1

Why a Quantum Tool in Classical Contexts? (Part II)

The reader may wonder where *Part I* is. In fact, there is no *Part I* here. Part I is in [1]. When I wrote that book I felt strongly the responsibility to justify my approach, since it was, in fact, rather unusual, and the reaction of most referees, when submitting a research paper of mine, was quite often the same: "Why are you adopting this technique? What is wrong with a *classical approach*?" However, since 2012, I realized that this approach was not so crazy, and I discovered that many people, in many different fields of research, were adopting similar strategies, using quantum ideas and, in particular, mathematical tools deeply connected with quantum mechanics, to deal with problems that are not necessarily related to the microscopic world. For this reason I do not really feel anymore the necessity of justifying myself. However, I think that giving some words of explanation can still be useful for readers, and this is what the next few sentences are about.

The driving idea behind my approach is that the lowering and raising operators related to the canonical (anti-)commutation relations (CCR or CAR) can be used in the description of processes where some relevant quantities change discontinuously. To cite a few examples, stock markets, migration processes or some biological systems: multiples of share are exchanged, in a market; one, two or more people (and not just *half of a person*) move from one place to another; and one cell duplicates producing two cells. This suggests that objects labeled by natural numbers are important, in some situations. People with a background in quantum mechanics know that nonnegative integers can be seen as eigenvalues of some suitable number operator constructed, in a natural way, using ladder operators. Also, ladder operators can be efficiently used to describe systems where discrete quantities of some kind are exchanged between different agents. Hence ladder operators, and some combinations of them, can be used in the description of particular systems. We refer to [1] for some results and models in this direction. But, due to the fact that the *observables* of a system $\mathcal{S}$ are now operators, one of the main questions to be answered is the following: *How should we assign a time evolution* *to these observables?* The answer which I have adopted so far is, in my opinion, completely natural: I use a Hamiltonian-like operator H, which describes the interactions occurring between the various elements of $\mathcal{S}$ and their *free energy*, and then I adopt the Heisenberg or, equivalently, the Schrödinger equation of motion borrowed from quantum mechanics to deduce the time evolution for $\mathcal{S}$. More specifically, I use the first when I am interested to describe the time evolution of the observables of $\mathcal{S}$, while I adopt the second to find the time evolution of the state of the system. Both choices are useful, and we know that they are equivalent [2, 3], at least under suitable conditions for H.[1] Quite often, in my previous applications, I have used the Heisenberg representation, so that I have dealt with differential equations with unknowns that are time-dependent operators. However, and this has proved to be particularly useful from a technical point of view, I have also used the Schrödinger representation, in particular when some explicit time dependence was to be included in the Hamiltonian of $\mathcal{S}$. This was the case, for instance, in [4], where my operator strategy was adopted in connection of some recently available demographic data for prehistoric South America, and proved to work well in reproducing some aspects of these data, providing some possible scenarios for the agents considered by the model: humans and the natural resources.

Despite of their nature, ladder operators are not necessarily restricted to the analysis of cases in which the observables can only have discrete variations: the same general settings was adopted in other situations, to describe the time evolution of densities in some biological or sociological contexts [5–8], or the *degrees of affection* in love affairs [9, 10], or yet in the description of some decision-making processes [11–16], some of which will be described in this book. We will see how, in many explicit situations, the dynamics of a physical system can be deduced not from a set of ordinary or partial differential equations, but from a single operator living in a non-commuting world. Of course, this approach makes particular sense *if it works*, i.e., if the dynamics deduced in this way describes what is observed in the real world, or, even better, if we can make predictions out of the model. However, I should admit that, in my opinion, a similar approach is also interesting in itself, since it suggests the possibility of using an unusual way to look at a certain system. And it is a way that is quite promising. In fact, a huge literature has been produced, and is still being produced, all having as its focus *quantum ideas outside quantum mechanics*. Just to cite few recent monographs, we refer to the following contributions by several authors: [17–22], in which these quantum ideas are applied in quite different contexts.

The reader should be aware of the fact that, within the book, more so than mathematical rigor, we will be interested in the possibility of getting concrete (exact, or at

[1] In particular, this is true if $H = H^\dagger$, but not when H and its adjoint $H^\dagger$ are different, which is the case in some of the applications discussed in these notes.

least approximated) results. However, we will try to stress the various approximations done all along the book, and to state explicitly whether these approximations and assumptions are *under control*, or if they are only useful to take some steps toward the solution of the problem. In other words, we will do our best to clarify to what extent, within the context of the model we are considering, our results can be trusted or should be refined further.

1.1 Organization of the Book

The book is divided into two main sections. In Part I, I will discuss useful facts from quantum dynamics that will be used later. Some of these facts are well-known, and can be also found in many textbooks, but some others are not, and they are needed for a better comprehension of some of the applications considered in Part II of the book. In particular, in Section 2.8 I will introduce and describe a sort of algebraic dynamics that is not only driven by a given Hamiltonian H, but that is also linked to other features of the system not (easily) included in any such H.

Looking at the table of contents the reader can see that the applications considered in Part II are really different. I will discuss applications to politics, to biology, to economics, to social sciences and to decision-making. This should not be surprising, since all the models proposed in all these areas will be considered as different *dynamical systems*. And we all know that dynamical systems cover quite different areas in research and in the real world. I will also apply quantum ideas to two different (non-dynamical) problems, typical in decision-making: the description of compatible and incompatible questions and the analysis of order effects. In this case, the natural non-commutativity of the observables in quantum mechanics will be the relevant tool adopted to describe these effects.

Going more into detail, in Chapter 3 I will describe a problem of alliances in politics, as a sort of decision-making procedure driven by the interactions of the political parties with a base of electors. In Chapter 4, an application to ecology will be discussed: desertification in a two-dimensional region, and a possible way to control it. A dynamical approach to escape strategies is the content of Chapter 5. In Chapter 6, I will describe two closed ecological systems, with one or two different garbages, and with several *kinds of organism*, discussing, in particular, the existence of some equilibrium, corresponding to a long-time survival of the system. Another biologically oriented application is the topic of Chapter 7, where a simple model of tumor cell proliferation is proposed. Chapter 8 contains a possible extension of the Game of Life, where quantum rules and quantum evolution are adopted. A somewhat *unexpected* application of quantum techniques to prehistoric data miming is given in Chapter 9, while in Chapter 10, I discuss a way to introduce the information in a simple model of stock market. Information is also the core

Cambridge University Press
978-1-108-49212-6 — Quantum Concepts in the Social, Ecological and Biological Sciences
Fabio Bagarello
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of my interest in Chapter 11, in connection with decision-making. In Chapter 12, a different application of quantum tools to a problem of order and compatibility of questions is discussed, again in the context of decision-making. My final remarks are given in Chapter 13. The variety of topics discussed in Part II is evident. To me, this is a nice indication that the framework I am going to describe here is really promising. Also, it is not hard to imagine that more, and more refined, applications can be considered. So, there is still a lot of work to do.

But now: let's start!