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Part I

Special Relativity

Minkowski Space-Time

In this first part, after a review of inertial and non-inertial frames in the non-relativistic Galilei space-time, I will study such frames in the Minkowski space-time of special relativity (SR).

In Newtonian physics, time and space are absolute notions whose metrological units are defined by means of standard clocks and rods, whose structure is not specified. This is satisfactory for the non-relativistic quantum mechanics used in molecular physics and in quantum information, where gravitation effects are described by Newtonian gravity.

However, in atomic physics one needs the description of light, whose quantum nature gives rise to the notion of massless photons whose trajectories do not exist in Galilei space-time. Moreover particle physics has to face high-speed objects. As a consequence, the Minkowski space-time of SR has to be introduced and a new type of metrology has been developed with different standards for length and time [31]. See references [32]–[38] for updated reviews on relativistic metrology on Earth, in the Solar System, and in astronomy.

The fundamental theoretical scale for time is the SI (International System of Units) atomic second, which is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium 133 atom at rest at a temperature of 0K. In practice one uses the International Atomic Time (TAI), defined as a suitable weighted average of the SI kept by (mainly cesium) atomic clocks in about 70 laboratories worldwide.

To introduce a convention for the synchronization of distant clocks one uses the notion of two-way (or round-trip) velocity of light c involving only one clock: The observer emits a ray of light that is reflected somewhere and then reabsorbed by the observer, so that only the clock of the observer is used to measure the time of flight of the ray. It is this velocity that is isotropic and constant in SR (the light postulate) and replaces the standard of length in relativistic metrology. The one-way velocity of light from one observer A to an observer B has a meaning only after a choice of a convention for synchronizing the clock in A with the one in B.

One uses the conventional value $c = 299\,792\,458$ m s^{-1} for the two-way velocity of light. To measure the 3-distance between two objects in an inertial frame, one puts an atomic clock in the first object, then sends a ray of light to the second object, where it is reflected and then reabsorbed by the first object, whose measure of the flight time allows finding the 3-distance. As a consequence, the meter is the length of the path traveled by light in a vacuum in an inertial frame during a time interval of $1/c$ of a second.

1

Galilei and Minkowski Space-Times

In this chapter we review the properties the non-relativistic Galilei space-time and of the relativistic Minkowski one. See Refs. [39–43] for a detailed study of the rotation group and of the kinematical Galilei and Poincaré groups connecting the inertial frames of the respective space-times.

1.1 The Galilei Space-Time of Non-Relativistic Physics and Its Inertial and Non-Inertial Frames

In Newtonian physics the notions of time and space are absolute, so that the chrono-geometrical structure of Galilei space-time is not dynamical. One has at each instant of the absolute time t, registered by an ideal clock, an instantaneous Euclidean 3-space R^3_t with the standard notion of Euclidean distance, measured with ideal rods. The clocks in each point of R^3_t are synchronized at the time t, so that Galilei space-time has the structure $R \times R^3$, where R denotes the time axis and R^3 is an abstract Euclidean 3-space associated to the fixed stars of astronomy. As a consequence, one can parametrize Galilei space-time as the straight trajectory of an inertial observer (the time axis) endowed with a foliation of Euclidean 3-spaces orthogonal to the time axis.

The Galilei relativity principle assumes the existence of preferred rigid inertial frames of reference in uniform translational motion, one with respect to the other with inertial Cartesian coordinates (t, x^i) centered on an inertial observer, whose trajectory is the time axis. In these frames, free bodies move along straight lines (Newton's first law) and Newton's equations take the simplest form. The laws of nature are covariant and there is no preferred observer. The connection between different inertial frames is realized with the kinematical Galilei transformations: $t^{'} = t + \epsilon$, $x^{'i} = R^{ij}\, x^j + v^i\, t + \epsilon^i$, where ϵ and ϵ^i are the time and space rigid translations, R ($R^{-1} = R^T$, the transposed matrix) is the O(3) matrix describing rigid rotations, and v^i are the parameters of the rigid Galilei boosts.

Due to the absolute nature of Newtonian time, the points on a $t =$ const. section of Galilei space-time are all simultaneous (instantaneous absolute 3-space), whichever inertial system we are using. As a consequence, the causal notions of before and after a certain event are absolute.

A particle of mass m has the trajectory described by inertial Cartesian 3-coordinates $x^i_m(t)$ in Galilei space-time. In the Hamiltonian phase space it has the momentum $p_i = m\,\delta_{ij}\,\frac{d\,x^i(t)}{dt}$. For a free particle the Galilei generators are $H = \vec{p}^2/2\,m$ (energy), p_i (momentum), $K_i = m\,\delta_{ij}\,x^j_m - t\,p_i$ (boost), $J_i = \epsilon_{ijk}\,\delta_{jh}\,x^h_m\,p_k$ (rotation). ϵ^{ijk} is the completely antisymmetric tensor.

For a system of mutually interacting N particles of mass m_k, trajectory $x^i_k(t)$, momenta $p_{k\,i}(t) = m_k\,\delta_{ij}\,\frac{d\,x^j_k(t)}{dt}$, $k = 1, \dots, N$, the Galilei generators are $H = \sum_{k=1}^N \frac{\vec{p}_k^2}{2\,m_k} + V(\vec{x}_h(t) - \vec{x}_k(t))$, $\vec{P} = \sum_{k=1}^N \vec{p}_k$, $\vec{J} = \sum_{k=1}^N \vec{x}_k(t) \times \vec{p}_k(t)$, $\vec{K} = \sum_{k=1}^N (t\,\vec{p}_k(t) - m_k\,\vec{x}_k(t)) = t\,\vec{P} - m\,\vec{x}$. Here, $\vec{x} = \sum_{k=1}^N \frac{m_k}{m}\,\vec{x}_k(t)$ $(m = \sum_{k=1}^n m_k)$ is the Newton center of mass, whose conjugate variable is $\vec{P}$. Therefore, the conserved Galilei boosts identify the Newtonian center of mass.

Usually the interacting potential depends only on the relative distances of the particles (and not on their velocities) and appears only in the energy (the Hamiltonian) and not in the boosts differently from what happens at the relativistic level with the Poincaré group.

For isolated N-body systems the ten generators of the Galilei group are Noether constants of motion. The Abelian nature of the Noether constants (the 3-momentum) associated to the invariance under translations allow making a global separation of the center of mass from the relative variables (usually the Jacobi coordinates, identified by the centers of mass of subsystems, are preferred): In phase space this can be done with canonical transformations of the point type both in the coordinates and in the momenta.

Instead, the non-Abelian nature of the Noether constants (the angular momentum) associated with the invariance under rotations implies that there is no unique separation [44] of the relative variables in six orientational ones (the body frame in the case of rigid bodies) and in the remaining vibrational (or shape) ones. As a consequence, an isolated deformable body or a system of particles may rotate by changing the shape (the falling cat, the diver).

In Refs. [45, 46] there is a kinematical treatment of non-relativistic N-body systems by means of canonical spin bases and of dynamical body frames, which can be extended to the relativistic case in which the notions of Jacobi coordinates, reduced masses, and tensors of inertia are absent and can be recovered only when extended bodies are simulated with multipolar expansions [47].

Another non-conventional aspect of non-relativistic physics is the many-time formulation of classical particle dynamics [48] with as many first-class constraints as particles. Like in the special relativistic case, a distinction arises between physical positions and canonical configuration variables and a non-relativistic version of the no-interaction theorem (see Chapter 3) emerges.

See Refs. [49, 50] for Newtonian gravity, where the Newton equivalence principle states the equality of inertial and gravitational mass, as a gauge theory of the Galilei group.

To define rigid non-inertial frames, let us consider an arbitrary accelerated observer whose Cartesian trajectory is $y^i(t)$ and let us introduce the rigid non-inertial coordinates (t, σ^i) by imposing $x^i = y^i(t) + R^{ij}(t)\,\sigma^j$, where $R(t)$ is a time-dependent rotation matrix, which can be parametrized with three Euler angles. It describes the rigid rotation of the non-inertial frame. It is convenient to write the 3-velocity of the accelerated observer in the form $v^i(t) = R^{ij}(t)\,\frac{d\,y^j(t)}{dt}$. The angular velocity of the rotating frame is $\Omega^i(t) = \frac{1}{2}\,\epsilon^{ijk}\,\Omega^{jk}$ with $\Omega^{jk}(t) = -\Omega^{kj} = (\frac{d\,R(t)}{dt}\,R^T(t))^{jk}$.

A particle of mass m with trajectory given by the Cartesian 3-coordinates $x^i_m(t)$ is described in the rigid non-inertial frames by 3-coordinates $\eta^r(t)$ such that $x^i_m(t) = y^i(t) + R^{ij}(t)\,\eta^j(t)$.

As shown in every book on Newtonian mechanics, a particle moving in an external potential $V(t, x^k_m(t)) = \tilde{V}(t, \eta^r(t))$ has the equation of motion $m\,\frac{d^2\,x^i_m(t)}{dt^2} = -\frac{\partial\,V(t, x^k_m(t))}{\partial\,x^i_m}$, whose form in the rigid non-inertial frames becomes

$$m\,\frac{d^2\,\vec{\eta}(t)}{dt^2} = -\frac{\partial\,\tilde{V}(t, \eta^k(t))}{\partial\,\vec{\eta}} - m\,\Big[\frac{d\vec{v}(t)}{dt} + \vec{\omega}(t)\times\vec{v}(t) + \frac{d\,\vec{\omega}(t)}{dt}\times\vec{\eta}(t)$$
$$+2\,\vec{\omega}(t)\times\frac{d\,\vec{\eta}(t)}{dt} + \vec{\omega}(t)\times[\vec{\omega}(t)\times\vec{\eta}(t)]\Big]. \tag{1.1}$$

In this equation there are the standard Euler, Jacobi, Coriolis, and centrifugal inertial (or fictitious) forces, proportional to the mass of the body, associated with the acceleration of the non-inertial observer and with the angular velocity of the rotating rigid non-inertial frame.

The extension to non-rigid non-inertial frames with coordinates (t, σ^i) (σ^r are global non-Cartesian 3-coordinates) is done in Ref. [51] by putting the Cartesian 3-coordinates x^i equal to arbitrary functions $\mathcal{A}^i(t, \sigma^r)$, well behaved at spatial infinity: $x^i = \mathcal{A}^i(t, \sigma^r)$. This coordinate transformation must be invertible with inverse $\sigma^r = S^r(t, x^i)$. The invertibility conditions are $\det J(t, \sigma^r) > 0$, where $J^a_r(t, \sigma^u) = \frac{\partial\,\mathcal{A}^a(t, \sigma^u)}{\partial\,\sigma^r}$ is the three-dimensional Jacobian, whose inverse is $\tilde{J}^r{}_a(t, \sigma^u) = \Big(\frac{\partial\,S^r(t, x^u)}{\partial\,x^a}\Big)_{x^b = \mathcal{A}^b(t, \sigma^u)}$ $(J^a{}_r(t, \sigma^u)\,\tilde{J}^r{}_b(t, \sigma^u) = \delta^a_b$, $\tilde{J}^s{}_a(t, \sigma^u)\,J^a{}_r(t, \sigma^u) = \delta^s_r)$. The group of Galilei transformations connecting inertial frames is replaced by some subgroup of the 3-diffeomorphisms of the Euclidean 3-space connecting the non-inertial ones. The quantum mechanics of particles in non-rigid non-inertial frames is studied in Ref. [51].

1.2 The Minkowski Space-Time: Inertial Frames, Cartesian Coordinates, Matter, Energy–Momentum Tensor, and Poincaré Generators

The Minkowski space-time of special relativity (SR) is an affine 4-manifold isomorphic to R^4 with Lorentz signature in which neither time nor space are absolute notions. As a consequence there is no unique notion of instantaneous 3-space and one needs some metrological convention about time and space to be

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able to formulate particle physics in the laboratories on Earth in the approximation of neglecting gravity. The only intrinsic structure of Minkowski space-time is the conformal one connected with the Lorentz signature: It defines the light-cone as the locus of incoming and outgoing radiation.

There is no absolute notion of simultaneity: Given an event, all the points outside the light-cone with vertices in that event are not causally connected with that event (they have space-like separation from it), so that the notions of before and after an event become observer-dependent. Therefore there is no notion of an instantaneous 3-space, of a spatial distance, and of a one-way velocity of light between two observers (the problem of the synchronization of distant clocks). Instead, as already said, there is an absolute chrono-geometrical structure: the light postulate saying that the two-way (or round-trip) velocity of light c (only one clock is needed for its definition) is (1) constant and (2) isotropic. Let us remark that the clocks are assumed to be standard atomic clocks measuring proper time [52–54].

Einstein relativity principle privileges the inertial frames of Minkowski space-time centered on inertial observers endowed with an atomic clock: Their trajectories are the time axis in Cartesian coordinates $x^\mu = (x^o = c\,t; x^i)$ where the flat metric tensor with Lorentz signature is ${}^4\eta_{\mu\nu} = \epsilon\,(1; -1, -1, -1)$. These inertial frames are in uniform translational motion, one with respect to the other. All special relativistic physical systems, defined in the inertial frames of Minkowski space-time, are assumed to be manifestly covariant under the transformations of the kinematical Poincaré group connecting the inertial frames. The laws of physics are covariant and there is no preferred observer.

The $x^o = \text{const.}$ hyper-planes of inertial frames are usually taken as Euclidean instantaneous 3-spaces, on which all the clocks are synchronized. They can be selected with Einstein's convention for the synchronization of distant clocks to the clock of an inertial observer. This inertial observer A sends a ray of light at x^o_i to a second accelerated observer B, who reflects it toward A. The reflected ray is reabsorbed by the inertial observer at x^o_f. The convention states that the clock of B at the reflection point must be synchronized with the clock of A when it signs $\frac{1}{2}\,(x^o_i + x^o_f)$. This convention selects the $x^o = \text{const.}$ hyper-planes of inertial frames as simultaneity 3-spaces and implies that only with this synchronization the two-way (A–B–A) and one-way (A–B or B–A) velocities of light coincide and the spatial distance between two simultaneous point is the (3-geodesic) Euclidean distance. However, if observer A is accelerated, the convention can break down due to the possible appearance of coordinate singularities.

Relativistic matter is defined in the relativistic inertial frames of Minkowski space-time centered on inertial observers using Cartesian 4-coordinates. It is in these frames that one defines the matter Lagrangian when it is known. Once one has the Lagrangian $\mathcal{L}(x)$ of a matter system the energy–momentum tensor is defined by replacing the flat 4-metric ${}^4\eta_{\mu\nu}$ appearing in the Lagrangian with a 4-metric ${}^4g_{\mu\nu}(x)$ like the one used in general relativity (GR), so that one gets a new Lagrangian $\mathcal{L}_g(x)$ and by using the definition $T^{\mu\nu}(x) = -\frac{2}{\sqrt{-det^4 g(x)}}\,\frac{\delta\, S_g}{\delta^4 g_{\mu\nu}(x)}$, where $S_g = \int d^4x\, \mathcal{L}_g(x)$. In inertial frames with Cartesian 4-coordinates x^μ, the Poincaré generators, assumed finite due to suitable boundary conditions at spatial infinity, have the following expression: $P^\mu = \int d^3x\, T^{\mu o}(x^o, \vec{x})$, $J^{\mu\nu} = \int d^3x\, [x^\mu\, T^{\nu o}(x^o, \vec{x}) - x^\nu\, T^{\mu o}(x^o, \vec{x})]$.

In Appendix A there are some properties of the Poincaré algebra and group. At the Hamiltonian level the canonical Poincaré generators P^μ, $J^{\mu\nu}$ satisfy the Poisson algebra $\{P^\mu, P^\nu\} = 0$, $\{P^\mu, J^{\alpha\beta}\} = \eta^{\mu\alpha}\, P^\beta - \eta^{\mu\beta}\, P^\alpha$, $\{J^{\alpha\beta}, J^{\mu\nu}\} = C^{\alpha\beta\mu\nu}_{\rho\sigma}\, J^{\rho\sigma}$ ($C^{\alpha\beta\mu\nu}_{\rho\sigma} = \eta^{\alpha\mu}\, \delta^\beta_\rho\, \delta^\nu_\sigma + \eta^{\beta\nu}\, \delta^\alpha_\rho\, \delta^\mu_\sigma - \eta^{\alpha\nu}\, \delta^\beta_\rho\, \delta^\mu_\sigma - \eta^{\beta\mu}\, \delta^\alpha_\rho\, \delta^\nu_\sigma$). If $J^r = -\frac{1}{2}\, \epsilon^{ruv}\, J^{uv}$ is the generator of space rotations and $K^r = J^{ro}$ one of the boosts, the form of the canonical Poincaré algebra becomes $\{J^r, J^s\} = \epsilon^{rst}\, J^t$, $\{K^r, K^s\} = \epsilon^{rsu}\, J^u$, $\{J^r, K^s\} = \{K^r, J^s\} = \epsilon^{rsu}\, K^u$.

To describe point particles with spin, with electric charge and with antiparticles of negative mass in a way that avoids self-reaction divergences at the classical level and gives the correct quantum theory after quantization, one needs a semiclassical approach, named pseudo-classical mechanics, in which these degrees of freedom are described with Grassmann variables. In Appendix B there is a review of this approach and of the needed mathematical tools.

For the detailed mathematical properties of Minkowski space-time, see any book on SR, such as the recent ones of Gourgoulhon Ref. [12, 13].

1.3 The 1+3 Approach to Local Non-Inertial Frames and Its Limitations

Since the actual time-like observers are accelerated, we need some statement correlating the measurements made by them to those made by inertial observers, the only ones with a general framework for the interpretation of their experiments based on Einstein convention for the synchronization of clocks. This statement is usually the hypothesis of locality, which can be expressed in the following terms [55–60]: An accelerated observer at each instant along its world-line is physically equivalent to an otherwise identical momentarily comoving inertial observer, namely a non-inertial observer passes through a continuous infinity of hypothetical momentarily comoving inertial observers.

While this hypothesis is verified in Newtonian mechanics and in those relativistic cases in which a phenomenon can be reduced to point-like coincidences of classical point particles and light rays (geometrical optic approximation), its validity is questionable with moving continuous media (for instance the constitutive equations of the electromagnetic field inside them in non-inertial frames are still unknown) and in the presence of electromagnetic fields when their wavelength is comparable with the acceleration radii of the observer (the observer is not "static" enough to be able to measure the frequency of such a wave). See Refs. [61, 62] for a review of these topics.

The fact that we can describe phenomena only locally near the observer and that the actual observers are accelerated leads to the 1+3 point of view (or threading splitting) [63–70], which tries to solve this problem starting from the local properties of an accelerated observer, whose world-line is assumed to be the time axis of some frame. Given the world-line γ of the accelerated observer, we describe it with Lorentzian coordinates $x^\mu(\tau)$, parametrized with an affine parameter τ, with respect to a given inertial system. Its unit 4-velocity is $u^\mu_\gamma(\tau) = \dot{x}^\mu(\tau)/\sqrt{\epsilon\, \dot{x}^2(\tau)}$ $[\dot{x}^\mu = \frac{dx^\mu}{d\tau}]$. The observer proper time $\tau_\gamma(\tau)$ is defined by $\epsilon\, \dot{\tilde{x}}^2(\tau_\gamma) = 1$ if we use the notations $x^\mu(\tau) = \tilde{x}^\mu(\tau_\gamma(\tau))$ and $u^\mu(\tau) = \tilde{u}^\mu(\tau_\gamma(\tau)) = d\,\tilde{x}^\mu(\tau_\gamma)/d\,\tau_\gamma$, and it is indicated by a comoving standard atomic clock.

By a conventional choice of three spatial axes $E^\mu_{(a)}(\tau) = \tilde{E}^\mu_{(a)}(\tau_\gamma(\tau))$, $a = 1, 2, 3$, orthogonal to $u^\mu(\tau) = E^\mu_{(o)}(\tau) = \tilde{u}^\mu(\tau_\gamma(\tau)) = \tilde{E}^\mu_{(o)}(\tau_\gamma(\tau))$, the non-inertial observer is endowed with an ortho-normal tetrad $E^\mu_{(\alpha)}(\tau) = \tilde{E}^\mu_{(\alpha)}(\tau_\gamma(\tau))$, $\alpha = 0, 1, 2, 3$. This amounts to a choice of three comoving gyroscopes in addition to the comoving standard atomic clock. Usually the spatial axes are chosen to be Fermi–Walker transported as a standard of non-rotation, which takes into account the Thomas precession (see [71]).

Since only the observer 4-velocity is given, this only allows identification of the tangent plane of the vectors orthogonal to this 4-velocity in each point of the world-line. Since there is no notion of a 3-space simultaneous with a point of γ and whose tangent space at that point is $R_{\tilde{u}(\tau_\gamma)}$, this tangent plane is identified with an instantaneous 3-space both in SR and GR (it is the local observer rest-frame at that point). This identification is the basic limitation of this approach because the hyper-planes at different times intersect each other at a distance from the world-line depending on the acceleration of the observer so that the approach works only in a world-tube whose radius is this distance. See Refs. [63–70] for the definition of the (linear and rotational) acceleration radii of the observer. At each point of γ with proper time $\tau_\gamma(\tau)$, the tangent space to Minkowski space-time in that point has the 1+3 splitting of vectors in vectors parallel to $\tilde{u}^\mu(\tau_\gamma)$ and vectors lying in the three-dimensional (so-called local observer rest-frame) subspace $R_{\tilde{u}(\tau_\gamma)}$ orthogonal to $\tilde{u}^\mu(\tau_\gamma)$.

Then Fermi normal coordinates [72–76] are defined on each hyper-plane orthogonal to the observer unit 4-velocity $u^\mu(\tau)$ and are used to define a notion of spatial distance. On each hyper-plane one considers three space-like geodesics as spatial coordinate lines. However, this produces only local coordinates and a notion of simultaneity valid only inside the world-tube. See Refs. [77–79] for variants of this approach, all unable to avoid the coordinate singularity on the world-tube.

To this type of coordinate singularities we have to add the singularities shown by all the rotating coordinate systems (the problem of the rotating disk): In all the proposed uniformly rotating coordinate systems the induced 4-metric expressed in these coordinates has pathologies (the component ${}^4g_{oo}$ vanishes) at the distance R from the rotation axis where $\omega\, R = c$ with ω being the constant angular velocity of rotation. This is the horizon problem: At R the time-like 4-velocity of a disk point becomes light-like, even if there is no real horizon as happens for Schwarzschild black holes. Again, given the unit 4-velocity

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field of the points of the rotating disk, there is no notion of an instantaneous 3-space orthogonal to the associated congruence of time-like observers, due to the non-zero vorticity of the congruence [71] (see Section 2.1 for the definition of vorticity). Due to the Frobenius theorem, the congruence is (locally) hyper-surface orthogonal, i.e., locally synchronizable [71], if and only if the vorticity vanishes. Moreover, an attempt to use Einstein convention to synchronize the clocks on the rim of the disk fails and one finds a synchronization gap (see Refs. [80–84] and the bibliographies of Refs. [61, 62] for these problems).

One does not know how to define the 3-geometry of the rotating disk, how to measure the length of the circumference, and which time and notion of simultaneity has to be used to evaluate the velocity of (massive or massless) particles in uniform motion along the circumference.

The other important phenomenon connected with the rotating disk is the Sagnac effect (see again Refs. [61, 62, 80–84]), namely the phase difference generated by the difference in the time needed for a round-trip by two light rays, emitted at the same point, one co-rotating and the other counter-rotating with the disk. This effect, which has been tested for light, X-rays, and matter waves (Cooper pairs, neutrons, electrons, and atoms) and must be taken into account for the relativistic corrections to space navigation, has again an enormous number of theoretical interpretations (both in SR and GR). Here the lack of a good notion of simultaneity leads to problems of time discontinuities or desynchronization effects when comparing clocks on the rim of the rotating disk.

In conclusion, in SR inertial frames are a limiting theoretical notion since, also disregarding GR, all the observers on Earth are non-inertial. According to the IAU 2000 Resolutions [32–35], for the physics in the solar system one can consider the Solar System Barycentric Celestial Reference System (BCRS) centered on the barycenter of the Solar System (with the axes identified by fixed stars (quasars) of the Hypparcos catalog) as a quasi-inertial frame. Instead, the Geocentric Celestial Reference System (GCRS), with origin in the center of the geoid, is a non-inertial frame whose axes are non-rotating with respect to the Solar Frame. Instead, every frame centered on an observer fixed on the surface of Earth (using the yet non-relativistic International Terrestrial Reference System [ITRS]) is both non-inertial and rotating. All these frames use notions of time connected to TAI.

Let us also remark that the physical protocols (think of GPS) can establish a clock synchronization convention only inside future light-cone of the physical observer defining the local 3-spaces only inside it, in accord with the 1+3 point of view.

This state of affairs and the need for predictability (a well-posed Cauchy problem for field theory) lead to the necessity of abandoning the 1+3 point of view and shifting to the 3+1 one. In this point of view, besides the world-line of an arbitrary time-like observer, it is given a global 3+1 splitting of Minkowski space-time, namely a foliation of it whose leaves are space-like hyper-surfaces.

Each leaf is both a Cauchy surface for the description of physical systems and an instantaneous Riemannian 3-space, namely a notion of simultaneity implied by a clock synchronization convention different from Einstein's one. Even if it is unphysical (i.e., non-factual) to give initial data (the Cauchy problem) on a non-compact space-like hyper-surface, this is the only way to be able to use the existing existence and uniqueness theorems for the solutions of partial differential equations like the Maxwell ones, needed to test the predictions of the theory.[1] Once we have given the Cauchy data on the initial Cauchy surface (an unphysical process), we can predict the future with every observer receiving the information only from his/her past light-cone (retarded information from inside it; electromagnetic signals on it). As emphasized by Havas [86], the 3+1 approach is based on Møller's formalization [87, 88] of the notion of simultaneity.

For non-relativistic observers the situation is simpler, but the non-factual need to give the Cauchy data on a whole initial absolute Euclidean 3-space is present also in this case for non-relativistic field equations like the Euler equation for fluids.

Moreover, to study relativistic Hamiltonian dynamics one has to follow its formulation given by Dirac [89] with the instant, front (or light), and point forms and the associated canonical realizations of the Poincaré algebra. In the instant form, the simultaneity hyper-surfaces (Cauchy surfaces) defining a parameter for the time evolution are space-like hyper-planes $x^o = \text{const.}$, in the front form hyper-planes $x^- = \frac{1}{2}(x^o - x^3) = \text{const.}$ tangent to future light-cones, while in the point form the future branch of a two-sheeted hyperboloid $x^2 > 0$. In a $6N$-dimensional phase space for N scalar particles the ten generators of the Poincaré algebra are classified into kinematical generators (the generators of the stability group of the simultaneity hyper-surface) and dynamical generators (the only ones to be modified with respect to the free case in the presence of interactions) according to the chosen concept of simultaneity. While in the instant and point forms there are four dynamical generators (in the former energy and boosts, in the latter the 4-momentum), the front form has only three of them.

We will see that the 3+1 approach is the natural framework to implement the instant form of relativistic Hamiltonian dynamics.

Let us add the final remark that both in the 1+3 and in the 3+1 approach we call observer an idealization by means of a time-like world-line whose tangent vector in each point is the 4-velocity of the observer. If the 4-velocity is completed with a spatial triad to form a tetrad in each point of the world-line, we get an idealized observer with both a clock and a gyroscope. While this notion is compatible with the absolute metrology of SR, in GR it corresponds to a test

[1] As far as we know the theorem on the existence and unicity of solutions has not yet been extended starting from data given only on the past light-cone. See Ref. [85] for an attempt to rephrase the instant form of dynamics in a form employing only data from the causal past light-cone of the observer.

observer. To describe dynamical observers we need a model with dynamical matter in both cases. Therefore, an observer, or better a mathematical observer, is an idealization of a measuring apparatus containing an atomic clock and defining, by means of gyroscopes, a set of spatial axes (and then a, maybe orthonormal, tetrad with a convention for its transport) in each point of the world-line. See Ref. [90] for properties of mathematical and dynamical observers.