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978-1-108-83183-3 — The Discrete Mathematical Charms of Paul Erdős
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Excerpt
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Introduction

The lecture notes that form the basis of this book have been distributed to graduate students, but the model readers I had in mind when writing them were contestants in a mathematical olympiad. As I did not want to intimidate such youngsters, I chose to include prerequisites that they might not be familiar with even if these prerequisites could be taken for granted when addressing graduates. (They are collected in Appendix A.) The term ‘simple’ in the subtitle of the book is the operative word: I aimed to make the material accessible to as wide an audience as possible.

I used the lecture notes in a course that consisted of twelve 90-minute lectures and a screening of George Csicsery’s brilliant documentary “N is a number” [92]. In each of its editions, I covered at most nine of the following eleven chapters (and once also a large part of Appendix A) at a leisurely pace.

Here is how I arrived at the order of the chapters.

1. Erdős’s first important achievement, his 1932 paper proving Bertrand’s postulate, seemed a logical choice for the first chapter.
2. His next widely acclaimed result, published in a 1935 paper co-authored with George Szekeres, was the Happy Ending Theorem. Erdős’s proof of it, chronologically second and quantitatively far superior to the first, is the starting point of Chapter 2. Its geometric nature suggests continuing with another early geometric interest of Paul Erdős, his conjecture that was confirmed by Tibor Gallai (né Grünwald) and became known as the Sylvester–Gallai theorem. As pointed out by Erdős in 1943, this theorem has a pretty corollary involving points and lines in the plane. In a 1948 paper, Erdős and Nicolaas de Bruijn proved a combinatorial theorem that subsumes this corollary and extends it far beyond the reaches of geometry. This De Bruijn–Erdős theorem and its several proofs round up Chapter 2.
3. Not to leave the reader in suspense for too long, we then backtrack to the Happy Ending Theorem and present its proof by Szekeres. This is done in Chapter 3, whose main theme is Ramsey’s theorem. At the end of this chapter, I indulge myself by discussing my second joint paper with Erdős.
4. Another instance of such self-indulgence comes in Chapter 4, where I point out how a qualitative version of the Erdős–Rado theorem on Δ-systems can be viewed as a corollary of Ramsey’s theorem. This observation is linked to a

conjecture of Erdős and Lovász on weak and strong Δ-systems, whose beautiful proof by Michel Deza concludes the chapter.

5. The Erdős–Rado theorem on Δ-systems opens the gates of extremal set theory, which is the subject of Chapter 5. One of the two results closing this chapter is Erdős's lower bound on the number of hyperedges in a k-uniform hypergraph of chromatic number greater than s.
6. This bound subsumes not only Erdős's lower bound on diagonal Ramsey numbers but also a lower bound on van der Waerden numbers, and so van der Waerden's theorem on arithmetic progressions is treated in Chapter 6.
7. In Chapter 7, we return to extremal set theory and survey its rich autonomous branch, extremal graph theory.
8. Chapter 8 stands out by having no links to other chapters. It begins with the Friendship Theorem of Erdős, Alfréd Rényi, and Vera Sós. Its proof by Herbert Wilf connects it to strongly regular graphs and the dazzling theorem on Moore graphs of diameter two by Alan Hoffman and R. R. Singleton.
9. After the detour, the next chapter begins with a reference to the Erdős–Stone–Simonovits formula of Chapter 7, which features the chromatic number of a graph. This invariant is the sole subject of Chapter 9. Several proof techniques used there are early instances of what has become known as the probabilistic method, and so it seems natural to continue with graph theory and probability.
10. The first two sections of Chapter 10 reproduce two fragments of the Erdős–Rényi theory of random graphs; the next section reports without proofs the fascinating results on the evolution of random graphs, with an emphasis on the double jump and its critical window; the concluding section puts the preceding material in its natural context of finite probability spaces.
11. Chapter 11 is more of an appendix than a genuine chapter: its theme, Hamiltonian graphs, was far from central among Erdős's interests in discrete mathematics. I have taken the liberty of recounting in its first section how a result of mine was directly inspired by Erdős's delightful algorithmic proof of Turán's theorem and presenting in the second section my first joint paper with Erdős. (Please note that I have displayed admirable restraint by not mentioning our third joint paper anywhere in this book. Except here.) A brief survey of results on Hamilton cycles in random graphs rounds up this chapter.

I regret the omission of two brilliant and important results, Lovász Local Lemma [249] and Szemerédi's Regular Partition Lemma [257]. I could not find a way of weaving them smoothly into the narrative.

Non-mathematical parts of the text are set in sans serif against a lightly shaded background like this.

Definitions that are used more than once are collected in Appendix B.