

## RELATIVISTIC FLUID DYNAMICS IN AND OUT OF EQUILIBRIUM

The past decade has seen unprecedented developments in the understanding of relativistic fluid dynamics in and out of equilibrium, with connections to astrophysics, cosmology, string theory, quantum information, nuclear physics, and condensed matter physics. Romatschke and Romatschke offer a powerful new framework for fluid dynamics, exploring its connections to kinetic theory, gauge/gravity duality, and thermal quantum field theory. Numerical algorithms to solve the equations of motion of relativistic dissipative fluid dynamics as well as applications to various systems are discussed. In particular, the book contains a comprehensive review of the theory background necessary to apply fluid dynamics to simulate relativistic nuclear collisions, including comparisons of fluid simulation results to experimental data for relativistic lead–lead, proton–lead, and proton–proton collisions at the LHC. The book is an excellent resource for students and researchers working in nuclear physics, astrophysics, cosmology, quantum many-body systems, and string theory.

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# Relativistic Fluid Dynamics In and Out of Equilibrium

And Applications to Relativistic Nuclear Collisions

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## Preface

In 2007, relativistic viscous fluid dynamics was formulated from first principles in an effective field theory framework, based entirely on the knowledge of symmetries and long-lived degrees of freedom. In the same year, numerical simulations for the matter created in relativistic heavy-ion collision experiments became first available, providing constraints on the shear viscosity in quantum chromodynamics (QCD). The field has come a long way since then, yet no textbook covering these modern developments exists to date. Thus the aim of this book is to present the current status of the theory of nonequilibrium fluid dynamics, including the divergence of the fluid dynamic gradient expansion, resurgence, nonequilibrium attractor solutions, the inclusion of thermal fluctuations as well as their relation to microscopic theories.

A key driver for the recent theory developments in relativistic fluid dynamics has been the experimental nuclear collision program both at CERN and Brookhaven National Laboratory. For this reason, this text also includes a theory review for applying numerical fluid dynamics to relativistic nuclear collisions and comparisons of modern simulations to experimental data for nucleus–nucleus, nucleus–proton, and proton–proton collisions. It should be emphasized that we focus on numerical algorithms for nonequilibrium (viscous) fluids rather than reviewing the extensive literature on numerical solvers for purely ideal (perfect) relativistic fluid dynamics. For ideal relativistic fluid dynamic solvers, as well as for most of their applications to astrophysics and cosmology, the interested reader is advised to consult a dedicated textbook on this topic.

In writing this text, we have benefited from the combined knowledge of many more people than could be acknowledged here. Of these, we are particularly grateful to Sašo Grozdanov, Jamie Nagle, Björn Schenke, Michal Spaliński, Prithwish Tribedy, Wilke van der Schee, and Bill Zajc for their help in preparing, and comments on, an early version of this text.

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