

AN INTRODUCTION TO RESERVOIR SIMULATION USING MATLAB/GNU OCTAVE

This book provides a self-contained introduction to the simulation of flow and transport in porous media, written by a developer of numerical methods. The reader will learn how to implement reservoir simulation models and computational algorithms in a robust and efficient manner. The book contains a large number of numerical examples, all fully equipped with online code and data, allowing the reader to reproduce results and use them as a starting point for their own work. All of the examples in the book are based on the MATLAB Reservoir Simulation Toolbox (MRST), an open-source toolbox popular in both academic institutions and the petroleum industry. The book can also be seen as a user guide to the MRST software. It will prove invaluable for researchers, professionals, and advanced students using reservoir simulation methods. This title is also available as Open Access on Cambridge Core at <https://doi.org/9781108591416>.

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Knut-Andreas Lie

Frontmatter

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AN INTRODUCTION TO RESERVOIR
SIMULATION USING
MATLAB/GNU OCTAVE

User Guide for the MATLAB Reservoir Simulation
Toolbox (MRST)

KNUT-ANDREAS LIE

SINTEF



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Preface

There are many books that describe mathematical models for flow in porous media and present numerical methods used to discretize and solve the corresponding systems of partial differential equations; a comprehensive list can be found in the References. However, neither of these books fully describes how you should implement the models and numerical methods to form a robust and efficient simulator. Some books may present algorithms and data structures, but most leave it up to you to figure out all the nitty-gritty details you need in order to get your implementation up and running. Likewise, you may read papers presenting models or computational methods that may be exactly what you need for your work. After the initial enthusiasm, however, you very often end up quite disappointed, or at least I do when I realize that the authors have not presented all the details of their methods, or that it will probably take me weeks or months to get my own implementation working.

In this book, I try to be a bit different and give a reasonably self-contained introduction to the simulation of flow and transport in porous media that also discusses how to implement the models and algorithms in a robust and efficient manner. In the presentation, I have tried to let the discussion of models and numerical methods go hand in hand with numerical examples that come fully equipped with codes and data, so that you can rerun and reproduce the results yourself and use them as a starting point for your own research and experiments. You will get most out of the book if you continuously switch between reading and experimenting with the many code snippets and tutorial examples on your own computer. All examples in the book are based on the MATLAB Reservoir Simulation Toolbox (MRST), which has been developed by my group and published online as free open-source code under the GNU General Public License since 2009.

The book can alternatively be seen as a comprehensive user-guide to MRST. Over the years, MRST has become surprisingly popular. At the time of writing (July 2018), the software has more than 17,000 unique downloads, 120 students have used MRST in their master or PhD theses, and more than 190 papers written by authors outside of SINTEF include numerical experiments run in MRST. This book tries to give an in-depth introduction to MRST and explain the two different programming paradigms you can find in the software. The book is up to date with respect to the latest developments in data structures and syntax, both for the original procedural approaches that primarily focus on incompressible flow, as

well as for the more recent object-oriented, automatic-differentiation (AD-OO) framework we developed to simulate compressible, multiphase flow.

The book has grown much longer than I anticipated when I started writing. Initially, my ambition was to provide introductory material on single-phase and two-phase incompressible flow, and discuss how such models could be implemented in a flexible and efficient manner by use of MATLAB. While I was writing, the software expanded rapidly and I had a hard time keeping pace. Inspired in part by the many people who have downloaded (and cited) the preliminary editions I have published online, but also as a result of numerous requests, I decided to expand the book and include a detailed discussion of the physics underlying industry-standard black-oil simulators, and give a thorough introduction to how such models have been implemented in MRST. Now that this is done, it is time to stop. However, MRST has more to offer, including solvers for various enhanced oil recovery (EOR) models, compositional flow, fractured media, geomechanics, geochemistry, as well as a comprehensive set of tools for modeling CO₂ storage in large-scale aquifer systems, but documentation of these will have to be another book, or perhaps also another author. At the moment, I am more than happy that I finally managed to finish this book.

First of all, I am very grateful to Equinor for the generous grant that enabled Gold Open Access publication of this book. I would also like to thank my current and former colleagues at SINTEF with whom I have collaborated over many year to develop MRST; primarily Olav Møyner, Halvor Møll Nilsen, Stein Krogstad, Jostein R. Natvig, Odd Andersen, Bård Skaflestad, and Xavier Raynaud. I have also had a great number of inspiring discussions with Alf Birger Rustad and Vegard Kippe at Equinor over the past decade. The chapter on flow diagnostics is partially the result of many discussions with Brad Mallison from Chevron. I am grateful to the University of Bergen and the Norwegian University of Science and Technology (NTNU) for funding through my Professor II positions. Victor Calo and Yalchin Efendiev invited me to KAUST, where important parts of the chapters on grids and petrophysics were written. Likewise, Margot Gerritsen invited me to Stanford and gave me the opportunity to develop Jolts videos that complement the first chapters of the book. Significant parts of the chapter on black-oil models were written during the three weeks I participated in the Long Program on Computational Issues in Oil Field Applications at UCLA in 2017. Let me also thank all colleagues and students who have given suggestions, pointed out errors and misprints, asked questions, and given me inspiration to continue working. Even though your name is not mentioned explicitly, I have not forgotten your important contributions. Last, but not least, I thank my wife Anne: the many evenings, late nights, and early mornings it took to write this book would never have been possible without your support and understanding.

Finally to the reader: As you may understand from the book, I strongly believe in reproducible computational science and sharing your work. I hope the book manages to convince you that the combination of a high-level scripting language like MATLAB/GNU Octave and development of reusable open-source software is an excellent approach to both being productive and having a significant impact on others; I elaborate more on this in an essay I wrote to my former supervisor, Helge Holden, for his 60th birthday [189]. Whereas

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the book is static, the MRST software will continue to develop in new directions as a result of continued research and contributions from external users. Your help is invaluable to making this happen; please do not hesitate to contact me if you have suggestions for improvements or new functionality, or if you have developed your independent add-on functionality and want advice on how to best share it with the general community.

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