Cambridge University Press 978-1-108-47701-7 — Classical Field Theory Hora□iu Năstase Table of Contents <u>More Information</u>

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