science and economics underlying the most important results on market design and introduces the main algorithmic questions and combinatorial structures. Methodologies and applications from both the pre-Internet and post-Internet eras are covered in detail. Key chapters discuss the basic notions of efficiency, fairness, and incentives and the way in which market design seeks solutions guided by normative criteria borrowed from social choice theory.

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The cover image is meant to be a caricature of the housing market of Shapley and Shubik, appearing in Chapter 3. The picture portrays a comparison of the value of a house to that of a rock. In this case, the rock wins!

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108831994
DOI: 10.1017/9781108937535

First published 2023

Printed in the United Kingdom by TJ Books Limited, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

A Cataloging-in-Publication data record for this book is available from the Library of Congress.

ISBN 978-1-108-83199-4 Hardback

Electronic version password: OMBMD_CUP

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Foreword

This volume is a tribute to the interdisciplinarity (if that's a word) of matching markets and market design. It is also an invitation to pursue the many important open questions concerning theory, computation, and practical market design that are surveyed here from the perspectives of economics, computer science, and operations research.

Two of the founding papers of this literature are due to Gale and Shapley [3] and Shapley and Scarf [11]. Both demonstrated algorithms, for two crisply defined discrete models, that showed constructively that the core of the game – the set of outcomes that can't be disrupted by dissatisfied coalitions – is non-empty. That is, both papers demonstrated algorithms that could use information about the preferences of participants to identify outcomes with desirable efficiency and stability properties.

Gale and Shapley introduced what has become a canonical model of two-sided matching, which they called the marriage problem, involving two disjoint sets of players (e.g., "men" and "women"), each of whom has preferences over players on the other side. (They also sketched a many-to-one generalization that they called the college admissions problem.) An outcome of the game is a matching of men and women. They defined stable matchings as those that no man – woman pair not matched to each other, and no unhappily matched individual, would prefer to disrupt and introduced the deferred acceptance algorithm, which finds a stable matching with respect to any preferences. Under the rules that any willing pair of players from opposite sides may be matched to one another if and only if they both agree, the set of stable matchings is the core of the resulting game. But the set of stable matchings has proved of great interest even in models in which it differs from the core. And the deferred acceptance algorithm has sparked a literature of its own, not least in computer science, where it became well known following Donald Knuth's 1976 monograph in French, *Mariages Stables*.[1]

Shapley and Scarf introduced a model in which each agent initially possesses a single unit of an indivisible good, which they called a house. Agents have preferences over all the houses, which can be traded. But no money can be used: trades have to be house swaps, among cycles of any length. They introduced the top trading cycles algorithm (which they attributed to David Gale) and showed that, for any preferences over houses, it produces an allocation in the core of the game, i.e., one that no coalition can improve upon by trading among its own members. This is a

[1] For the English translation, see [5].

"one-sided" model: any player can trade with any other (unlike the case of the two-sided marriage model). Another way in which this and other models are one-sided rather than two-sided is that players (who have preferences, and for whom we have welfare concerns) are matched to objects, not to other players.

While both of these foundational papers were boundary-busting in how they combined game theory with algorithms, they both started from the point of view of cooperative game theory. The object of cooperative game theory (which was thought of as the study of a class of games in which players could reach binding agreements) was to identify desirable or likely outcomes of the games studied, whose rules were specified in terms of what coalitions could achieve by agreement, not in terms of what specific actions individual players could take. In contrast, non-cooperative game theory was thought of as studying games in which no binding agreements could be reached, and rules were specified in terms of the strategies that individuals could independently employ.

It seemed natural to think of Gale – Shapley and Shapley – Scarf as suggesting the designs of centralized clearinghouses, which would use information about the market to suggest market outcomes to participants. But while data on participants and resources could be observed and incorporated into the design of a clearinghouse, preferences are the private information of individuals. If we were interested in actually designing a centralized clearinghouse built around the deferred acceptance algorithm or top trading cycles, how would we obtain the preferences needed as inputs? In Roth [6], [7] I began to study when it would be safe for participants who were asked to state their preferences to state them truthfully. In game-theoretic terms, I was studying what was then thought of as part of non-cooperative game theory, namely when and for whom it could be made a *dominant strategy* to state preferences truthfully. I found that the top trading cycles algorithm makes it a dominant strategy to state preferences truthfully in the Shapley and Scarf model, but that no mechanism that always produces stable matchings can make the truthful revelation of preferences a dominant strategy for all players in the marriage model. However, it is possible to make it a dominant strategy for one side of the marriage market to state true preferences, and this has in some applications been sufficient, particularly in light of many subsequent results on the difficulty and low prevalence of profitable opportunities for agents on the other side to misrepresent their preferences in naturally occurring markets.[2]

These approaches showed that matching markets can be thought of both as cooperative games and as non-cooperative games, and today we no longer think of those two kinds of game theory as necessarily studying different games. Rather, coalitional models from cooperative game theory and strategic models from non-cooperative game theory answer different kinds of market design questions about a given market (see Roth and Wilson [10]). For example, the papers Roth [8] and Roth and Peranson [10] each studied the clearinghouse for new American doctors from the point of view of when stable matchings exist, and when truthful preferences can be safely elicited.

In the years since those beginnings, market design has continued to break boundaries, including those between theory, computation, and application. Market design has become an engineering discipline, in which game theory, computation,

[2] Dubins and Freedman [2] and Bergstrom and Manning [1] independently investigated closely related questions about the marriage model.

optimization, observation, and a healthy dose of trial and error combine to create new designs, which have had some notable practical successes in being implemented and maintained.

This volume opens up a window on much that has been accomplished so far. For readers new to the field, it provides an easy entryway, and the introductory essays by the editors provide helpful orientation. And (if my own experience is any indication) even grizzled veterans will find much to learn here, in the chapters on theory, on empirics and design, and on new boundaries to cross. This is a volume to read and study, and to let yourself be invited and recruited into the theory of matching and the practice of market design.

Alvin E. Roth

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Preface

The topic of this book is the rich, multi-faceted, and multi-disciplinary field of matching-based market design. Although the home discipline of this field is economics, it has been intimately connected to the discipline of algorithm design right from its birth[3] and also shares boundaries with operations research. With chapters contributed by over 50 top researchers, from all three disciplines, this volume is unique in its breadth and depth of coverage while still retaining the feel of a cohesive, unified textbook.

The importance of this field arises from its highly successful applications, having economic as well as sociological impact. From the viewpoint of applications and algorithmic methodology, the field consists of two distinct eras – pre-Internet and post-Internet. Methodologies and applications from both eras are covered in detail in this book.

The book covers the dominant ideas from computer science and economics that underlie the most important results on market design. It introduces readers to the main algorithmic questions raised by matching markets, as well as to the key combinatorial structures that underlie such questions. It discusses the basic notions of efficiency, fairness, and incentives and the way in which market design seeks solutions that are guided by normative criteria borrowed from social choice theory. Because of its broad sweep of introductory as well as advanced topics, it will be valuable for the uninitiated as well as the expert.

The text is suitable for use in a wide variety of courses across several disciplines, as will be described next. A basic semester-long course on the topics of the book, suitable for upper-level undergraduates and beginning graduate students, would cover the four chapters of Part One, most of the chapters from Part Two, and a selection of the rest, based on the instructor's preferences. For a graduate course in economics, the book offers cutting-edge results on the most important areas of research on these topcis today, e.g., school choice, the AdWords and other online marketplaces, the organ donation market, large markets, and machine learning and pseudo-markets. A course on the economic theories of market design would concentrate on Parts One and Three, against a backdrop of other relevant topics. Readers interested in experimental economics, applied economics, or operations research will find relevant material in Parts Two and Four, and Part Five will appeal to those interested in new directions and advanced topics.

[3] The main result of the 1962 paper of Gale and Shapley, which initiated this field, was an *efficient algorithm* for the stable matching problem, obtained three years before polynomial time solvability was formally defined!

Because of the groundswell of fundamental algorithmic ideas, presented from first principles, this book is also suitable for use as a supplementary text in basic undergraduate and graduate courses on algorithm design. The first three chapters of Part One are particularly suitable for this.

Multiple thanks are due. First, to the chapter authors for producing very high quality chapters in a timely manner. Second, to Simons Institute for running a program on the same topic as the title of the book, in Fall 2019; it provided a scintillating environment in which the detailed structure of this book evolved. Third, to Lauren Cowles for her expert advice throughout the two years in which this book took shape.

We hope this book will contribute to the rapid growth of this field, not only as a pedagogic tool but also via the large number of open problems and issues discussed in the more advanced chapters. It is our intention that it will be in active use for several decades to come.

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